# International Journal of Mathematical Archive-5(1), 2014, 1-6 IMAAvailable online through www.ijma.info <br> ISSN 2229-5046 <br> ON GRAPHS AND TOPOLOGICAL SPACES 

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#### Abstract

In this paper, we will introduce the isomorphism between graph and topological space. The relation between folding of topological space and The folding of graph are obtained. Some operations on the topological space are achieved. The induced operations on the corresponding graph are deduced.


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## INTRODUCTION

The folding of Riemannian manifolds is introduced by S.A-Robertson 1977[10]. More studies on the folding of real manifolds are studied by E.El-kholy [9]. and M.EL-Ghoul [2,3,4,5,6,7]. The folding of dynamical fuzzy topological space is defined in [8]. The folding of graph is discussed in [4, 5]. In this article we will introduce the relations between graph and topological space

## DEFINITIONS AND BACKGROUND

We will give some definitions which we will need them in this paper:
(1) Let $X$ be a non empty set. A class $\tau$ of subsets of $X$ is a topology on $X$ iff $\tau$ satisfies the following:
(i) $X$ and $\phi \in \tau$.
(ii) The union of any number of sets in $\tau$ belong to $\tau$.
(iii) The Intersection of any two sets in $\tau$ belong to $\tau$.
(2) A graph is a diagram consisting of points, called vertices, joined together by edges; each edge joins exactly two vertices [1].
(3) $\mathrm{A} "$ simple graph" is a graph with no loops or no multiple edges [13].
(4) Two or more edges joining pair of vertices are called "multiple edges" and edge joining a vertex to itself is called a" loop". A" multiple" graph allows multiple edges. Sometimes the multiple graph is called a general graph or, simply a graph [12, 13, 14].
(5) Map $f: M \rightarrow N$, where $M, N$ are $C^{\infty}$ - Riemannian manifolds of dimensions $m, n$, respectively is said to be an isometric folding of $M$ into $N$, if and only if for any piecewise geodesic path $\gamma: J \rightarrow N$, the induced path $f \circ \gamma: J \rightarrow M$ is piecewise geodesic and of the same length as $\gamma, \gamma=[0,1][10]$.

If $f$ not preserves lengths then $f$ is a topological folding.
(6) The intersection of two graph: If $V_{1} \cap V_{2} \neq \phi$, the graph $G=(V, E)$, where $V=V_{1} \cap V_{2}$ and $E=E_{1} \cap E_{2}$, is the intersection of $G_{1}$ and $G_{2}$ and is written as $G_{1} \cap G_{2}$ [11].

## THE MAIN RESULTS

Aiming to our study we will introduce some definition:
(1) Let $X=\{a, b, c\}$ and $\tau_{1}=\{X, \varphi,\{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\}\}$ then this topology $\tau_{1}$ is isomorphic to the graph $G_{1}$ whose vertices are $V_{1}=\{a, b, c\}$ and edges are $E_{1}=\{a b, a c, b c\}$ see Fig. (1).


Fig. - 1
But the topology $\tau_{2}=\{X, \varphi,\{a\},\{b\},\{a, b\}\}$ is isomorphic the graph $G_{2}$ which vertices are $V_{2}=\{a, b, c\}$ and edge is $E_{2}=\{a b\}$ see Fig.(2).


Fig. -2
But the topology $\tau_{3}=\{X, \varphi\}$ is isomorphic the graph $G_{3}$ its vertices are $V_{3}=\{a, b, c\}$ and no edges $E_{3}=\{\phi\}$ see Fig. (3).
a.
b. .c

Fig. -3
But the topology $\tau_{4}=\{X, \varphi,\{a\},\{b\},\{a, b\},\{a, c\}\}$ is isomorphic the graph $G_{4}$ its vertices are $V_{4}=\{a, b, c\}$ and edges are $E_{4}=\{a b, a c\}$ see Fig. (4).


Fig. -4
If $X=\{a, b, c, d\}$ there exist many topologies for example
$\tau=\{X, \varphi,\{a\},\{b\},\{c\},\{d\},\{a, b\},\{a, c\},\{b, d\},\{d, c\},\{a, b, c\},\{a, b, d\},\{a, c, d\},\{b, c, d\}\}$ which is isomorphic the graph $G$ its vertices $V=\{a, b, c, d\}$ and edges $E=\{a b, a c, b d, d c\}$ see Fig. (5).


Fig. - 5
(2) Let $X=\{a, b, c\}$ be another set, then there exist many topologies for example $\tau_{1}=\{X, \varphi,\{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\}\}, \tau_{2}=\{X, \varphi,\{a\},\{b\},\{a, b\}\}$ and the intersection between two topologies $\tau_{1}$ and $\tau_{2}$ is $\tau_{1} \cap \tau_{2}=\{X, \varphi,\{a\},\{b\},\{a, b\}\}$ and the intersection between two graphs $G_{1}$ and $G_{2}$ is $G_{1} \cap G_{2}, V_{1} \cap V_{2}=\{a, b, c\}$ and $E_{1} \cap E_{2}=\{a b\}$ see Fig. (6).


Fig. - 6
Then there exist isomorphism between the intersection of two topologies and the intersection of two graphs.
Theorem: 1 The intersection of two topologies is isomorphic to the intersection of the two corresponding graphs.
Proof: Let $\left(X_{1}, \tau_{1}\right),\left(X_{2}, \tau_{2}\right)$ are any two topologies, the two corresponding graphs are $\left(X_{1}, G_{1}\right),\left(X_{2}, G_{2}\right)$. If $f: \tau_{1} \rightarrow \tau_{2}$ be an isomorphism then there is an induced isomorphism $\bar{f}: G_{1} \rightarrow G_{2}$ and if $\tau_{1} \cap \tau_{2}=\tau_{3}$ the induced graph is $G_{1} \bigcap G_{2}=G_{3}$ and $G_{3}$ is the corresponding graph of $\tau_{3}$.

And if $f\left(\tau_{1} \cap \tau_{2}\right)=f\left(\tau_{1}\right) \bigcap f\left(\tau_{2}\right) \Rightarrow \bar{f}\left(G_{1} \bigcap G_{2}\right)=\bar{f}\left(G_{1}\right) \bigcap \bar{f}\left(G_{2}\right)$,

$$
\begin{aligned}
& f\left(\tau_{1} \cap \tau_{2}\right)=f\left(\tau_{1}\right) \cap \tau_{2} \Rightarrow \bar{f}\left(G_{1} \cap G_{2}\right)=\bar{f}\left(G_{1}\right) \cap G_{2} \text { and if } \\
& f\left(\tau_{1} \cap \tau_{2}\right)=\tau_{1} \cap f\left(\tau_{2}\right) \Rightarrow \bar{f}\left(G_{1} \cap G_{2}\right)=G_{1} \cap \bar{f}\left(G_{2}\right)
\end{aligned}
$$

(3) The folding of the topology $\tau_{1}=\{X, \varphi,\{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\}\}$ into itself defined
$f:\left(X_{1}, \tau_{1}\right) \rightarrow\left(X_{2}, \tau_{2}\right)$ as follows:
$f: X \rightarrow\{a, b\} \quad, \quad f: \phi \rightarrow \phi$
$f:\{a\} \rightarrow\{a\} \quad, \quad f:\{b\} \rightarrow\{b\}$
$f:\{c\} \rightarrow \phi \quad, \quad f:\{a, b\} \rightarrow\{a\}$
$f:\{a, c\} \rightarrow\{a\}, f:\{b, c\} \rightarrow\{b\}$

The image is $\tau_{2}=\{\{a, b\}, \varphi,\{a\},\{b\}\}$, then the topology $\tau_{1}$ is isomorphic to $G_{1}$ whose vertices $V_{1}=\{a, b, c\}$ and edge $E_{1}=\{a b, a c, b c\}$ see Fig.[7].


Fig. - 7

But the image of $\tau_{1}$ is $\tau_{2}=\{\{a, b\}, \varphi,\{a\},\{b\}\}$ is isomorphic $G_{2}$ whose vertices $V_{1}=\{a, b\}$ and edge $E_{1}=\{a b\}$ see Fig. [8].


Fig. - 8
Also if $f:\left(X_{2}, \tau_{2}\right) \rightarrow\left(X_{3}, \tau_{3}\right)$ such that $f\left(\tau_{2}\right)=\{\{a\}, \varphi\}$ or $f\left(\tau_{2}\right)=\{\{b\}, \varphi\}$ then the graphs are


But the folding of the graph $G_{1}$ is the identity see Fig. [9].


Fig. - 9
From the above discussion there exist an isomorphism between the folding of the topological spaces and the folding of the graphs and the following diagram is commutative:

s.t: $\bar{i} \circ f=\bar{f} \circ i$

This can be generalized for a chain of folding as follows:

s.t: $i_{n+1} \circ f_{n}=\bar{f}_{n} \circ i_{n}$

Example: 1 Let $X=\{a, b, c, d\}$ there exist many topologies for example
$\tau=\{X, \varphi,\{a\},\{b\},\{c\},\{d\},\{a, b\},\{a, c\},\{b, d\},\{d, c\},\{a, b, c\},\{a, b, d\},\{a, c, d\},\{b, c, d\}\}$ and let $f$ is the folding of $\tau$ into itself:
$f: X \rightarrow\{a, b, c\} \quad, \quad f: \varphi \rightarrow \varphi$
$f:\{a\} \rightarrow\{a\} \quad, \quad f:\{b\} \rightarrow\{b\}$
$f:\{c\} \rightarrow\{a\} \quad, \quad f:\{d\} \rightarrow \varphi$
$f:\{a, b\} \rightarrow\{a, b\} \quad, \quad f:\{a, c\} \rightarrow\{a, c\}$
$f:\{b, d\} \rightarrow\{a, c\} \quad, \quad f:\{d, c\} \rightarrow\{a, b\}$
$f:\{a, b, c\} \rightarrow\{a, b\}, \quad f:\{a, b, d\} \rightarrow\{a, b\}$
$f:\{a, c, d\} \rightarrow\{a, c\}, \quad f:\{b, c, d\} \rightarrow\{b\}$
The image $\tau^{\prime}=\{\{a, b, c\}, \varphi,\{a\},\{b\},\{a, b\},\{a, c\}\}$ which is a topology.
$\tau \cong G_{1}$ which vertices $V_{1}=\{a, b, c, d\}$ and edge $E_{1}=\{a b, a c, b d, d c\}$ see Fig [10].


Fig. - 10
Let $\tilde{f}: G_{1} \rightarrow G_{2}$ defined as following:
$\tilde{f}: a b \rightarrow a b, \tilde{f}: a c \rightarrow a c$
$\tilde{f}: b d \rightarrow a c, \tilde{f}: d c \rightarrow b a$
See Fig [11].


Fig. - 11
In this example there is exist isomorphic between the folding of the topological spaces and the folding of the graphs.
From this example we arrive to this theorem:
Theorem: 2 The folding of the topological spaces induce a sequence of foldings of the corresponding graphs .
Proof: Let $f:\left(X, \tau_{1}\right) \rightarrow\left(X, \tau_{2}\right)$ then there are induced folding $\bar{f}: G_{1} \rightarrow G_{2}$ also any $\bar{f}: G_{1} \rightarrow G_{2}$ induce $f:\left(X, \tau_{1}\right) \rightarrow\left(X, \tau_{2}\right)$.
And let $f_{1}:\left(X_{1}, \tau_{1}\right) \rightarrow\left(X_{2}, \tau_{2}\right), X_{2} \subset X_{1}$

$$
\begin{aligned}
& f_{2}:\left(X_{2}, \tau_{2}\right) \rightarrow\left(X_{3}, \tau_{3}\right), X_{3} \subset X_{2} \\
& \cdot \\
& \cdot \\
& f_{n}:\left(X_{n}, \tau_{n}\right) \rightarrow\left(X_{n+1}, \tau_{n+1}\right), X_{n+1} \subset X_{n}
\end{aligned}
$$

Then there is an induced sequence of foldings:
$\bar{f}_{1}: G_{1} \rightarrow G_{2}, G_{2} \subset G_{1}$
$\bar{f}_{2}: G_{2} \rightarrow G_{3}, G_{3} \subset G_{2}$
$\bar{f}_{n}: G_{n} \rightarrow G_{n+1}, G_{n+1} \subset G_{n}$
and all $f_{i}$ isomorphic to $\bar{f}_{i}$

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