International Journal of Mathematical Archive-5(1), 2014, 1-6
Available online through www.ijma.info ISSN 2229 – 5046

ON GRAPHS AND TOPOLOGICAL SPACES

H. I. Attiya*

Basic Science Department, College of Industrial Education, Bine-suef University, Egypt.

(Received on: 20-11-13; Revised & Accepted on: 06-01-14)

ABSTRACT

In this paper, we will introduce the isomorphism between graph and topological space. The relation between folding of topological space and The folding of graph are obtained. Some operations on the topological space are achieved. The induced operations on the corresponding graph are deduced.

Keywords: *Folding, Graphs.*

2000 Mathematics Subject Classification: *51H10, 57N20.*

INTRODUCTION

The folding of Riemannian manifolds is introduced by S.A-Robertson 1977[10]. More studies on the folding of real manifolds are studied by E.El-kholy [9]. and M.EL-Ghoul [2,3,4,5,6,7]. The folding of dynamical fuzzy topological space is defined in [8]. The folding of graph is discussed in [4, 5]. In this article we will introduce the relations between graph and topological space

DEFINITIONS AND BACKGROUND

We will give some definitions which we will need them in this paper:
(1) Let X be a non empty set. A class τ of subsets of X is a topology on X iff τ satisfies the following:
(i) $X \text{ and } \phi \in \tau$.
(ii) The union of any number of sets in τ belong to τ .
(iii) The Intersection of any two sets in τ belong to τ .

(2) A graph is a diagram consisting of points, called vertices, joined together by edges; each edge joins exactly two vertices [1].

(3) A" simple graph" is a graph with no loops or no multiple edges [13].

(4) Two or more edges joining pair of vertices are called "multiple edges" and edge joining a vertex to itself is called a" loop". A" multiple" graph allows multiple edges. Sometimes the multiple graph is called a general graph or, simply a graph [12, 13, 14].

(5) Map $f: M \to N$,where M, N are C^{∞}- Riemannian manifolds of dimensions m, n ,respectively is said to be an isometric folding of M into N, if and only if for any piecewise geodesic path $\gamma: J \to N$, the induced path $f \circ \gamma: J \to M$ is piecewise geodesic and of the same length as $\gamma, \gamma = [0,1]$ [10] .

If f not preserves lengths then f is a topological folding.

(6) The intersection of two graph: If $V_1 \cap V_2 \neq \phi$, the graph $G = (V, E)$, where $V = V_1 \cap V_2 \text{ and } E = E_1 \cap E_2$, is the intersection of $G_1 \text{ and } G_2$ and is written as $G_1 \cap G_2$ [11].

Corresponding author: H. I. Attiya*
Basic Science Department, College of Industrial Education, Bine-suef University, Egypt.

THE MAIN RESULTS

Aiming to our study we will introduce some definition:

(1) Let $X = \{a, b, c\}$ and $\tau_1 = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$ then this topology τ_1 is isomorphic to the graph G_1 whose vertices are $V_1 = \{a,b,c\}$ and edges are $E_1 = \{ab, ac, bc\}$ see Fig. (1).

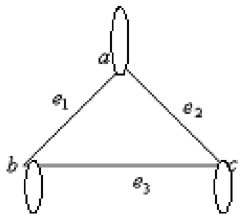


Fig. – 1

But the topology $\tau_2 = \{X, \varphi, \{a\}, \{b\}, \{a,b\}\}$ is isomorphic the graph G_2 which vertices are $V_2 = \{a,b,c\}$ and edge is $E_2 = \{ab\}$ see Fig.(2).

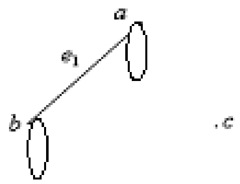


Fig. – 2

But the topology $\tau_3 = \{X, \varphi\}$ is isomorphic the graph G_3 its vertices are $V_3 = \{a,b,c\}$ and no edges $E_3 = \{\phi\}$ see Fig. (3).

Fig. – 3

But the topology $\tau_4 = \{X, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ is isomorphic the graph G_4 its vertices are $V_4 = \{a,b,c\}$ and edges are $E_4 = \{ab, ac\}$ see Fig. (4).

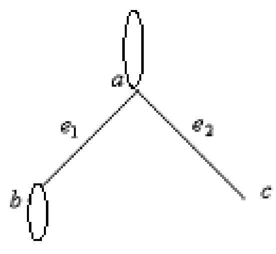


Fig. – 4

If $X = \{a, b, c, d\}$ there exist many topologies for example

$\tau = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{b,d\}, \{d,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$ which is isomorphic the graph G its vertices $V = \{a,b,c,d\}$ and edges $E = \{ab, ac, bd, dc\}$ see Fig. (5).

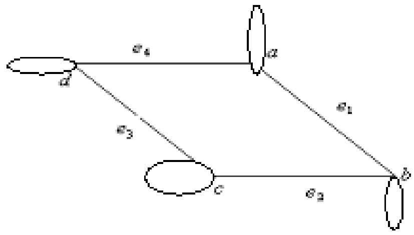


Fig. – 5

(2) Let $X=\{a,b,c\}$ be another set, then there exist many topologies for example $\tau_1=\{X,\varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$, $\tau_2=\{X,\varphi,\{a\},\{b\},\{a,b\}\}$ and the intersection between two topologies τ_1 *and* τ_2 is $\tau_1\cap\tau_2=\{X,\varphi,\{a\},\{b\},\{a,b\}\}$ and the intersection between two graphs G_1 *and* G_2 is $G_1\cap G_2$, $V_1\cap V_2=\{a,b,c\}$ and $E_1\cap E_2=\{ab\}$ see Fig. (6).

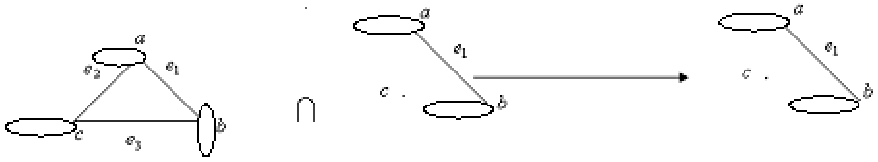


Fig. - 6

Then there exist isomorphism between the intersection of two topologies and the intersection of two graphs.

Theorem: 1 The intersection of two topologies is isomorphic to the intersection of the two corresponding graphs.

Proof: Let (X_1,τ_1), (X_2,τ_2) are any two topologies, the two corresponding graphs are (X_1,G_1), (X_2,G_2). If $f:\tau_1\to\tau_2$ be an isomorphism then there is an induced isomorphism $\bar{f}:G_1\to G_2$ and if $\tau_1\cap\tau_2=\tau_3$ the induced graph is $G_1\cap G_2=G_3$ and G_3 is the corresponding graph of τ_3.

And if $f(\tau_1\cap\tau_2)=f(\tau_1)\cap f(\tau_2)\Rightarrow\bar{f}(G_1\cap G_2)=\bar{f}(G_1)\cap\bar{f}(G_2)$,
$f(\tau_1\cap\tau_2)=f(\tau_1)\cap\tau_2\Rightarrow\bar{f}(G_1\cap G_2)=\bar{f}(G_1)\cap G_2$ and if
$f(\tau_1\cap\tau_2)=\tau_1\cap f(\tau_2)\Rightarrow\bar{f}(G_1\cap G_2)=G_1\cap\bar{f}(G_2)$.

(3) The folding of the topology $\tau_1=\{X,\varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ into itself defined
$f:(X_1,\tau_1)\to(X_2,\tau_2)$ as follows:
$f:X\to\{a,b\}$, $f:\phi\to\phi$
$f:\{a\}\to\{a\}$, $f:\{b\}\to\{b\}$
$f:\{c\}\to\phi$, $f:\{a,b\}\to\{a\}$
$f:\{a,c\}\to\{a\}$, $f:\{b,c\}\to\{b\}$

The image is $\tau_2=\{\{a,b\},\varphi,\{a\},\{b\}\}$, then the topology τ_1 is isomorphic to G_1 whose vertices $V_1=\{a,b,c\}$ and edge $E_1=\{ab,ac,bc\}$ see Fig.[7].

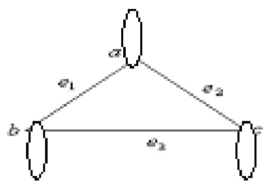


Fig. - 7

But the image of τ_1 is $\tau_2 = \left\{\{a, b\}, \varphi, \{a\}, \{b\}\right\}$ is isomorphic G_2 whose vertices $V_1 = \{a, b\}$ and edge $E_1 = \{ab\}$ see Fig. [8].

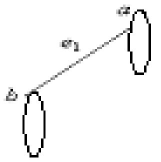
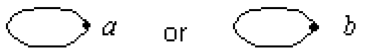


Fig. - 8

Also if $f : (X_2, \tau_2) \to (X_3, \tau_3)$ such that $f(\tau_2) = \left\{\{a\}, \varphi\right\}$ or $f(\tau_2) = \left\{\{b\}, \varphi\right\}$ then the graphs are



But the folding of the graph G_1 is the identity see Fig. [9].

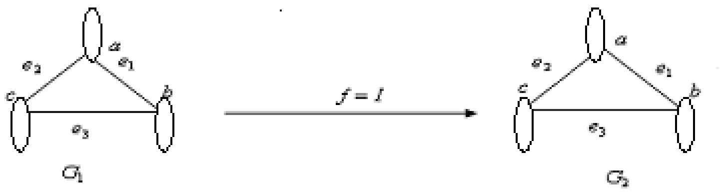


Fig. - 9

From the above discussion there exist an isomorphism between the folding of the topological spaces and the folding of the graphs and the following diagram is commutative:

$$\begin{tikzcd}
\tau_1 \arrow[r, "f"] \arrow[d, "i"'] & \tau_2 \arrow[d, "\bar{i}"] \\
G_1 \arrow[r, "\bar{f}"'] & G_2
\end{tikzcd}$$

s.t: $\bar{i} \circ f = \bar{f} \circ i$

This can be generalized for a chain of foldings as follows:

$$\begin{tikzcd}
\tau_1 \arrow[r, "f_1"] \arrow[d, "i_1"'] & \tau_2 \arrow[r, "f_2"] \arrow[d, "i_2"] & \tau_3 \cdots \arrow[r, "f_n"] \arrow[d, "i_3"] & (X, \varphi) \arrow[d, "i_{n+1}"] \\
G_1 \arrow[r, "\bar{f}_1"'] & G_2 \arrow[r, "\bar{f}_2"'] & G_3 \cdots \arrow[r, "\bar{f}_n"'] & \varphi
\end{tikzcd}$$

s.t: $i_{n+1} \circ f_n = \bar{f}_n \circ i_n$

Example: 1 Let $X=\{a,\, b,\, c,\, d\}$ there exist many topologies for example
$\tau=\{X,\varphi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{b,d\},\{d,c\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and let f is the folding of τ into itself:

$f\ : X \to \{a\,,b,c\}$, $f:\varphi\to\varphi$

$f\ :\{a\}\to\{a\}$, $f:\{b\}\to\{b\}$
$f\ :\{c\}\to\{a\}$, $f:\{d\}\to\varphi$
$f\ :\{a\,,b\}\to\{a,b\}$, $f:\{a\,,c\}\to\{a,c\}$
$f\ :\{b\,,d\}\to\{a,c\}$, $f:\{d,\,c\}\to\{a,b\}$
$f\ :\{a\,,b,c\}\to\{a,b\}$, $f:\{a,\,b,\,d\}\to\{a,b\}$
$f\ :\{a\,,c,d\}\to\{a,c\}$, $f:\{b,c,d\}\to\{b\}$

The image $\tau'=\{\{a,b,c\},\varphi,\{a\},\{b\},\{a,b\},\{a,c\}\}$ which is a topology.
$\tau\cong G_1$ which vertices $V_1=\{a\,,\,b,\,c,\,d\}$ and edge $E_1=\{ab,\,ac,bd\,,dc\}$ see Fig [10].

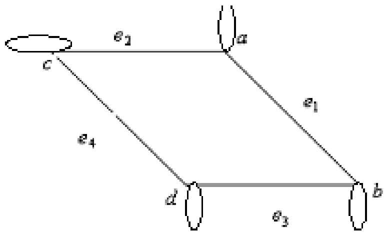


Fig. - 10

Let $\tilde{f}:\ G_1\ \to G_2$ defined as following:
$\tilde{f}:ab\ \to ab$, $\tilde{f}:\ ac\ \to ac$
$\tilde{f}:bd\to ac$, $\tilde{f}:\ dc\ \to ba$

See Fig [11].

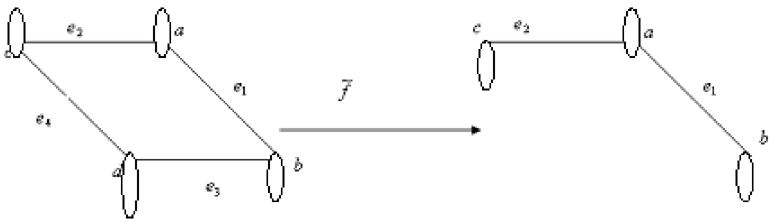


Fig. - 11

In this example there is exist isomorphic between the folding of the topological spaces and the folding of the graphs.

From this example we arrive to this theorem:

Theorem: 2 The folding of the topological spaces induce a sequence of foldings of the corresponding graphs .

Proof: Let $f:(X,\,\tau_1)\to(X,\,\tau_2)$ then there are induced folding $\bar{f}:G_1\to G_2$ also any $\bar{f}:G_1\to G_2$ induce $f:(X,\,\tau_1)\to(X,\,\tau_2)$.
And let $f_1:(X_1\,,\,\tau_1)\to(X_2\,,\,\tau_2),X_2\subset X_1$

$$f_2 : (X_2 , \tau_2) \to (X_3 , \tau_3), X_3 \subset X_2$$

.
.
.

$$f_n : (X_n , \tau_n) \to (X_{n+1} , \tau_{n+1}), X_{n+1} \subset X_n$$

Then there is an induced sequence of foldings:

$$\bar{f}_1 : G_1 \to G_2, G_2 \subset G_1$$

$$\bar{f}_2 : G_2 \to G_3, G_3 \subset G_2$$

.
.
.

$$\bar{f}_n : G_n \to G_{n+1}, G_{n+1} \subset G_n$$

and all f_i isomorphic to $\bar{f}_i$

REFERENCES

[1] R. Balakrishnam, K. Ranganathan*: A text book of Graph theory*, Springer-verlag new York, Inc (2000)

[2] M. El-Ghoul: *Folding of manifolds*, ph. D, Thesis, Tanta Univ., Egypt, (1985).

[3] M. El-Ghoul: *Unfolding of Riemannian manifold*, Commune, Fac. Sic. University of Ankara. 37, 1-4, (1988).

[4] M. El-Ghoul: *Folding of fuzzy graphs and fuzzy spheres*, Fuzzy Sets and Systems, Germany, 58, 355-363, (1993).

[5] M. El-Ghoul: *Folding of fuzzy torus and fuzzy graphs*, Fuzzy Sets and Systems, Germany, 80, 389-396, (1995).

[6] M. El-Ghoul: *The limit of folding of a manifold and its deformation retract*, Journal of Egyptian, Mathematic Society, 5, 133-140, (1997)

[7] M. El-Ghoul: *The most general set chaos graph:* Chaos, solitons and Fractals, UK, 13, 833-838, (2002).

[8] M. El-Ghoul, H.I. Attiya: *The dynamical fuzzy topological space: The* Journal of fuzzy mathematics Vol.12 No.3, Los Angeles, U.S.A, 685-693, (2004).

[9] E.M. El-Kholy: *Isometric and topological folding of manifold*, ph. D. Thesis university of South Thampton, UK (1981).

[10] S. A. Robertson: *Isometric Folding of Riemannian Manifolds*, Proc. Roy. Soc. Edinburgh, 77, 275-289, (1977).

[11] J. Robbin wll-son, J.John Watkins: *GRAPHS an introductory approach*. John Wiley & sons, Inc, (1990).

[12] A.T.White :*Graphs, groups and surfaces.* Amsterdam, North-Holland Publishing Company 1973.

[13] R.J.Wilson: *Introduction to graph theory*. Oliver& Boyed, Edinburgh 1972.

[14] R.J.Wilson; J.J.Watkins: *Graphs, an introductory approach, a first course in discrete mathematics.* Jon Wiley& Sons Inc, Canada 1990.

Source of support: Nil, Conflict of interest: None Declared