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CONNECTED TWO- OUT DEGREE EQUITABLE DOMINATION NUMBER IN DIFFERENT GRAPHS

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ABSTRACT

Let G=(V,E) be a graph. A dominating set D in V of a graph G is called two out degree equitable dominating set if for any two vertices u, v ∈ D, $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of a two-out degree equitable dominating set is called two-out degree equitable domination number, and is denoted by $\gamma_{2oe}(G)$. An two-out degree equitable dominating set D is said to be connected two-out degree equitable dominating set if the sub graph <D> induced by D is connected. The minimum cardinality of connected two –out degree dominating set is called connected two out degree equitable domination number and is denoted by $\gamma_{c2oe}(G)$. In this paper we introduce the connected two-out degree equitable domination number and connected two-out degree equitable domatic in graphs and its exact values for some standard graphs.

***Key words**: Two-out degree equitable domination number, connected two-out degree equitable domination set, connected two-out degree equitable domination number. Connected two out degree equitable domatic Number.*

1. INTRODUCTION

By a graph G= (V, E), We mean a finite, undirected with neither loops nor multiple edges. The order and size of G are denoted by m and n respectively. For graph theoretic terminology we refer to Harray[5]. A subset D of V is called a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The minimum cardinality of a dominating set of G is called domination number of G and it is denoted by$\gamma(G)$. An Excellent treatment of the fundamentals of dominion is given in the book by Haynes *et al* [8]. Various types of domination have defined and studied by several authors and more than 75 models of domination are listed in the appendix of Haynes et al. [8]. Sampath Kumar and Waliker [4] introduced the concept of connected domination in graph. Let G= (V, E) be a graph and let v ∈ V the open neighborhood and the closed neighborhood of v are denoted by N(v) and N[v]=N(v) U v respectively. If D⊆V then N(D)=$U_{v \in D} N(v)$ and N[D]= N(D) U D. If D⊆V and u ∈ D the private neighbor set of u with respect to D is defined by P_n[u,D]={v: N[v]∩ D={u}}.

A dominating set D of G is called a connected dominating set if the induced sub graph <D> is connected. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by γ_c(G).

A subset D of V is called an equitable dominating set if for every v ∈ V-D there exist a vertex u∈ D such that uv∈ E(G) and |deg(u)-deg(v)|≤ 1. The minimum cardinality of such an equitable dominating set is denoted by γ_e(G) and is called the equitable domination number of G. A vertex u is said to be degree equitable with a vertex v∈V if |deg(u)-deg(v)|≤ 1. An equitable set D is said to be a minimal dominating set if no proper subset of D is an equitable dominating set. . Let G= (V, E) be a graph and let u∈ V the equitable neighbood of u dentoed by $N_e(u)$ is defined as $N_e(u)$={v∈ V: |v∈ N(u), |deg(u)-deg(v)|≤ 1}. Let v ∈ V the open equitable neighborhood and the closed equitable neighborhood of v are denoted by $N_e(u)$ and $N_e[u]$ =$N_e(u)$ U v respectively. If D⊆V then $N_e(D)$ =$U_{v \in D} N_e(u)$ and $N_e[D]$] = $N_e(D)$ U D. A double star is the tree obtained from two disjoint stars $K_{1,n}$ and $K_{1,m}$ by connecting their centers.

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If G is connected graph, then a vertex cut of G is a subset R of V with the property that the sub graph of G induced by V-R is disconnected. If G is not complete graph, the vertex connectivity number k (G) is the minimum cardinality of a vertex cut. It is a convention that if G is complete graph k_m it is know that k(G)=m-1,

Definition: Let G=(V,E) be a graph, Let D⊆ V and v be any vertex is D. The out degree of v with respect to D denoted by $od_D(v)$, is defined as $od_D(v) = |N(\text{v}) \cap (V - D)|$.

Definition: Let D be a dominating set of a graph G= (V, E). For v ∈ D, let $od_D(v) = |N(\text{v}) \cap (V - D)|$. Then D is called an equitable dominating set of type 1 if $|od_D(u) - od_D(v)| \leq 1$ for all u, v ∈ D. The minimum cardinality of such a dominating set is denoted by γ_{eq1}(G) and is called the 1- equitable domination number of G[1].

Definition: A dominating set D in a graph G is called a two-out degree equitable dominating set if for any two vertices u,v ∈ D, $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of a two-out degree equitable dominating set is called the two-out degree equitable domination number of G, and is denoted by γ_{2oe}(G)[2]. A sub set D of V is a minimal two-out degree equitable dominating set if no proper subset of D is a two –out degree equitable dominating set.

2. CONNECTED TWO- OUT DEGREE EQUITABLE DOMINATION IN GRAPHS

Definition: Let G= (V, E) be a connected graph. A two-out degree equitable dominating set D of a graph G is called the connected two-out degree equitable dominating set if the induced sub graph <D> is connected. The minimum cardinality of a connected two-out degree equitable domination number of G and is denoted by γ_{c2eq}(G)

We supposed that G is connected because if the graph has more than one component the two out degree equitable dominating set has at least one vertex from every component of G then <D> is not connected, and conversely if G has a minimum connected two out degree equitable dominating set D and hence connected two out degree equitable domination number then<D> is connected that means G is connected according to that we state the following observation.

Theorem: 2.1 A connected two out degree equitable dominating set exist for a graph if and only if G is connected.

Example: 2.2 Let G be a graph as in the figure 1, we can find the two out degree equitable domination number & connected two out degree equitable domination number.

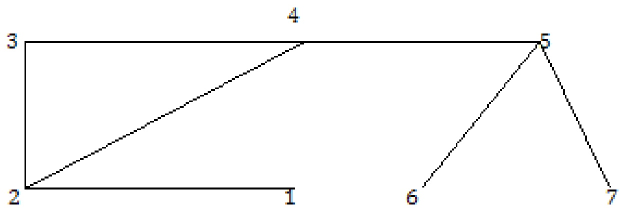

Fig 1:

Consider the set D={2,4,5} be a dominating set and V-D={1,3,6,7}

$od_D(2)$ = $|N(2) \cap$ {1,3,6,7}|

= |{1,3,4} ∩ {1,3,6,7}|.= 2

$od_D(4)$ = $|N(4) \cap$ {1,3,6,7}|

= |{2,3,5} ∩ {1,3,6,7}|.= 1

$od_D(5)$ = $|N(5) \cap$ {1,3,6,7}|

= |{4,6,7} ∩ {1,3,6,7}|.= 2

From the above any two vertices u,v ∈ D such that $|od_D(u) - od_D(v)| \leq 2$ and also induced sub graph <D> is connected. Therefore {2, 3, 5} is connected two-out degree equitable dominating set and

Similarly D={2,3,4,5} are also connected two-out degree equitable dominating set and {2,3,5} is connected two-out degree equitable dominating set with minimum cardinality so $\gamma_{c2oe}(G)$ =3.

Remark: 2.1 Let D be the dominating set is connected two out-degree equitable dominating set if and only if D is two out-degree equitable dominating set and connected dominating set

The following theorem gives the relationship between domination number, two out degree domination number and connected two out degree domination number.

Theorem: 2.2 For any connected graph G**,** $\gamma(G) \leq \gamma_{2oe}$(G)≤ $\gamma_{c2oe}(G)$.

Proof: From the definition of connected two-out degree equitable dominating set in graph G, it is clearly that for any graph G any connected two-out degree equitable dominating set D is also an two-out degree equitable dominating set and every two-out degree equitable dominating set is also a dominating set.

Hence $\gamma(G) \leq \gamma_{2oe}$(G)≤ $\gamma_{c2oe}(G)$.

The following theorem gives the relationship between, connected two out degree domination number and connected dominating set.

Theorem: 2.3 For any connected graph G**,** γ_c(G)≤ $\gamma_{c2oe}(G)$.

Proof: Since every connected two-out degree equitable dominating set for any connected graph G is connected dominating set,

Thus γ_c(G)≤ $\gamma_{c2oe}(G)$.

Result: 2.4 Let G is a graph with n vertices then **2** $\leq \gamma_{c2oe}(G) \leq n$-1

Result: 2.5 For any connected graph G, $\gamma_{c2oe}(G) = 2$ if and only if $\gamma_{2oe}(G) = 2$

3. CONNECTED TWO –OUT DEGREE EQUITABLE DOMINATION NUMBER FOR SOME STANDARD GRAPHS

Theorem: 3.1 For the complete graph k_m, the connected two –out degree equitable domination number is:
$\gamma_{c2oe}(k_m)$ =2

Proof: Let u, v ∈ D. Since G is complete and u ∈ V, N(u)= $V - \{u\}$

$od_D(u)$= |N(u)∩ V-D|

= $m - 2$,

For any u, v ∈ D, $od_D(u) = m - 2$, $od_D(v) = m - 2$

So $|od_D(u) - od_D(v)| = 0 \leq 2$

Therefore D forms a two degree equitable dominating set.

Since G is complete, D={u, v} is connected.

Hence $\gamma_{c2oe}(K_m)$ =2.

Theorem: 3.2 For the star$k_{1,m}$, the connected two –out degree equitable domination number is: $\gamma_{c2oe}(k_{1,m})$ =m-2.

Proof: Let {v,u_1, u_2, u_3…u_m} be the set of vertices in $k_{1,m}$.

Let D={ v,u_1, u_2, u_3 ... u_{m-2}} and V-D={u_{m-1}, u_m}

By the definition of star $k_{1,m}$, N(u_i)=v for i=1,2,…m and N(u_i) ∩ V-D = ∅.

Therefore $od_D(u)$= |N(u)∩ V-D| $= 0$.

Then N(v)={ u_1, u_2, u_3----------u_m} and V-D ⊆ $N(v)$ so N(v) ∩ V-D = V-D.

Therefore $od_D(u)$= $|N$(u)∩ V-D| $= |V - D|$ =2.

So $|od_D(u) - od_D(v)| = 2 \leq 2.$

So D is two degree equitable dominating set and induced sub graph of a D is connected.

Hence $\gamma_{c2oe}(k_{1,m})$ =m-2.

Theorem: 3.3 For the complete bipartite graph $K_{m,n}$, the connected two –out degree equitable domination number is:
$\gamma_{c2oe}(K_{m,n}) = \begin{cases} 2 & if\ |n-m| \leq 2 \\ not\ exists & Otherwise \end{cases}$

Proof: Let $V_1$and V_2 be the partition of vertices in $K_{m,n}$ such that $|V_1|$= m and $|V_2|$=n

And V_1= $\{u_1, u_2, u_3 \dots \dots u_m\}$, V_2 =$\{v_1, v_2 v_3 \dots \dots v_n\}$ be vertices in V

$$\deg(u) = \begin{cases} n & if\ v \in V_1 \\ m & if\ v \in V_2 \end{cases}$$

Let D= $\{u_i, v_i\}$ be a dominating set and <D> is connected.

Claim D is two out degrees connected equitable dominating set.

Here $u_i \in V_1$ then N(u_i) =V_2 and

V-D= $\{u_1, u_2, u_3 \dots \dots u_{i-1}, u_{i+1} \dots \dots u_m, v_1, v_2 v_3 \dots \dots v_{i-1}, v_{i+1} \dots \dots v_n\}$
$N(u_i)$∩ V-D= $\{ v_1, v_2 v_3 \dots \dots v_{i-1}, v_{i+1} \dots \dots v_n\}$

So $od_D(u_i)$= $|N(u_i)$∩ V-D|= n-1

Here $v_i \in V_2$ then N(v_i) = $V_1$and

V-D= $\{u_1, u_2, u_3 \dots \dots u_{i-1}, u_{i+1} \dots \dots u_m, v_1, v_2 v_3 \dots \dots\ v_{i-1}, v_{i+1} \dots \dots v_n\}$
$N(v_i)$∩ V-D=$\{u_1, u_2, u_3 \dots \dots u_{i-1}, u_{i+1} \dots \dots\ u_m\ \}$

so $od_D(v_i)$= $|N(v_i)$∩ V-D|= m-1. then

$|od_D(u) - od_D(v)| = |n - 1 - (m - 1)|$ =|n-m| ≤ 2.

So D is connected two out degree equitable dominating set $\gamma_{c2oe}(K_{m,n})$ = 2 $\quad if\ |n-m| \leq 2.$

If |n-m|≥ 2, we can't find a connected two out degree equitable dominating set.

Theorem: 3.4 For the Path graph P_n, the connected two –out degree equitable domination number is:
$\gamma_{c2oe}(P_m)$ =m-2. m≥ 2

Proof: Since the degree of any vertex in P_m is 2 except the initial and terminal vertices.

Let D be a set except initial and terminal vertices. So V-D contains initial and terminal vertices.

Let u, v ∈ D, deg (u) = |N (u)|=2 and $od_D(u)$= $|N$(u)∩ V-D| $\leq 2.$

Therefore D is two degree equitable dominating set and <D> is connected.

$\gamma_{c2oe}(P_m)$ =m-2.

Theorem: 3.5 For the Cycle C_m, the connected two –out degree equitable domination number is : $\gamma_{c2oe}(C_m)$ =m-2. m≥ 3.

Proof: Since the degree of any vertex in C_m is 2. And $\gamma(C_m)$ =m-2. Let D be a dominating set with minimum cardinality is n-2.

We have to prove that D is connected two degree equitable dominating set.

Since the maximum degree of a vertex in C_m is 2. |N(v)|=2 for all v in G.

Therefore $od_D(u)$= |N(u)∩ V-D| ≤ 2.

So D is two degree equitable dominating set and induced sub graph of a cycle is connected.
$\gamma_{c2oe}(C_m)$ =m-2. m≥ 3

Theorem: 3.6 For the Wheel W_m, the connected two –out degree equitable domination number is:

$$\gamma_{c2oe}(W_m) = \begin{cases} 2 & if\ m = 4,5 \\ 3 & if\ \ m = 6 \\ m-2 & if\ m \geq 7 \end{cases}$$

Proof: Let W_m be a when with m-1 vertices on the cycle and a single vertex at the center.

Let V (W_m) = {v,u_1, u_2, u_3 ... ... u_{m-1}}, where u is the center and v_i (1≤ $i \leq m-1$) is on the cycle.

Clearly deg (v_i) = for all 1≤ $i \leq m-1$ and deg (u) =m-1.

Clearly m≥ 4. We have the following cases:

Case: 1. m=4 and 5

If m=4 then W_4 forms a complete graph then by theorem 3.1 $\gamma_{c2oe}(W_4) = 2$

Suppose n=5. Let us take D= {u, v_i,} and since u is adjacent with v_i for all i 1≤ $i \leq 4$, V-D ⊂ N(u) so N(u)∩ V-D ⊂ V-D

$od_D(u)$= |N(u)∩ V-D| = |V-D|=3

Now for v_i

Since deg (v_i) =3 and vi is adjacent to u∈D then N(v_i)= {u, v_j, v_k} and $N(v_i)$∩ V-D= {v_j, v_k}

$od_D v_i$= |N(u)∩ V-D| = 2.

$|od_D(u) - od_D(v_i)| = 1 \leq 2$ and clearly <D>= <u, v_i> is connected

So D is connected two degree equitable dominating set

Hence $\gamma_{c2oe}(W_n)$ =2.

Case: 2. m=6

In this case deg (u)=5, while deg(v_i)=3 for all i, 1≤ $i \leq 5$,

Let us take D={ u, v_j, v_k} i≠j be a dominating set and since u is adjacent with v_i for all i 1≤ $i \leq 5$, V-D ⊂ N(u) so N(u)∩ V-D ⊂ V-D $od_D(u)$= |N(u)∩ V-D| = 3

Now for v_i and v_j

If v_i and v_j is adjacent N(v_i)= {u, v_j, v_k} and N(u)∩ V-D= {v_k}}

$od_D(v_i)$= |N(u)∩ V-D| = 1

If v_i and v_j are not adjacent but v_i and v_j are adjacent with u so N(u)∩ V-D contains two elements so $od_D(v_i)$= |N(u)∩ V-D| = 2

So for any elements u, v ∈ D

$|od_D(u) - od_D(v)| \leq 2$ and clearly <D>= < u, v_j, v_k> is connected

So D is connected two degree equitable dominating set

Hence $\gamma_{c2oe}(W_n)$=3

Case: 3. m$\geq$ 7

In this case deg (u) ≥ 6, while deg(v_i)=3 for all I, 1≤ $i \leq m-1$,

Consider a subset D of V contains with n-2 vertices

Since u is adjacent with v_i for all i 1≤ $i \leq 5$, V-D ⊂ N(u) so N(u)∩ V-D ⊂ V-D

$od_D(u)$= |N(u)∩ V-D| = 2.

Let $v_i \in D$, then N(vi) ⊆ D then N(u)∩ V-D=∅

$od_D(v_i)$= |N(u)∩ V-D| = 0

If N(v_i) not contains D then N(u)∩ V-D contains only one vertices

$od_D(v_i)$= |N(u)∩ V-D| = 1

So for any elements u, v ∈ D

$|od_D(u) - od_D(v)| \leq 2$ and clearly <D> is connected.

So D is connected two degree equitable dominating set.

Hence $\gamma_{c2oe}(W_m)$=m-2

Theorem: 3.7 The connected two-out degree equitable domination number of Peterson graph is 5.

Theorem: 3.8 For the double star $S_{m,n}$, the connected two –out degree equitable domination number is:

$$\gamma_{c2oe}(S_{m,n}) = \begin{cases} 2 & if\ |n-m| \leq 2 \\ not\ exist & otherwise \end{cases}$$

Proof: By the definition of double star Let $\{\boldsymbol{u}, u_1, u_2, u_3 \dots \dots u_n\}$, and $\{v, v_1, v_2 v_3 \dots \dots v_m\}$ are the vertices of $S_{m,n}$ and take D={u,v} connected and dominating set and then by definition $N(u)$∩ V-D={ $u_1, u_2, u_3 \dots \dots u_n$},

u_m so $od_D(u)$= |$N(u)$∩ V-D|= n and $N(v)$∩ V-D={ $v_1, v_2 v_3 \dots \dots v_m$},
so $od_D od_D(u)$= |$NN(uv)$∩ V-D|= m

then$|od_D(u) - od_D(v)| \leq |m-n] \leq 2$ therefore D={u,v} is connected two-out degree equitable dominating set.

Clearly connected two-out degree equitable dominating set is not exists if $|m-n| \geq 2$.

4. CONNECTED TWO OUT DEGREE EQUITABLE DOMATIC NUMBER

Definition: 4.1 A partition P= $\{V_1, V_2 \dots \dots V_l\}$ of V(G) is called a connected two-out degree equitable domatic partition of V_i is a connected two-out degree equitable dominating set for every 1≤ $i \leq l$.

Example:

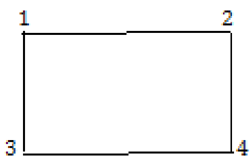

Figure 2:

{{1, 2}, {3, 4}} is a connected two-out degree equitable domatic partition of G

Definition: 4.2 The connected two-out degree equitable domatic number of G is the maximum cardinality of a connected two-out degree equitable domatic partition of G and is denoted by $d_{c2oe}(G)$.

Theorem: 4.3 For any graph G, $d_{c2oe}(G) \leq d_c(G) \leq d(G)$, where d(G), $d_c(G)$, are the domatic and connected domatic number of G respectively .

Proof: Let G=(V,E) be a graph. Since it is clear that any partition of V into connected two out degree equitable domination set is also partition of V into connected dominating set, and also any partition of V in to dominating set.

Hence $d_{c2oe}(G) \leq d_c(G) \leq d(G)$,

We now proceed to compute $d_{c2oe}(G)$ for some standard graphs.

Theorem; 4.4

(1) For the complete graph K_m, $d_{c2oe}(K_m) = m$
(2) For the cycle C_m, m ≥ 4, $d_{c2oe}(C_m) = 2$
(3) For the path P_m, $d_{c2oe}(P_m) = 2$
(4) For the star $K_{1,m}$, $d_{c2oe}(K_{1,m}) = 2$

Theorem: 4.5 For any connected graph with m vertices and with vertex connectivity k(G)≤ m-1, if R its vertex cut and D is the connected two out degree equitable dominating set, then D$\cap R \neq \emptyset$.

Proof: Let $C_{1,}C_{2,} \dots \dots C_{k,}$ be the component s of <V-D> and evidently k≥ 2.

Suppose that D$\cap R = \emptyset$.

Since <D> is connected, then <D> is a sub graph of C_i for some i∈ $\{1,2 \dots \dots k\}$.

Let v∈ C_j for j≠ i.

The v∈ $V - D$ and no vertex in D is adjacent to v.

Then D is not dominating set , which is contradiction.

Hence D$\cap R \neq \emptyset$.

Theorem: 4.6 For any graph G of order n , $d_{c2oe} \leq \frac{m}{\gamma_{c2oe}}$

Proof: Suppose that $d_{c2oe} = t$, for some positive integer t.

Let P= $\{D_{1,}D_{2,} \dots \dots D_{k,}\}$ be the connected two out degree equitable domatic partion of G.

Obviously, |V(G)|=$\sum_{i=1}^{t} |D_i|$ and frp, the definition of connected two out degree equitable domination number $\gamma_{c2oe}(G)$, we have $\gamma_{c2oe} \leq |D_i|$, i=1,2,3------t,

Hence n=$\sum_{i=1}^{t} |D_i| \geq t\gamma_{c2oe}(G)$. that implies $m \geq t\gamma_{c2oe}(G)$

Thus $d_{c2oe} \leq \frac{m}{\gamma_{c2oe}}$

5. CONCLUSION

In this paper we define new parameter called connected two-out degree equitable domination in graphs. We can extend this concept of a two-out degree equitable domination in graphs to connected neighborhood two-out degree equitable domination, non split two-out degree equitable domination etc and study the characteristic of these parameters.

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