International Journal of Mathematical Archive-4(12), 2013, 4-6
Available online through www.ijma.info ISSN 2229 – 5046

SOME STUDIES ON UNIT GRAPH ASSOCIATED WITH CONNECTED RINGS

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(Received on: 17-11-13; Revised & Accepted on: 19-12-13)

ABSTRACT

Ashrafi studies the properties of the unit graph associated with rings. In this paper we present the properties of the unit graph of a connected ring. If R is a connected ring and U (R)is the set of unit elements of R, then the unit graph of R denoted by G (R) is the graph obtained by setting all the elements of R to the vertices and defining distinct vertices x and y to be adjacent if and only if $x+y \in U(R)$. *We prove that if R is a finite ring, then the following statements hold for the unit graph of R. (a) If* $2 \notin U(R)$ *then the unit graph G (R) is a* $|U(R)|$*- regular graph (b) If* $2 \in U(R)$*, then for every* $x \in U(R)$ *we have* $\deg(x) = |U(R)|-1$ *and for every* $x \in R/U(R)$ *we have* $\deg(x)= |U(R)|$.

Key words: *Connected ring, degree, unit graph, regular graph.*

INTRODUCTION

Ashrafi *et al.* [1] studied the properties of the unit graph regarding connectedness, chromatic, index, diameter, girth and planarity. In this paper we present some properties of unit graph with connected rings. Throughout this paper U (R) is the set of unit elements of R. The unit graph of R denoted by G (R), is the graph obtained by setting all the elements of R to the vertices and defining distinct vertices x and y to be adjacent if and only if $x+y \in$ U(R). If we omit the word "distinct" in the definition. We obtained the closed unit graph denoted $\bar{G}$ (R). This graph hare loops. We note that if $2 \notin U(R)$, then $\bar{G}$ (R) = G(R), then graphs in fig 1 are the unit graphs of the rings indicated. A graph in which all vertices are of the same degree is called regular graph. In an r-regular graph all the vertices are of degree r, the then number r is called the regularity of a regular graph. Regular graphs are undirected.

The graphs in Fig. 1 are the unit graphs of the rings indicated. It is easy to see that, for give rings R and S, if $R \cong S$ as rings, then $G(R) \cong G(S)$ as graphs. This point is illustrated in Fig. 2, for the unit graphs of two isomorphic rings $Z_3 \times Z_2$ and Z_6.

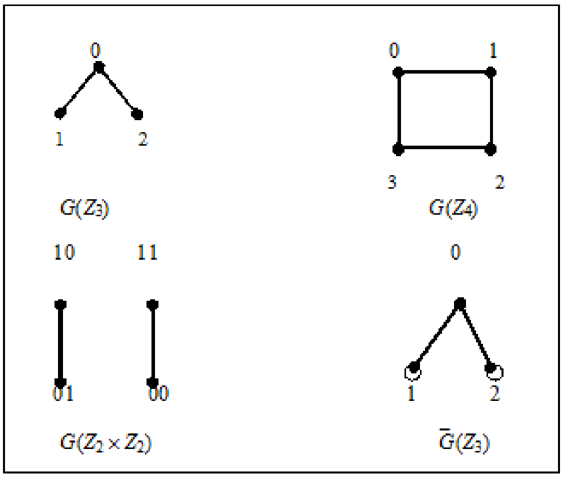

Fig. 1: The Unit graphs of some specific rings.

Next we give additional basic notation and define graph products.

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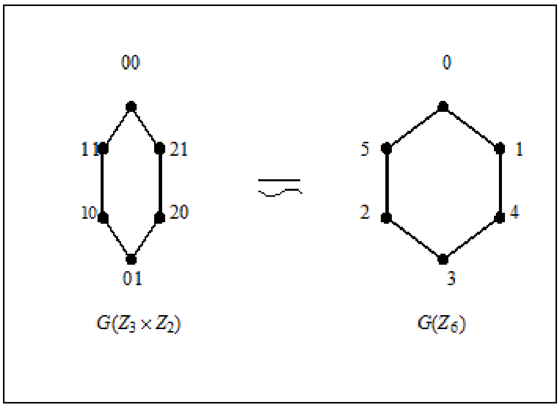

Fig. 2: The Unit graphs of two isomorphic rings.

Definition and Remarks 1: For a graph G, let $V(G)$ denote the set of vertices, and Let $E(G)$ denote the set of edges let G_1, and G_2 be two vertex – disjoint graphs. The category product of G_1 and G_2 is denoted by G_1 x G_2. That is, $V(G_1$ x $G_2) = V(G_1)$ x $V(G_2)$, two distinct vertices (x, y) and (x', y') are adjacent if and only if x is adjacent to x' in G_1 and y is adjacent to y' in G_2. Clearly, for given rings R_1 and R_2 two distinct vertices (x, y), $(x', y') \in V(\bar{G}(R_1)$ x $(\bar{G}(R_2))$ are adjacent if and only if x is adjacent to x' in $(G(R_1)$ and y is adjacent to y' in $\bar{G}(R_2)$.This implies and $(G(R_1) \times \bar{G}(R_2)) \cong G(R_1 \times R_2)$.

In Fig. 3 we illustrate the above point for the direct product of Z_2 and Z_3. We now state some basic properties of unit graphs. First we give more definitions.

Definitions 2: For a graph G and vertex $x \in V(G)$, the degree of x, denoted by deg(x), is the number of edges of G incident with x. For every non negative integer r, the graph G is called r-regular if the degree of each vertex of G is equal to r. Also, for a given vertex $x \in V(G)$, the neighbour set of x is the set $N_G(x) = \{V \in V(G) / V$ is adjacent to $x\}$.

Moreover, if G has a loop at vertex x then we always assume that $x \in N_G(x)$. The closed neighbour set of x, is the set $N_G[x] = N_G(x) \cup \{x\}$.

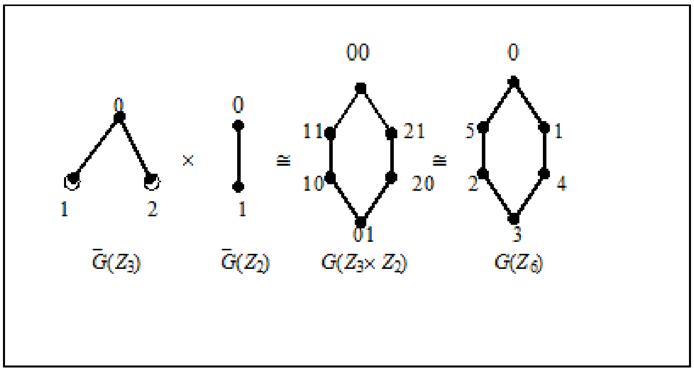

Fig. 3: The category product of two closed rings.

Theorem 1: Let R be a finite ring. Then the following statements hold for the unit graph of R.

(a) If $2 \notin U(R)$, then the unit graph $G(R)$ is a $|U(R)|$ - regular graph

(b) If $2 \in U(R)$, then for every $x \in U(R)$ we have deg $(x) = |U(R)| - 1$ and for every $x \in R/U(R)$ we have deg $(x) = |U(R)|$.

Proof: For the proof of both (a) and (b), Suppose that the vertex $x \in R$ is given. We have $R + x = R$, and so for every $u \in U(R)$ there exists an element $x_u \in R$ such that $x_u + x = u$. Clearly, x_U is uniquely determined by u.

First, suppose that $2 \notin U(R)$. In this case $x_u \neq x$ and so x_u is adjacent to x in $G(R)$. Therefore, f: $U(R) \rightarrow N_{G(R)}(x)$ give by $f(u) = x_u$ is a well – defined function. It is easy to see that f is a bijection and, therefore, $deg\ (x) = |\ N_{G(R)}(x)| = |U(R)|$. This shows that, in this case, the unit graph $G(R)$ is a $|U(R)|$ - regular graph. This completes the proof of (a).

Second, suppose that $2 \in U(R)$ and $x \in R \setminus U(R)$. In this case, we have again $x_u \neq x$, and so x_u is adjacent to x in $G(R)$.

Therefore, the above observation is still valid, which shows that $deg\ (x) = |U(R)|$.

Finally, suppose that $2 \in U(R)$ and $x \in U(R)$. In this case $2x \in U(R)$ and we have $x_u \neq x$ for $u \neq 2x$ and $x_{2x} = x$. Now x_u adjacent to x in $G(R)$ for $u \neq 2x$. Therefore, $f: U(R) \to N_{G(R)}[x]$ given by $f(u) = x_u$ is a well – defined function. It is easy to see that f is a bijection and therefore, $deg(x) = |N_{G(R)}(x)| = |N_{G(R)}[x] - 1 = |U(R)| - 1$ .Thus (b) holds.

Theorem 1 shows that, if 2 is not a unit element in the ring R, then the unit graph of R is a $|U(R)|$ – regular graph, the unit graph is not $|U(R)|$ – regular when 2 is a unit of R.

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Source of support: Nil, Conflict of interest: None Declared