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MAGNETOHYDRODYNAMIC FLOW A POLAR FLUID

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ABSTRACT

The flow of an incompressible and electrically conducting polar fluid in the presence of a magnetic field is considered. The polar fluid is moving over a porous moving plate and the magnetic Reynolds number is not small enough so that the induced magnetic field is not negligible. The numerical calculations are given for various values of the material parameters characterizing the polarity of the fluid and the magnetic parameter.

Keywords: *Magnetohydrodynamics, polar fluids.*

INTRODUCTION

Polar fluids are defined as fluids that sustain couple stresses. Examples of fluids that can be modeled as polar fluids are mud, crude oil, body fluids and lubricants with polymer additives. The study of the magnetohydrodynamics flow for a polar fluid past a plate has attracted the interest in view of its applications in many engineering problems. Soundalgekar and Aranake [1] have studied the effects of the polar fluids on the oscillatory flow past a plate with constant suction. Hiremath and Patil [2], Patil [3] and Chamkha and Aly [4] have examined the effects of a polar fluid through a porous medium. Magnetohydrodynamic flow of a incompressible and electrically conducting polar fluid past a plate has been studied by Helmy [6], Kim [7], Ogulu [8] and Mohamed at al. [9] when the magnetic Reynolds number is small so that the induced magnetic field is negligible.

The object of the present paper is the study of the steady two dimensional magnetohydrodynamics flow of an incompressible polar fluid. The polar fluid is moving over a moving porous plate and the magnetic Reynolds number is not small enough so that the induced magnetic field is not negligible.

ANALYSIS

Consider the steady two dimensional flow of an electrically conducting polar fluid past an infinite moving porous plate. The fluid is viscous and incompressible and the plate absorbs the fluid with a constant velocity. The $x'-$axis is taken along the plate and the $y'-$axis normal to it. The plate is electrically non-conducting and the applied magnetic field is such that in the region of the plate $\vec{H}' = (H'_{x'}, H_0^*, 0)$, where $H'_{x'}$ is the induced magnetic field and H_0^* is the constant component of the magnetic field perpendicular to the plate.

The equations governing the problem are [3], [7], [9]

Continuity equation

$$\frac{\partial \upsilon'}{\partial y'} = 0 \tag{1}$$

Momentum equation

$$\upsilon' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho'}\frac{\partial P'}{\partial x'} + (\nu + \nu_r)\frac{\partial^2 u'}{\partial y'^2} + 2\nu_r \frac{\partial \omega'}{\partial y'} + \frac{\mu_0}{\rho'} H_0^* \frac{\partial H'_{x'}}{\partial y'} \tag{2}$$

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Angular momentum equation

$$\upsilon' \frac{\partial \omega'}{\partial y'} = \frac{\gamma}{I} \frac{\partial^2 \omega'}{\partial y'^2} \tag{3}$$

Equation of induced magnetic field

$$\upsilon' \frac{\partial H'_{x'}}{\partial y'} = H_0^* \frac{\partial u'}{\partial y'} + \frac{1}{\sigma \mu_0} \frac{\partial^2 H'_{x'}}{\partial y'^2} \tag{4}$$

where u', υ' are the velocity components along of the velocity along the x' and y' directions respectively, ρ' is the density of the fluid, P' is the pressure, ν is the kinematic viscosity, ν_r is the rotational kinematic viscosity, ω' is the angular velocity component of rotation of the particles, μ_0 is the magnetic permeability, I is a scalar constant of dimension to that of the moment of inertia per unit mass, $\gamma = \frac{c_a + c_d}{I}$, where c_a and c_d are coefficients of the couple stress viscosities and σ is the electrical conductivity.

The boundary conditions are

$$\left. \begin{array}{lllll} u' = U, & \frac{\partial \omega'}{\partial y'} = -\frac{\partial^2 u'}{\partial y'^2}, & \frac{\partial H'_{x'}}{\partial y'} = 0, & at & y' = 0 \\ u' \to 0, & \omega' \to 0, & H'_{x'} \to 0, & as & y' \to \infty \end{array} \right\} \tag{5}$$

where U is the velocity of the plate.

From equation (1) we have

$$\upsilon' = -\upsilon_0 \tag{6}$$

where υ_0 is the constant suction velocity and the negative sign indicates that the suction velocity is directed towards the plate.

Far away of the plate we have from equation (2)

$$-\frac{1}{\rho'} \frac{\partial P'}{\partial x'} = 0 \tag{7}$$

On taking into account equations (6) and (7), equations (2), (3) and (4) become

$$-\upsilon_o \frac{\partial u'}{\partial y'} = (\nu + \nu_r) \frac{\partial^2 u'}{\partial y'^2} + 2\nu_r \frac{\partial \omega'}{\partial y'} + \frac{\mu_0}{\rho'} H_0^* \frac{\partial H'_{x'}}{\partial y'} \tag{8}$$

$$-\upsilon_0 \frac{\partial \omega'}{\partial y'} = \frac{\gamma}{I} \frac{\partial^2 \omega'}{\partial y'^2} \tag{9}$$

$$-\upsilon_0 \frac{\partial H'_{x'}}{\partial y'} = H_0^* \frac{\partial u'}{\partial y'} + \frac{1}{\sigma \mu_0} \frac{\partial^2 H'_{x'}}{\partial y'^2} \tag{10}$$

On introducing the following non-dimensional quantities

$$y = \frac{y' \upsilon_0}{\nu}$$

$$u = \frac{u'}{U}$$

$$\mathrm{a} = \frac{\nu_r}{\nu} \qquad \text{(material parameter characterizing the polarity of the fluid)}$$

$$\beta = \frac{I \nu}{\gamma} \qquad \text{(material parameter characterizing the polarity of the fluid)} \tag{11}$$

$$\omega = \frac{\omega' \nu}{U \upsilon_0} \qquad \text{(dimensionless angular velocity)}$$

$$H = \left(\frac{\mu_0}{\rho}\right)^{1/2} \frac{H'_{x'}}{U}$$

$P_m = \nu\sigma\mu_0$ (magnetic Prandtl number)

$M = \left(\frac{\mu_0}{\rho}\right)^{1/2} \frac{H_0^*}{\upsilon_0}$ (magnetic parameter)

into equations (8), (9) and (10) we get

$$(1+\mathrm{a})u'' + u' + 2\mathrm{a}\,\omega' + MH' = 0 \tag{12}$$

$$\omega'' + \beta\omega' = 0 \tag{13}$$

$$H'' + P_m H' + MP_m u' = 0 \tag{14}$$

The boundary conditions (5) now become

$$\begin{aligned} &u = 1, \quad \omega' = -u'', \quad H' = 0, \quad at \quad y = 0 \\ &u \to 0, \quad \omega \to 0, \quad H \to 0, \quad as \quad y \to \infty \end{aligned} \tag{15}$$

In the above equations the primes denote the differentiation with respect to y.

RESULTS AND DISCUSSION

Equations (12)-(14) with boundary conditions (15) have been solved numerically using Runge- Kutta and shooting technique and the results are the following:

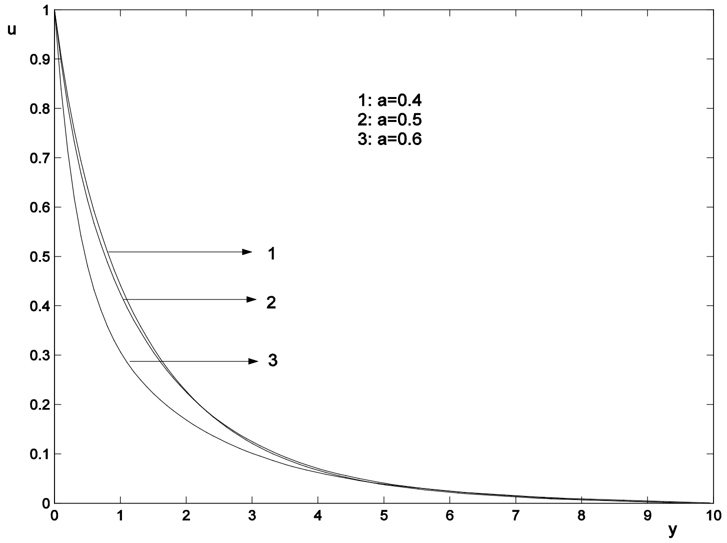

Fig.1: Velocity profiles for various values of the parameter a

In Fig. 1 we have plotted the velocity profiles showing the effect of the parameter a , when $\beta = 3.0, M = 0.2, P_m = 0.3$.

We see that the velocity decreases when the parameter a increases.

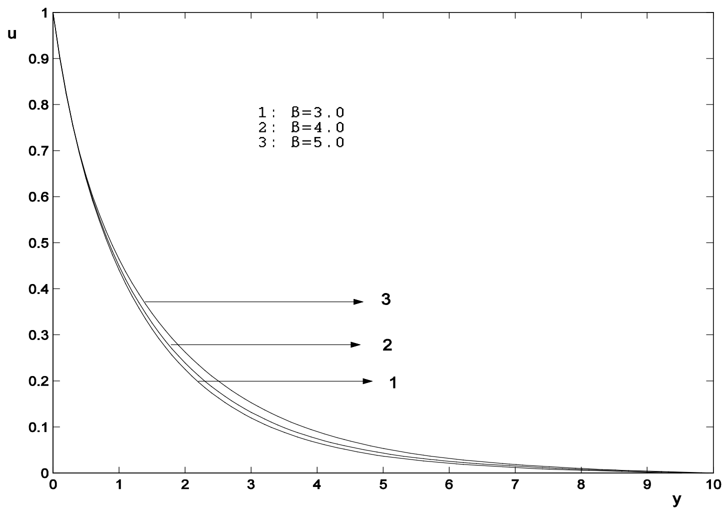

Fig.2: Velocity profiles for various values of the parameter β

In Fig. 2 we have plotted the velocity profiles showing the effect of the parameter β, when $a = 0.4, M = 0.2, P_m = 0.3$. We see that the velocity increases when the parameter β increases.

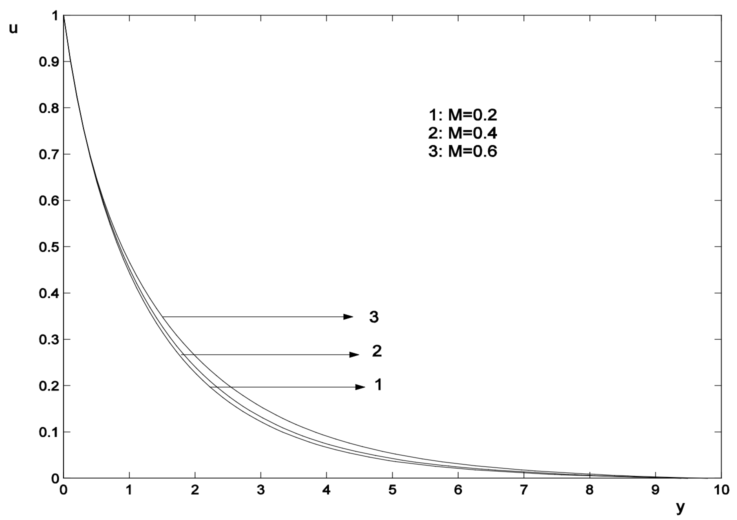

Fig.3: Velocity profiles for various values of the parameter M

In Fig. 3 we have plotted the velocity profiles showing the effect of the parameter M, when $a = 0.4, \beta = 3.0, P_m = 0.3$. We see that the velocity increases when the parameter M increases.

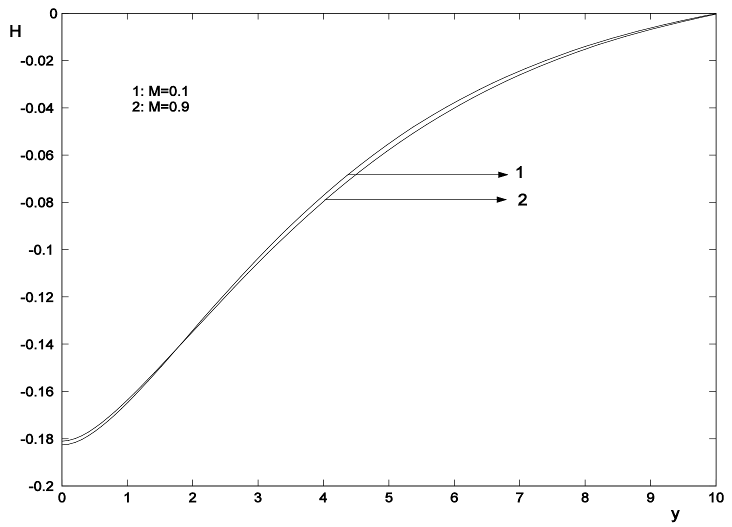

Fig.4: Induced magnetic field for various values of the parameter M

In Fig. 4 we have plotted the induced magnetic field showing the effect of the parameter M, when $a = 0.4, \beta = 3.0, P_m = 0.3$. We see that the induced magnetic field decreases when the parameter K increases.

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