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GENERALIZATION OF ROUGH FUZZY SETS USING SEMI- APPROXIMATION

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ABSTRACT

The objective of this paper is to provide a new generalization of rough fuzzy sets in a generalized approximation space. In the present paper, a pair of fuzzy semi-lower and fuzzy semi-upper generalized approximation operators are defined by using the more familiar notion of binary relations. The connections between relations and fuzzy semi-approximation operators are examined. Moreover, some results, examples and counter examples are provided. Finally, the relationships between fuzzy semi-approximation spaces and fuzzy pretopological spaces are studied.

Keywords: *generalized approximation space, rough fuzzy set, fuzzy semi-lower approximation, fuzzy semi-upper approximation, fuzzy pretopological space.*

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1. INTRODUCTION

In Pawlak's rough set model [11], an equivalence relation is a key and primitive notion. This equivalence relation, however, seems to be a very stringent condition that may limit the application domain of the rough set model. To solve this problem, several authors have generalized the notion of approximation operators by using nonequivalence binary relations [6], [13], [20]-[22]. This has led to various other approximation operators [1], [2], [5], [8], [14], [19], [23]-[25]. More general frameworks can be obtained which involve the approximations of fuzzy sets based on crisp and fuzzy nonequivalence binary relations [7],[9],[10],[12],[15]-[18].

The present paper studies a new approach of generalization of rough fuzzy set by introducing the concepts of fuzzy semi-lower and fuzzy semi-upper approximation operators. The resulting semi-rough fuzzy sets are proper generalization of generalized rough fuzzy sets [15], [16], [18], rough fuzzy sets[4], generalized rough sets [20],[22] and Pawlak rough set[11].

2. BASICS OF ROUGH FUZZY SETS

By using the equivalence relations, Dubois and Prade introduced the notion of rough fuzzy sets as a pair of fuzzy sets resulting from the approximation of a fuzzy set in a crisp approximation space [4]. Wu *et al.* [16] generalized Dubois' concept in the case of general relations.

Definition: 2.1[16] Let $G = (U, R)$ be generalized approximation space. That is, R is an arbitrary binary relation on U. xR is the successor neighborhood of x. For any fuzzy set X, define the lower and upper approximations of X in G as follows :

$$\underline{R}(X)(x) = \min\{X(y) \mid y \in xR\}$$
$$\overline{R}(X)(x) = \max\{X(y) \mid y \in xR\}.$$

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Then $\underline{R}(X)$ and $\overline{R}(X)$ are called lower and upper approximations of fuzzy set X about the generalized approximation space. Furthermore, $\underline{R}(X)$ and $\overline{R}(X)$ are called lower and upper approximation operators from $F(U)$ to $F(U)$ (where $F(U)$ denotes all the fuzzy subsets on U). If $\underline{R}(X) = X = \overline{R}(X)$, then X is called exact fuzzy set, otherwise X is a rough fuzzy set.

The basic properties of the fuzzy lower and fuzzy upper approximations are given by the following propositions.

Proposition: 2.1[16] Let $G = (U, R)$ be a generalized approximation space, with a binary relation Ron U, then the approximation operators have the following properties for all $X, Y \in F(U)$

L_1. $\underline{R}(X) = (\overline{R}(X^c))^c$, where X^c denotes the complement of X in U .

L_2. $\underline{R}(U) = U$.

L_3. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$.

L_4. $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$.

L_5. $X \subseteq Y \Longrightarrow \underline{R}(X) \subseteq \underline{R}(Y)$.

U_1. $\overline{R}(X) = (\underline{R}(X^c))^c$.

U_2. $\overline{R}(\emptyset) = \emptyset$.

U_3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$.

U_4. $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$.

U_5. $X \subseteq Y \Longrightarrow \overline{R}(X) \subseteq \overline{R}(Y)$.

Proposition: 2.2 [16],[18] Let $G = (U, R)$ be a generalized approximation space and X be a fuzzy subset of U. Then

(1) R is serial $\Leftrightarrow LU.\ \underline{R}(X) \subseteq \overline{R}(X)$.
$\Leftrightarrow L_6.\ \underline{R}(\emptyset) = \emptyset$,
$\Leftrightarrow U_6.\ \overline{R}(U) = U$.

(2) R is reflexive $\Leftrightarrow L_7.\ \underline{R}(X) \subseteq X$,
$\Leftrightarrow U_7.\ X \subseteq \overline{R}(X)$.

(3) R is symmetric $\Leftrightarrow L_8.\ X \subseteq \underline{R}(\overline{R}(X))$,
$\Leftrightarrow U_8.\ \overline{R}(\underline{R}(X)) \subseteq X$.

(4) R is transitive $\Leftrightarrow L_9.\ \underline{R}(X) \subseteq \underline{R}(\underline{R}(X))$,
$\Leftrightarrow U_9.\ \overline{R}(\overline{R}(X)) \subseteq \overline{R}(X)$.

(5) R is Euclidean $\Leftrightarrow L_{10}.\ \overline{R}(\underline{R}(X)) \subseteq \underline{R}(X)$,
$\Leftrightarrow U_{10}.\ \overline{R}(X) \subseteq \underline{R}(\overline{R}(X))$.

Proposition: 2.3 Let $G = (U, R)$ be a generalized approximation space and X be a fuzzy subset of U. Then the following are hold

(1) If R is symmetric then

$$\underline{R}(X) = \underline{R}\left(\overline{R}\left(\underline{R}(X)\right)\right);$$
$$\overline{R}(X) = \overline{R}(\underline{R}(\overline{R}(X))) \quad .$$

(2) If R is serial and transitive then

$$\underline{R}(X) \subseteq \underline{R}\left(\overline{R}\left(\underline{R}(X)\right)\right);$$
$$\overline{R}(X) \supseteq \overline{R}(\underline{R}(\overline{R}(X))) \quad .$$

(3) If R is serial and Euclidean then

$$\underline{R}(X) \supseteq \underline{R}(\overline{R}(\underline{R}(X))) = \overline{R}(\overline{R}(\underline{R}(X))) = \overline{R}\left(\underline{R}(X)\right) = \underline{R}\left(\underline{R}(X)\right);$$
$$\overline{R}(X) \subseteq \overline{R}(\underline{R}(\overline{R}(X))) = \underline{R}(\underline{R}(\overline{R}(X))) = \underline{R}\left(\overline{R}(X)\right) = \overline{R}\left(\overline{R}(X)\right); \quad .$$

Proof: Obvious

3. SEMI-ROUGH FUZZY SETS GENERALIZATION

In this section we introduce a new generalization for the concept of rough fuzzy set by defining the fuzzy semi-lower approximation and fuzzy semi-upper approximation as follows:

Definition: 3.1 Let $G = (U, R)$ be generalized approximation space. That is, R is an arbitrary binary relation on U. For any fuzzy set X, define the fuzzy semi-lower and the fuzzy semi-upper approximations of X in G as follows:

$\underline{R_s}(X)(x) = \wedge_{y \in U}(X(y), \vee_{z \in yR}(\wedge_{w \in zR} X(w))),$

$$\overline{R}_s(X)(x) = \vee_{y\in U}(X(y), \wedge_{z\in yR}(\vee_{w\in zR} X(w))).$$

If $\underline{R}_s(X) = X = \overline{R}_s(X)$, then X is called semi-exact fuzzy set , otherwise X is a semi-rough fuzzy set.

Proposition: 3.1 Let $G = (U, R)$ be a generalized approximation space, for any fuzzy subset X of U, then

$$\underline{R}_s(X) = X \cap \overline{R}(\underline{R}(X)), \qquad \overline{R}_s(X) = X \cup \underline{R}(\overline{R}(X)) .$$

Proof: Since

$$\begin{aligned}\underline{R}_s(X)(x) &= \wedge_{y\in U}(X(y), \vee_{z\in yR}(\wedge_{w\in zR} X(w)))\\ &= \wedge_{y\in U}(X(y), \vee_{z\in yR}(\underline{R}(X)(z)))\\ &= \wedge_{y\in U}(X(y), \overline{R}(\underline{R}(X)(y)))\\ &= (X \cap \overline{R}(\underline{R}(X)))(x) .\end{aligned}$$

Therefore $\underline{R}_s(X) = X \cap \overline{R}(\underline{R}(X))$.

In the same manner we can prove $\overline{R}_s(X) = X \cup \underline{R}(\overline{R}(X))$.

Remark: 3.1 If R is an equivalence relation. There are $xR = [x]_R$, $\underline{R}(X) = \overline{R}(\underline{R}(X))$ and $\overline{R}(X) = \underline{R}(\overline{R}(X))$ hold. Then

$$\begin{aligned}\underline{R}_s(X) &= X \cap \overline{R}(\underline{R}(X))\\ &= X \cap \underline{R}(X)\\ &= \underline{R}(X)\end{aligned}$$

And

$$\begin{aligned}\overline{R}_s(X) &= X \cup \underline{R}(\overline{R}(X))\\ &= X \cup \overline{R}(X)\\ &= \overline{R}(X)\end{aligned}$$

Therefore, it is the rough fuzzy sets which are defined by Dubois [4].

And if we take X as a crisp set, we shall obtain the rough set in Pawlak approximation space [11].

Proposition: 3.2 Let $G = (U, R)$ be a generalized approximation space, for any fuzzy subsets X and Y of U, the fuzzy semi-lower and fuzzy semi-upper approximation operators on G have the following properties:

SL_1. $\underline{R}_s(X) = (\overline{R}_s(X^c))^c$.

SL_3. $\underline{R}_s(X \cap Y) \subseteq \underline{R}_s(X) \cap \underline{R}_s(Y)$.

SL_4. $\underline{R}_s(X \cup Y) \supseteq \underline{R}_s(X) \cup \underline{R}_s(Y)$.

SL_5. $X \subseteq Y \Longrightarrow \underline{R}_s(X) \subseteq \underline{R}_s(Y)$.

SL_6. $\underline{R}_s(\emptyset) = \emptyset$.

SL_7. $\underline{R}_s(X) \subseteq X$.

SU_1. $\overline{R}_s(X) = (\underline{R}_s(X^c))^c$.

SU_3. $\overline{R}_s(X \cup Y) \supseteq \overline{R}_s(X) \cup \overline{R}_s(Y)$.

SU_4. $\overline{R}_s(X \cap Y) \subseteq \overline{R}_s(X) \cap \overline{R}_s(Y)$.

SU_5. $X \subseteq Y \Longrightarrow \overline{R}_s(X) \subseteq \overline{R}_s(Y)$.

SU_6. $\overline{R}_s(U) = U$.

SU_7. $X \subseteq \overline{R}_s(X)$.

SLU. $\underline{R}_s(X) \subseteq \overline{R}_s(X)$.

Proof: The proof comes from Proposition 2.1 and Proposition 3.1 .

In the above proposition the inclusion in SL_3 and SU_3 can not be replaced by equality in general as the following example illustrating:

Example: 3.1 Let $U = \{a, b, c\}$, $R \subset (U \times U)$ be a binary relation defined as $R = \{(a,b),(a,c),(b,a),(b,b),(c,c)\}$, X and Y are fuzzy subsets of U defined as follows

$X(a) = 0.1,\ X(b) = 0.7,\quad X(c) = 0.8;$
$Y(a) = 0.8,\ Y(b) = 0.3,\quad Y(c) = 0.1.$

By easy computation it follows that

$$\underline{R}_s(X \cap Y) \neq \underline{R}_s(X) \cap \underline{R}_s(Y).$$

As similar as we have

$$\overline{R}_s(X \cup Y) \neq \overline{R}_s(X) \cup \overline{R}_s(Y).$$

The next proposition shows that the semi-lower and semi-upper approximations are proper generalization of the lower and upper approximations in the generalized approximation space.

Proposition: 3.3 Let $G = (U, R)$ be a generalized approximation space with reflexive relation, for any fuzzy subset X of U the following statements are true:
(1) $\underline{R}(X) \subseteq \underline{R}_s(X) \subseteq X \subseteq \overline{R}_s(X) \subseteq \overline{R}(X)$
(2) Every exact fuzzy set is a semi-exact fuzzy set
(3) Every semi-rough fuzzy set is a rough fuzzy set

Proof: It is straightforward.

Remark: 3.2 The converse of Proposition 3.3 need not be true. This can be shown by the following examples

Example: 3.2 Let $U = \{a, b, c, d, e\}$, $R \subset (U \times U)$ be a reflexive relation defined as
$R = \{(a,a),(a,c),(a,d),(b,b),(b,d),(b,e),(c,a),(c,b),(c,c),(d,b),(d,c),\ (d,d),(e,a),(e,c),(e,d),(e,e)\}$ and X be a fuzzy subset of U defined as follows

$$X(a) = 0.3,\ X(b) = 0.9,\ X(c) = 0.1,\ X(d) = 0.5,\ X(e) = 0.7.$$

By easy computation it follows that
$\underline{R}(X) \neq \underline{R}_s(X) \neq X$ also $X \neq \overline{R}_s(X) \neq \overline{R}(X)$.

Example: 3.3 Let $U = \{a, b, c, d, e\}$, $R \subset (U \times U)$ be a reflexive relation defined as
$R = \{(a,a),(a,c),(a,d),(a,e),(b,b),(c,a),(c,b),(c,c),(c,e),(d,a),(d,b),(d,c),\ (d,d),(e,e)\}$ and X be a fuzzy subset of U defined as follows

$$X(a) = 0.2,\ X(b) = 0.5,\ X(c) = 0.3,\ X(d) = 0.4,\ X(e) = 0.2.$$

By easy computation it follows that

$$\underline{R}_s(X) = X = \overline{R}_s(X) \quad \text{but} \quad \underline{R}(X) \neq X \neq \overline{R}(X)\ .$$

Thus X is a semi exact fuzzy set but not exact fuzzy set, also it is rough fuzzy but not semi-rough fuzzy.

Proposition: 3.4 Let $G = (U, R)$ be a generalized approximation space, then
R is serial $\Leftrightarrow SL_2.\ \underline{R}_s(U) = U,$
$\Leftrightarrow SU_2.\ \overline{R}_s(\emptyset) = \emptyset.$

Proposition: 3.5 Let $G = (U, R)$ be a generalized approximation space, then the properties $SL_9.\ \underline{R}_s(\underline{R}_s(X)) = \underline{R}_s(X)$ and $SU_9.\ \overline{R}_s(\overline{R}_s(X)) = \overline{R}_s(X)\ \forall X \in F(U)$ are hold if one of the following are hold:
(1) R is symmetric relation.
(2) R is serial and transitive relation.
(3) R is serial and Euclidean relation.

The proofs of the above two propositions come directly from Proposition 2.2 and Proposition 3.1.

Definition: 3.2 Let $G = (U, R)$ be a generalized approximation space, X and Y are two fuzzy subsets of U. If $\underline{R}_s(X) = \underline{R}_s(Y)$, then we called X is semi-lower rough fuzzy equal to Y, denote as $X \mathrel{\overset{=}{\approx}} Y$. If $\overline{R}_s(X) = \overline{R}_s(Y)$, then we called X is semi-upper rough fuzzy equal to Y, denote as $X \cong Y$. If X is both semi-lower and semi-upper rough fuzzy equal to Y, then it called X semi-rough fuzzy equal to Y, denote as $X \approx Y$.

Proposition: 3.6 Let $G = (U, R)$ be a generalized approximation space, for any fuzzy sets X and Y, they have the following properties:
(1) If $X \mathrel{\overline{\overline{\sim}}} \emptyset$ or $Y \mathrel{\overline{\overline{\sim}}} \emptyset$, then $(X \cap Y) \mathrel{\overline{\overline{\sim}}} \emptyset$.
(2) If $X \cong U$ or $Y \cong U$, then $(X \cup Y) \cong U$.
(3) If $X \subseteq Y$ and $Y \mathrel{\overline{\overline{\sim}}} \emptyset$, then $X \mathrel{\overline{\overline{\sim}}} \emptyset$.
(4) If $X \subseteq Y$ and $X \cong U$, then $Y \cong U$.

Proof: Obvious.

The next propositions examine the connection between the semi-lower and semi-upper approximation operators on the one hand and between some special types of relations on the other.

Proposition: 3.7 Let $G = (U, R)$ be a generalized approximation space with symmetric relation, for any fuzzy subset X of U the following statements are hold:
(1) $\underline{R}_s(X) = \overline{R}(\underline{R}(X))$; $\overline{R}_s(X) = \underline{R}(\overline{R}(X))$.
(2) $\underline{R}(\underline{R}_s(X)) = \underline{R}(X)$; $\overline{R}(\overline{R}_s(X)) = \overline{R}(X)$.
(3) $\underline{R}_s(\overline{R}(X)) = \overline{R}(X)$; $\overline{R}_s(\underline{R}(X)) = \underline{R}(X)$.
(4) $\overline{R}(\underline{R}(X)) \subseteq \underline{R}_s(\overline{R}_s(X)) \subseteq \underline{R}(\overline{R}(X))$; $\overline{R}(\underline{R}(X)) \subseteq \overline{R}_s(\underline{R}_s(X)) \subseteq \underline{R}(\overline{R}(X))$.

Proposition: 3.8 Let $G = (U, R)$ be a generalized approximation space with serial and transitive relation, for any fuzzy subset X of U the following statements are hold:
(1) $\underline{R}_s(\underline{R}(X)) = \underline{R}(X)$; $\overline{R}_s(\overline{R}(X)) = \overline{R}(X)$.
(2) $\underline{R}(\underline{R}_s(X)) = \underline{R}(X)$; $\overline{R}(\overline{R}_s(X)) = \overline{R}(X)$.
(3) $\underline{R}_s(\overline{R}(X)) = \overline{R}(\underline{R}(\overline{R}(X)))$; $\overline{R}_s(\underline{R}(X)) = \underline{R}(\overline{R}(\underline{R}(X)))$.
(4) $\overline{R}(\underline{R}_s(X)) = \overline{R}(\overline{R}(\underline{R}(X)))$; $\underline{R}(\overline{R}_s(X)) = \underline{R}(\underline{R}(\overline{R}(X))$.
(5) $\underline{R}(\overline{R}(X)) \subseteq \underline{R}_s(\overline{R}_s(X)) \subseteq \overline{R}(\underline{R}(\overline{R}(X); \underline{R}(\overline{R}(\underline{R}(X))) \subseteq \overline{R}_s(\underline{R}_s(X)) \subseteq \overline{R}(\underline{R}(X))$
(6) $\overline{R}_s(\underline{R}(X)) \subseteq \overline{R}_s(\underline{R}_s(X)) \subseteq \overline{R}(\underline{R}_s(X))$; $\underline{R}(\overline{R}_s(X)) \subseteq \underline{R}_s(\overline{R}_s(X)) \subseteq \underline{R}_s(\overline{R}(X))$.

Proposition: 3.9 Let $G = (U, R)$ be a generalized approximation space with serial and Euclidean relation, for any fuzzy subset X of U the following statements are hold:
(1) $\underline{R}_s(\underline{R}(X)) = \overline{R}(\underline{R}(\underline{R}(X)))$; $\overline{R}_s(\overline{R}(X)) = \underline{R}(\overline{R}(\overline{R}(X)))$.
(2) $\underline{R}(\underline{R}_s(X)) = \underline{R}(\overline{R}(\underline{R}(X)))$; $\overline{R}(\overline{R}_s(X)) = \overline{R}(\underline{R}(\overline{R}(X)))$
(3) $\underline{R}_s(\overline{R}(X)) = \overline{R}(X)$; $\overline{R}_s(\underline{R}(X)) = \underline{R}(X)$.
(4) $\overline{R}(\underline{R}_s(X)) = \overline{R}(\underline{R}(X))$; $\underline{R}(\overline{R}_s(X)) = \underline{R}(\overline{R}(X))$.
(5) $\underline{R}_s(\underline{R}_s(X)) = \underline{R}_s(X)$; $\overline{R}_s(\overline{R}_s(X)) = \overline{R}_s(X)$.

The above three propositions could be proven by from Propositions 2.1, 2.2, 2.3, 3.1 and Proposition 3.2.

In general, the inclusion can not be replaced by equality in the property (4) of Proposition 3.7 and the properties (5) and (6) of Proposition 3.8. This can be shown by the following examples:

Example:3.4 Let $U = \{a, b, c, d, e\}$, $R \subset (U \times U)$ be a symmetric relation defined as $R = \{(a,a), (a,b), (a,e), (b,a), (b,b), (b,c), (b,d), (c,b), (c,c), (d,b), (d,d), (e,a), (e,e)\}$ and X be a fuzzy subset of U defined as follows

$$X(a) = 0.5,\ X(b) = 0.2,\ X(c) = 0.7,\ X(d) = 0.1,\ X(e) = 0.8.$$

By easy computation it follows that
$\overline{R}(\underline{R}(X)) \neq \underline{R}_s(\overline{R}_s(X)) \neq \underline{R}(\overline{R}(X))$,
$\overline{R}(\underline{R}(X)) \neq \overline{R}_s(\underline{R}_s(X)) \neq \underline{R}(\overline{R}(X))$.

Example: 3.5 Let $U = \{a, b, c, d, e\}$, $R \subset (U \times U)$ be serial and transitive relation defined as $R = \{(a,a), (a,b), (a,c), (b,b), (c,c), (d,b), (d,c), (d,d), (d,e)\ (e,b), (e,e)\}$ and X be a fuzzy subset of U defined as follows

$$X(a) = 0.2,\ X(b) = 0.5,\ X(c) = 0.3,\ X(d) = 0.7,\ X(e) = 0.1.$$

By easy computation it follows that
$\underline{R}(\overline{R}(X)) \neq \underline{R}_s(\overline{R}_s(X)) \neq \overline{R}(\underline{R}(\overline{R}(X)))$, $\underline{R}(\overline{R}(\underline{R}(X))) \neq \overline{R}_s(\underline{R}_s(X)) \neq \overline{R}(\underline{R}(X))$,
$\overline{R}_s(\underline{R}(X)) \neq \overline{R}_s(\underline{R}_s(X)) \neq \overline{R}(\underline{R}_s(X)$ and $\underline{R}(\overline{R}_s(X)) \neq \underline{R}_s(\overline{R}_s(X)) \neq \underline{R}_s(\overline{R}(X))$.

4. FUZZY TOPOLOGICAL CONCEPTS ON SEMI-APPROXIMATIONS

The purpose of this section is to study the relationships between fuzzy semi-approximation operators and fuzzy pretopolgical spaces. It will be shown in this section that for a relation being serial and transitive it is enough to ensure that the semi-lower approximation and semi-upper approximation operators are fuzzy semi-interior and fuzzy semi-closure operators respectively.

Definition: 4.1[3] A fuzzy pretopological space is a pair(U, cl), where U is any set, and $cl : F(U) \to F(U)$ is a function, which verifies:
(P1) : $cl(\emptyset) = \emptyset$.
(P2) : $cl(X) \supseteq X,\ \forall X \in F(U)$.

Definition: 4.2[3] Let (U, cl) be a fuzzy pretopological space, and consider the following properties :
(P3) : $X \subseteq Y \Longrightarrow cl(X) \subseteq cl(Y),\ \forall X, Y \in F(U)$. (U, cl) is then said to be of type **I**.
(P4) : $cl(X \cup Y) = cl(X) \cup cl(Y),\ \forall X, Y \in F(U)$. (U, cl) is then said to be of type **D**.
(P5) $cl(cl(X)) = cl(X),\ \forall X \in F(U)$. (U, cl) is then said to be of type **S**.

Proposition: 4.1 Let $G = (U, R)$ be a generalized approximation space. Then $(U, \overline{R}_s)$ is a fuzzy pretopological space of type **I** if R is serial relation.

Proof:. Clear by Proposition 3.2, Proposition 3.4, Definition 4.1 and Definition 4.2.

Proposition: 4.2 Let $G = (U, R)$ be a generalized approximation space. Then $(U, \overline{R}_s)$ is a fuzzy pretopological space of type **I** and **S** if one of the following are hold:
(1) R is symmetric relation.
(2) R is serial and transitive relation.
(3) R is serial and Euclidean relation.

Proof: Obvious.

Proposition: 4.3 Let $G = (U, R)$ be a generalized approximation space. Then the operators $\underline{R}_s$ and $\overline{R}_s$ are respectively the fuzzy semi-interior and the fuzzy semi-closure operators of the fuzzy topology on U iff the relation R is serial and transitive.

Proof: Straightforward by propositions 3.2, 3.4, 3.5 and properties of fuzzy semi-interior and fuzzy semi-closure.

5. CONCLUSION

In this paper, we used topological concepts to introduce a new generalization of rough fuzzy sets .Employing the notion of binary relations has provided a definition of a pair of fuzzy semi-lower and fuzzy semi-upper generalized approximation operators. The paper examines the connections between relations and fuzzy semi-approximations operators and concludes by studying the relationships between fuzzy semi-approximation spaces and fuzzy pretopological spaces.

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