International Journal of Mathematical Archive-4(10), 2013, 323-328
IJMA Available online through www.ijma.info ISSN 2229 – 5046

# MHD FLOW OF DUSTY VISCO-ELASTIC [OLDROYD (1958) TYPE] FLUID THROUGH A LONG CURVILINEAR QUADRILATERAL CHANNEL UNDER THE INFLUENCE OF UNIFORM TRANSVERSE MAGNETIC FIELD

**Gaurav Mishra, Ravindra Kumar* and K. K. Singh**

***Department of Mathematics, Agra College, Agra (U.P.), India.***

***(Received on: 02-09-13; Revised & Accepted on: 01-10-13)***

***ABSTRACT***

*In the present paper the unsteady magnetohydrodynamic flow of a dusty electrically conducting visco-elastic [Oldroyd (1958) model] fluid through a long uniform channel, whose cross-section is a curvilinear quadrilateral bounded by the arc and radii of two concentric circles under the influence of time dependent pressure gradient and a uniform magnetic field applied perpendicularly to the flow of fluid has been studied. Analytical expressions for the velocities of the conducting liquid and non-conducting dust particles are obtained by the application of integral transforms. Some particular cases for pressure gradient have been discussed in detail. The results for dusty conducting visco-elastic (Kuvshiniski type) fluid, dusty conducting visco-elastic (Rivlin-Ericksen type) fluid and purely conducting dusty viscous fluid have also been deduced by taking limits (i)* $\mu_1 \to 0$ *,(ii)* $\lambda_1 \to 0$ *and (iii)* $\mu_1 \to 0$ , $\lambda_1 \to 0$ *respectively. If magnetic field is withdrawn i.e.* $M \to 0$ *, all corresponding results can be obtained.*

## INTRODUCTION

In hydromagnetic flow of fluid we study of the flow of electrically conducting fluid in presence of Maxwell electromagnetic field. The flow of conducting fluid is effectively changed by the presence of the magnetic field and the magnetic field is also perturbed due to the motion of the conducting fluid. This phenomena is therefore interlocking in character and the discipline of this branch of science is called magnetohydrodynamics and in short written as MHD. It is equally rich and wider applications in Engineering technology, Cosmology, Astrophysics and other applied sciences.

In recent years, interests in solving the problems of flow of dusty viscous and visco-elastic fluids have increased enormously for its great importance in technological areas such as petroleum industry, environmental pollution, purification of crude oil, fluidization, soil salvation by natural winds and so on. Saffman (1962) studied the stability for the laminar flow of a dusty gas with uniform distribution of dust particles. Using Saffman’s model equations several researchers such as Bagchi and Maiti (1980); Singh and Singh (1989); Kumar and Singh (1990); Singh and Singh (1990); Chaudhary and Singh (1990, 1991); Garg, Shrivastava and Singh (1994); Johari and Gupta (1999); Das (2004, 2005); Singh, Gupta and Varshney (2005); Jadon, Jha and Yadav (2006); Singh (2009); Singh (2010); Agrawal, Agrawal and Varshney (2012); Agrawal and Singh (2012) etc. have studied the unsteady flow of dusty visco-elastic fluid though channels of various cross-sections under the influence of transverse uniform magnetic field and time dependent pressure gradient.

The aim of this paper is to discuss the hydromagnetic unsteady flow of a dusty electically conducting visco-elastic Oldroyd (1958) type fluid through a long uniform channel, whose cross-section is a curvilinear quadrilateral bounded by the arcs and radii $\theta = 0, \theta = \alpha$ , of two concentric circles $r = a, r = b, (b > a)$ under the influence of time dependent pressure gradient and variable magnetic field applied perpendicularly to the flow of fluid. Dust particles are non-conducting small in size distributed uniformly in the electrically conducting fluid. Analytical expressions for the velocity of visco-elastic fluid and dust particles are obtained by the application of integral transforms. Two particular cases for pressure gradient have been discussed in detail. The results for dusty visco-elastic Kuvshiniski type fluid, dusty visco-elastic Rivlin-Ericksen type fluid and purely dusty viscous fluid under the influence of magnetic field have also been deduced by taking limits (i) $\mu_1 \to 0$ ,(ii) $\lambda_1 \to 0$ and (iii) $\mu_1 \to 0$ , $\lambda_1 \to 0$ respectively. If magnetic field is withdrawn i.e. $M \to 0$ , all corresponding results can be obtained.

***Corresponding author: Ravindra Kumar****
***Department of Mathematics, Agra College, Agra (U.P.), India.***

**FORMULATION OF THE PROBLEM AND SOLUTION**

Consider the cylindrical-polar coordinates $(r,\theta,z)$ with the *z*-axis along the axis of the channel. Let $u_r, u_\theta, u_Z$ and $v_r, v_\theta, v_{Z,}$ be the components of fluid velocity *u* and velocity of dust particles *v* in the radial, tangential and axial directions respectively. Since dusty visco-elastic fluid flowing in the direction of *z*-axis through a long uniform channel, therefore

$$u_r = 0, \quad u_\theta = 0, \quad u_Z = u_Z(r,t)$$

$$v_r = 0, \quad v_\theta = 0, \quad v_Z = v_Z(r,t)$$

The effects due to perturbation of the flow and due to induced magnetic field have been neglected.

Following Saffman (1962) the equations of motion of dusty visco-elastic Oldroyd type fluid under the influence of time dependent pressure gradient and the uniform magnetic field applied normally to the flow of field i.e., in perpendicular direction of z-axis, are given by

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \upsilon\left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}\right)$$
$$+\frac{kN_0}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)(v_z - u_z) - \frac{\sigma B_0^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)u_z \tag{1}$$

$$m\frac{\partial v_z}{\partial t} = k(u_z - v_z) \tag{2}$$

where *p* is the fluid pressure, *k* the stoke's resistance coefficients, *m* the mass of a particle, $N_0$ the number density of the particles, $\upsilon = \left(\frac{\mu}{\rho}\right)$ the Kinematic coefficient of viscosity, $\mu$ the coefficient of viscosity, $\rho$ the density of the fluid, $B_0$ the magnetic inductivity and $\sigma$ is the electric conductivity of the fluid.

Introducing the following non-dimensional quantities:

$$u^* = \frac{a}{\upsilon}u_z,\ v^* = \frac{a}{\upsilon}v_z,\ r^* = \frac{r}{a},\ z^* = \frac{z}{a},\ t^* = \frac{\upsilon}{a^2}t,\ p^* = \frac{a^2}{\rho\upsilon^2}p,\ \lambda_1^* = \frac{\upsilon}{a^2}\lambda_1,\ \mu_1^* = \frac{\upsilon}{a^2}\mu_1$$

in equations (1) and (2), we get after dropping the stars:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$
$$+\beta\left(1+\lambda_1\frac{\partial}{\partial t}\right)(v-u) - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)u \tag{3}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\gamma}(u-v) \tag{4}$$

where $\beta = \frac{f_0}{\gamma} = \frac{N_0 k a^2}{\rho},\ \ f_0 = \frac{N_0 m}{\rho}, r = \frac{m\upsilon}{ka^2},\ \ M = B_0 a\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number).

The initial and boundary conditions are:

$$t = 0,\ u(r,\theta,t) = 0 = v(r,\theta,t) \tag{5}$$

$$\left.\begin{aligned} t > 0,\ \ & u(r,\theta,t) = 0 = v(r,\theta,t) \ \ for\ \ r = 1 \ \ and\ r = \frac{b}{a} = c > 1,\ 0 \le \theta \le \alpha \\ & u(r,\theta,t) = 0 = v(r,\theta,t) \ \ for\ \ \theta = 0 \ \ and\ \theta = \alpha,\ 1 \le r \le c \end{aligned}\right\} \tag{6}$$

Let $\theta = \frac{\alpha}{\pi}\phi$ and $-\frac{\partial p}{\partial z} = f(t)$, equations (3) and (4) reduced to

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \left(1+\lambda_1\frac{\partial}{\partial t}\right) f(t) + \left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\pi^2}{\alpha^2 r^2}\frac{\partial^2 u}{\partial \varphi^2}\right)$$
$$+\beta\left(1+\lambda_1\frac{\partial}{\partial t}\right)(v-u) - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)u \tag{7}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\gamma}(u-v) \tag{8}$$

The conditions (5) and (6) become

$$t = 0,\ u(r,\varphi,t) = 0 = v(r,\varphi,t) \tag{9}$$

$$\left.\begin{aligned} t > 0,\ \ & u(r,\varphi,t) = 0 = v(r,\varphi,t) \ \ for\ \ r = 1 \ \ and\ r = c,\ 0 \le \varphi \le \pi \\ & u(r,\varphi,t) = 0 = v(r,\varphi,t) \ \ for\ \ \varphi = 0 \ \ and\ \varphi = \pi,\ 1 \le r \le c \end{aligned}\right\} \tag{10}$$

For solving equation (7) and (8) apply finite Fourier sine transform

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial \bar{u}}{\partial t} = \frac{2}{q_n}\left(1+\lambda_1\frac{\partial}{\partial t}\right) f(t) + \left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial r} - \frac{N^2}{r^2}\bar{u}\right)$$
$$+\beta\left(1+\lambda_1\frac{\partial}{\partial t}\right)(\bar{v}-\bar{u}) - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)\bar{u}$$

Or

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial \bar{u}}{\partial t} = \frac{2}{q_n}\left(1+\lambda_1\frac{\partial}{\partial t}\right) f(t) + \left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial r} - \frac{N^2}{r^2}\bar{u}\right)$$
$$+\beta\left(1+\lambda_1\frac{\partial}{\partial t}\right)(\bar{v}-\bar{u}) - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)\bar{u} \tag{11}$$

$$\frac{\partial \bar{v}}{\partial t} = \frac{1}{\gamma}(\bar{u}-\bar{v}) \tag{12}$$

where

$\bar{u} = \int_0^\pi u(r,\phi,t)\sin q_n \phi d\phi$ [Finite Fourier Sine transform]

$q_n = (2n+1),\ \ N^2 = \frac{\pi^2}{\alpha^2} q_n^2$

 

Now taking finite Hankel transform of equations (11) and (12) subject to boundary conditions $\bar{u}_H = 0$, $\bar{v}_H = 0$ $at$ $t = 0$, we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial \bar{u}_H}{\partial t} = \frac{2}{q_n}\left(1+\lambda_1\frac{\partial}{\partial t}\right) f(t)\int_1^c rB_N(\xi_i r)\,dr - \xi_i^2\left(1+\mu_1\frac{\partial}{\partial t}\right)\bar{u}_H + \beta\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\bar{v}_H - \bar{u}_H\right) - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)\bar{u}_H \tag{13}$$

$$\frac{\partial \bar{v}_H}{\partial t} = \frac{1}{\gamma}\left(\bar{u}_H - \bar{v}_H\right) \tag{14}$$

where

$\bar{u}_H = \int_1^c \bar{u}\, r\, B_N(\xi_i r)dr,\quad \bar{v}_H = \int_1^c \bar{v}\, r\, B_N(\xi_i r)dr$ [Finite Hankel transform]

$B_N(\xi_i r) = J_N(\xi_i r)Y_N(\xi_i) - J_N(\xi_i)Y_N(\xi_i r),$ $\xi_i$ are the roots of the equation:

$J_N(\xi_i c)Y_N(\xi_i) - J_N(\xi_i)Y_N(\xi_i c) = 0$

and $J_N(\xi_i r), Y_N(\xi_i r)$ are the Bessel functions of first and second kinds of order *N* respectively.

Taking Laplace transform of equations (13) and (14), we obtain

$$(1+\lambda_1 s)s\bar{\bar{u}}_H = \frac{2}{q_n}(1+\lambda_1 s)\bar{f}(s)\zeta - \xi_i^2(1+\mu_1 s)\bar{\bar{u}}_H + \beta(1+\lambda_1 s)\left(\bar{\bar{v}}_H - \bar{\bar{u}}_H\right) - M^2(1+\lambda_1 s)\bar{\bar{u}}_H \tag{15}$$

$$s\bar{\bar{v}}_H = \frac{1}{\gamma}\left(\bar{\bar{u}}_H - \bar{\bar{v}}_H\right) \tag{16}$$

where $\bar{\bar{u}}_H$, $\bar{\bar{v}}_H$ and $\bar{f}(s)$ are the Laplace transforms of $\bar{u}_H$, $\bar{v}_H$ and *f(t)* respectively, and

$$\zeta = \int_1^c r\, B_N(\xi_i r)dr$$

Solving equations (15) and (16) for $\bar{\bar{u}}_H$ $and$ $\bar{\bar{v}}_H$, we get

$$\bar{\bar{u}}_H = \frac{2\zeta(1+\gamma s)(1+\lambda_1 s)\bar{f}(s)}{q_n\left\{\lambda_1\gamma s^3 + \left(\lambda_1+\gamma+\mu_1\gamma\xi_i^2+\beta\gamma\lambda_1+\gamma\lambda_1 M^2\right)s^2 + \left(1+\gamma\xi_i^2+\mu_1\xi_i^2+\gamma\beta+\lambda_1 M^2+\gamma M^2\right)s + \xi_i^2 + M^2\right\}} \tag{17}$$

$$\bar{\bar{v}}_H = \frac{\bar{\bar{u}}_H}{1+\gamma s} \tag{18}$$

Now to obtain u and v from equation (17) and (18) we first invert Laplace transform by inversion theorem, apply inversion formula of Hankel transform and then inversion of finite Fourier sine transform, we get velocities of fluid and the dust particles:

$$u = \frac{8}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\zeta\xi_i^2 J_N^2(\xi_i c)B_N(\xi_i r)}{q_n\left\{J_N^2(\xi_i) - J_N^2(\xi_i c)\right\}}\sin q_n\phi\, Q(S_n^j)\int_0^t e^{s_n^i\lambda} f(t-\lambda)d\lambda \tag{19}$$

 

$$v=\frac{8}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\zeta\xi_i^2 J_N^2(\xi_i c)B_N(\xi_i r)}{q_n\left\{J_N^2(\xi_i)-J_N^2(\xi_i c)\right\}}\sin q_n\phi R(S_n^j)\int_0^t e^{s_n^i} f(t-\lambda)d\lambda \tag{20}$$

where $S_n^j$ are the roots of cubic equation:

$$\lambda_1\gamma(S_n^j)^3+(\lambda_1+\gamma+\mu_1\gamma\xi_i^2+\beta\gamma\lambda_1+\gamma\lambda_1 M^2)(S_n^j)^2+(1+\gamma\xi_i^2+\mu_1\xi_i^2+\gamma\beta+\lambda_1 M^2+\gamma M^2)(S_n^j)+\xi_i^2+M^2=0$$

$$Q\left(S_n^j\right)=\frac{(1+\lambda_1 S_n^j)(1+\gamma S_n^j)}{3\lambda_1\gamma(S_n^j)^2+2(\lambda_1+\gamma+\mu_1\gamma\xi_i^2+\beta\gamma\lambda_1+\gamma\lambda_1 M^2)(S_n^j)+(1+\gamma\xi_i^2+\mu_1\xi_i^2+\gamma\beta+\lambda_1 M^2+\gamma M^2)}$$

$$R\left(S_n^j\right)=\frac{(1+\lambda_1 S_n^j)}{3\lambda_1\gamma(S_n^j)^2+2(\lambda_1+\gamma+\mu_1\gamma\xi_i^2+\beta\gamma\lambda_1)(S_n^j)+(1+\gamma\xi_i^2+\mu_1\xi_i^2+\gamma\beta)}$$

**PARTICULAR CASES**

**I.** When pressure gradient is constant i.e. $f(t)=C$, From equations (19) and (20), we get

$$u=\frac{8C}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\zeta\xi_i^2 J_N^2(\xi_i c)B_N(\xi_i r)}{q_n\left\{J_N^2(\xi_i)-J_N^2(\xi_i c)\right\}}\sin q_n\phi Q(S_n^j)\left\{\frac{e^{(s_n^j)t}-1}{s_n^j}\right\} \tag{21}$$

$$v=\frac{8C}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\zeta\xi_i^2 J_N^2(\xi_i c)B_N(\xi_i r)}{q_n\left\{J_N^2(\xi_i)-J_N^2(\xi_i c)\right\}}\sin q_n\phi R(S_n^j)\left\{\frac{e^{(s_n^j)t}-1}{s_n^j}\right\} \tag{22}$$

**II.** When pressure gradient is exponentially decreasing function of time i.e.

$f(t)=Ce^{-\omega t}$.

From equations (19) and (20), we get

$$u=\frac{8C}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\zeta\xi_i^2 J_N^2(\xi_i c)B_N(\xi_i r)}{q_n\left\{J_N^2(\xi_i)-J_N^2(\xi_i c)\right\}}\sin q_n\phi Q(S_n^j)\left\{\frac{e^{(s_n^j)t}-e^{-\omega t}}{s_n^j+\omega}\right\} \tag{23}$$

$$u=\frac{8C}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\zeta\xi_i^2 J_N^2(\xi_i c)B_N(\xi_i r)}{q_n\left\{J_N^2(\xi_i)-J_N^2(\xi_i c)\right\}}\sin q_n\phi R(S_n^j)\left\{\frac{e^{(s_n^j)t}-e^{-\omega t}}{s_n^j+\omega}\right\} \tag{24}$$

**DEDUCTIONS**

(i) Taking limit $\mu_1\to 0$, all expressions for conducting dusty visco-elastic Kuvshiniski type fluid are obtained.

(ii) Taking limit $\lambda_1\to 0$, all expressions for conducting dusty visco-elastic Rivlin-Ericksen fluid are obtained.

(iii) Taking limit $\mu_1\to 0$, $\lambda_1\to 0$, all expressions for purely conducting dusty viscous fluid are obtained.

(iv) Taking limit $B_0\to 0$ i.e. magnetic field withdrawn, all expressions for dusty visco-elastic Oldroyd type fluid can be obtained.

(v) If mass of dust particles are very small, their influence on the fluid flow is reduced i.e. if limit $m\to 0$, all the corresponding results are obtained for visco-elastic Oldroyd type fluid in presence of variable magnetic field.

**REFERENCES**

[1] Bagchi, S. and Maiti, M.K. (1980): Acta Ciencia Indica, Vol. VI, No.3, p-130.

[2] Chaudhary, R.K.S. and Singh, K.K. (1990): Jour. MACT, Vol.23, p-23. (1991): Proc. Nat. Acad. Sci. India, Vol.61, Sec.A, Part II, p-233.

[3] Das, K.K. (2004): Ind. Jour. Theo. Phy., Vol.52, No.1, p-31. (2005): Ind. Jour. Theo. Phy., Vol.54, No.2, p-145.

[4] Garg, A., Shrivastava, R.K. and Singh, K.K. (1994): Proc. Nat. Acad. Sci. India, Vol. 64(A), Part III, p-355.

[5] Jadon, V.K., Jha, R. and Yadav, S.S. (2006): Acta Ciencia Indica, Vol. XXXII M, No.4, p-1713.

[6] Johri, R. and Gupta, G.D. (1999): Acta Ciencia Indica, Vol. XXV M, No.1, p-275.

[7] Kumar, G. and Singh, K.K. (1990): Jour. MACT, Vol.23, p-133. (1990): Pure Appl. Mathematika Sci., Vol. XXXII, No.1-2, p-65.

[8] Saffman, P.G. (1962): Jour. Fluid Mech., Vol.13, p-120.

[9] Sharma, J.P. and Singh, K.K. (1990): The Mathematics Edu., Vol. XXIV, No.1, p-13.

[10] Singh. J., Gupta, C.B. and Varshney, N.K. (2005): Ind. Jour. Theo. Phy., Vol.53, No.3, p-257.

[11] Singh, K. and Singh, K.K.(1989): Bull. Cal. Math. Soc., Vol.80, p-286.

[12] Singh, K.K. and Singh, D.P. (1990): Jour. Agri. Sci. Res., Vol. 32, p-83.

[13] Singh, N. (2009): Ph.D. Thesis, Dr. B. R. A. Univ. Agra, p-174.

[14] Singh, P. (2010): Ind. Jour. Theo. Phys., Vol.58, No.1, p-27.

**Source of support: Nil, Conflict of interest: None Declared**

 