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NEAR Z_{4p} - MAGIC LABELING

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ABSTRACT

For any non-trivial abelian group(moduloZ_4,+), a graph G(V,E) is said to be Z_4 –magic if there exists a function f from E(G) into Z_4-{0} where '0' is the additive identity element of modulo Z_4, induce a mapping f^+ from V(G) into Z_4 such that $f^+(v)$=$\sum f(uv)$ is a constant for all vertices $v \in V$. If $f^+(v)$=$\sum f(uv)$ is a constant for almost all vertices $v \in V$ and for one or atmost two vertices of v, $f^+(v)$ is not the same constant where the summation is taken over all the edges uv incident at v, then the labeling is called near Z_4 – magic labeling and the graph which admits near Z_4-magic labeling is called near Z_4 -magic graph. At the end we generalize near Z_4 -magic labeling into near Z_{4p}- magic labeling.

Mathematics Subject Classification 2000: *05C78.*

***Keywords**: NearZ_4 -magic labeling, near Z_4-magic graph and nearZ_{4p} –magic graph.*

1. INTRODUCTION

Throughout this paper by a graph *G(V,E)* we mean a finite, simple, undirected graph. Magic labeling were introduced by Sedlacek in 1963 Kong, Lee and Sun[4] used the term magic labeling for the labeling of edges with non-negative integers such that for each vertex v, the sum of labels of all edges incident at v is same for all v in particular the edge labels need not be distinct. For the abelian group (modulo $Z_4, +$), a graph G is said to be Z_4-magic if there exists a labeling f of the edges of G with non-zero elements of Z_4 which induce the vertex labeling f^+ such that $f^+(v)$=$\sum f(uv)$ where the summation is taken over all edges uv incident at v is a constant.

In near Z_4 magic graph $f^+(v)$=$\sum f(uv)$ where the summation is taken over all edges uv incident at v is a constant, for almost all vertices v of G and for one or atmost two vertices of G, $f^+(v)$ is not the same constant. It is different from the anti magic labeling which was introduced in 1990 by Hartsfield and Ringel [5].

2. BASIC DEFINITIONS

Definition 2.1: A walk of a graph G is an alternating sequence of vertices and edges $v_{0,}e_1, v_1, e_2, \dots, v_{n-1}, e_n v_n$ beginning and ending with vertices in which each edges incident with two vertices immediately proceeding and succeeding it. The number 'n' is called the length of the walk.

Definition 2.2: A Walk in which all the vertices are distinct is called a path.

Definition 2.3: If the end two vertices are same in a path it is called as a cycle. A path having 'n' vertices and n-1 length is denoted as P_n here. By C_n we mean a cycle consisting of 'n' vertices and 'n' edges.

Definition 2.4 [5]: A bipartite graph (or bigraph) G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins V_1 with V_2.

Definition 2.5: If G contains every line joining V_1 and V_2 then G is called complete bipartite graph. If V_1 and V_2 have m and n vertices, we write *G* =$K_{m,n}$.

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3. MAIN RESULTS

Theorem 3.1: P_n is near Z_4-magic graph $n \geq 4$.

Proof:
Case 1: n is even

Let $f: E(G) \rightarrow Z_4 - \{0\}$ be defined as

$f(u_{2i-1}u_{2i})$=1, $1\leq i \leq \frac{n}{2}$ and $f(u_{2i}u_{2i+1})$= 2, $1\leq i \leq \frac{n}{2} - 1$

Now, $f^+: V(G) \rightarrow Z_4$

We get $f^+(u_i) = f(u_{i-1}u_i) + f(u_iu_{i+1}) \equiv 3(mod4) = 3$, $2\leq i \leq n-1$

$f^+(u_1) = 1 = f^+(u_n)$

Here, only two vertices namely u_1 and u_n get a different constant.

Case 2: n is odd

Let $f: E(G) \rightarrow Z_4 - \{0\}$ be defined as

$f(u_{2i-1}u_{2i})$=1, $1\leq i \leq \frac{n-1}{2}$ and $f(u_{2i}u_{2i+1})$= 2, $1\leq i \leq \frac{n-1}{2}$

Now, $f^+: V(G) \rightarrow Z_4$

We get, $f^+(u_i) = f(u_{i-1}u_i) + f(u_iu_{i+1}) \equiv 3(mod4) = 3$, $2\leq i \leq n-1$

$f^+(u_1)$ = 1 and $f^+(u_n) = 2$

The vertices u_1 and u_n get a different constant.

Hence P_n admits near Z_4 - magic labeling and so P_n is near Z_4 – magic graph for $n \geq 4$.

Example 3.2: For n=5, 6 the near Z_4 - magic labeling of P_n are given below.

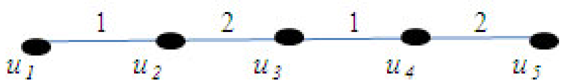

Fig – 1: *Near Z_4* – magic labeling of P_5

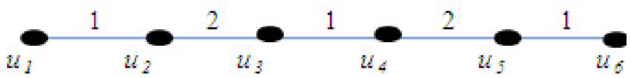

Fig – 2: *Near Z_4* – magic labeling of P_6

Theorem 3.3: C_n is near Z_4 - magic for n$\equiv 1(mod2)$

Proof: Let $f: E(C_n) \rightarrow Z_4 - \{0\}$ be defined as

$f(u_{2i-1}u_{2i})$=2, $1\leq i \leq \frac{n-1}{2}$ and $f(u_{2i}u_{2i+1})$=3, $1\leq i \leq \frac{n-1}{2}$

$f(u_nu_1)$=2.

Now, $f^+: V(C_n) \rightarrow Z_4$

We get, $f^+(u_i) = f(u_{i-1}u_i) + f(u_iu_{i+1}) \equiv (2+3)(mod4) = 1$, $2 \leq i \leq n$
$f^+(u_1) = f(u_1u_2) + f(u_nu_1) \equiv (2+2)(mod4) = 0$

Here only one vertex u_1 gets a different constant zero.

Therefore, C_n is near Z_4 - magic for $n \equiv 1(mod2)$.

Note 3.4 $u_{n+1} \equiv u_1$ (here).

Example 3.5:

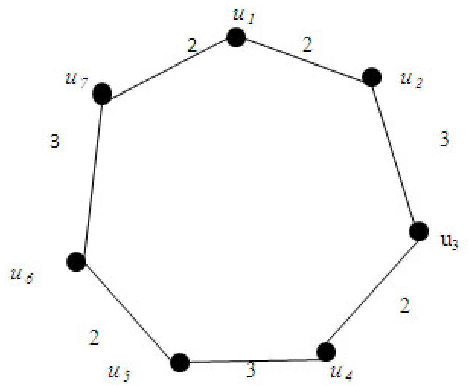

Fig – 3: *Near Z_4* – magic labeling of C_7

Remark 3.6: 1. If f(E(C_n)) is a constant in $Z_4 - \{0\}$ then C_n is Z_4 – magic for all n. Hence Z_{4p} - magic 2. C_n ,n $\equiv 0(mod2)$ is a Z_4 – magic graph for the labeling given in the theorem3.3.So it is Z_{4p} - magic

Definition3.7 [6]: The join G_1+G_2 of G_1 and G_2 consists of $G_1 \cup G_2$ and all edges joining V_1 with V_2. The graph P_n+K_1 is called a fan and it denoted as F_n.

Theorem 3.8: F_n is near Z_4 – magic graph, n≥ 3

Proof:

Case 1: n$\equiv 0,2,3(mod4)$

V(G) = { u} $\cup \{v_i/1 \leq i \leq n\}$ and E(G)=$\{uv_i/1 \leq i \leq n\} \cup \{v_i v_{i+1}/1 \leq i \leq n-1\}$

Let $f: E(F_n) \rightarrow Z_4 - \{0\}$ be defined as

$f(v_i v_{i+1})$=1, 1$\leq i \leq n-1$ and $f(uv_i)$=1, 1$\leq i \leq n-1$

$f(uv_1)$= $f(uv_n)$ = 2.

$f^+(v_i)$ =Now $f^+: V(F) \rightarrow Z_4$

$f^+(v_1)$= $f(uv_1) + f(v_1 v_2)$

$f^+(v_1) \equiv (2+1)(mod4) = 3 = f^+(v_n)$

$f^+(v_i) = f(v_i v_{i+1}) + f(v_{i-1} v_i) + f(uv_i)$

$f^+(v_i) \equiv (1+1+1)(mod\ 4)$ =3, 2$\leq i \leq n-1$

$f^+(u) = 2,\ for\ n \equiv 0(mod4)$

$f^+(u) = 0, for\ n \equiv 2(mod4)$

$f^+(u) = 1, for\ n \equiv 3(mod4)$

Here, only one vertex u gets a different constant.

Case 2: n ≡1(mod4)

Let $f: E(F_n) \to Z_4 - \{0\}$ be defined as

$f(v_{2i-1}v_{2i})$=2, $1 \leq i \leq \frac{n-1}{2}$ and $f(v_{2i}v_{2i+1})$= 1, $1 \leq i \leq \frac{n-1}{2}$

$f(uv_i)$=2, $2 \leq i \leq n-1$

$f(uv_1) = f(uv_n) = 3$

Now $f^+: V(F_n) \to Z_4$

$f^+(v_1) = f(uv_1) + f(v_1v_2)$

$f^+(v_1) \equiv (3+2)(mod4) = 1$

$f^+(v_i) \equiv (2+1+2)(mod4) = 1, \; 2 \leq i \leq n-1$

$f^+(v_n) \equiv (3+1)(mod4) = 0$

$$f^+(u) = \sum_{i=1}^{n} f(uv_i)$$

$$\equiv (3+2+2+\cdots+(n-2)times\ 2+3)(mod4) = 0$$

Only two vertices v_n and u get a different constant.

Therefore, F_n admits near Z_4 – magic labeling. for n≥ 3 .

Example 3.9: We shall verify near Z_4 -magic labeling for n=3, 4, 5.

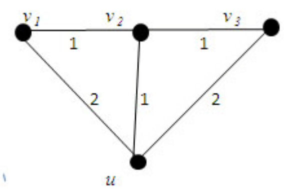

Fig – 4: *Near Z_4* – magic labeling of F_3

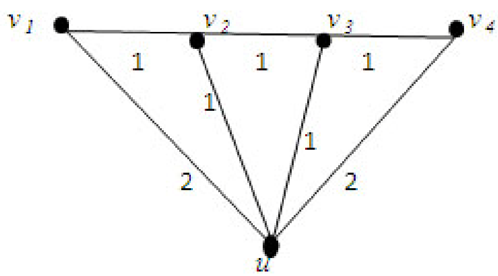

Fig – 5: *Near Z_4* – magic labeling of F_4

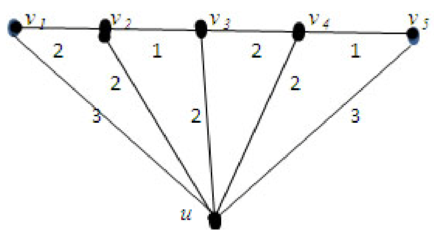

Fig – 6: *Near Z_4* – magic labeling of F_5

Definition 3.10[2]: The graph C_n+K_1 is called a wheel and it is denoted as W_n.

Theorem 3.11: W_n is near Z_4 –magic graph, n≥ 3 .

Proof:
$V(W_n) = \{u\} \cup [v_i/1 \leq i \leq n]$

$E(W_n) = \{uv_i/1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\}$

Case 1: n$\equiv 1(mod\ 2)$

Let $f: E(W_n) \rightarrow Z_4 - \{0\}$ be defined as

$f(v_i v_{i+1})$=1, 1$\leq i \leq n-1$

$f(v_n v_1)$= 1

$f(uv_i)$=2, 1$\leq i \leq n$

Now $f^+: V(W_n) \rightarrow Z_4$

$f^+(v_i) = f(v_i v_{i+1}) + f(uv_i) + f(v_{i-1} v_i)$ 1$\leq i \leq n$

$f^+(v_i) \equiv (2\ times\ 1 + 2)(mod 4)$ =0, 1$\leq i \leq n$ ($v_0 = v_n$ and $v_{n+1} = v_1$)

$$f^+(u) = \sum_{i=1}^{n} f(uv_i)$$

$$\equiv (2 + 2 + \cdots n\ times\ 2)(mod 4) = 2$$

The only vertex *u* gets a different constant in this case.

Case 2: n$\equiv 0(mod\ 2)$ n≥ 3 .

Let $f: E(W_n) \rightarrow Z_4 - \{0\}$ be defined as

$f(v_i v_{i+1})$ =1, 1$\leq i \leq n-1$

$f(v_n v_1)$= 1

$f(uv_i)$=3, 1$\leq i \leq n$

$f^+: V(W_n) \rightarrow Z_4$

We get $f^+(v_i) = f(v_i v_{i+1}) + f(uv_i) + f(v_{i-1} v_i)$, 1$\leq i \leq n$ ($v_0 = v_n$ and $v_{n+1} = v_1$)

$f^+(v_i) \equiv (2\ times\ 1 + 3)(mod 4)$ =1, 1$\leq i \leq n$

$$f^+(u) = \sum_{i=1}^{n} f(uv_i)$$

$$\equiv (3 + 3 + \cdots n\ times\ 3)(mod 4) = 2 \quad \text{for } n \equiv 2(mod 4)$$
$$= 0 \quad \text{for } n \equiv 0(mod 4)$$

Here vertex *u* gets a different constant .

Therefore in both the cases W_n admits near Z_4 –magic labeling.

Hence, W_n is near Z_4 –magic graph

Example 3.12:

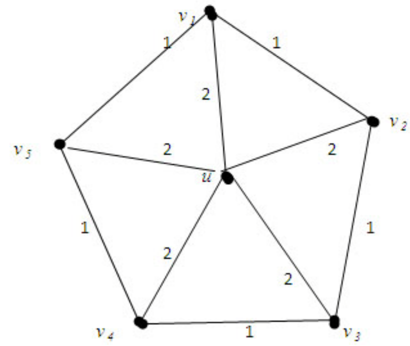

Fig – 7: *Near* Z_4 – magic labeling of W_5

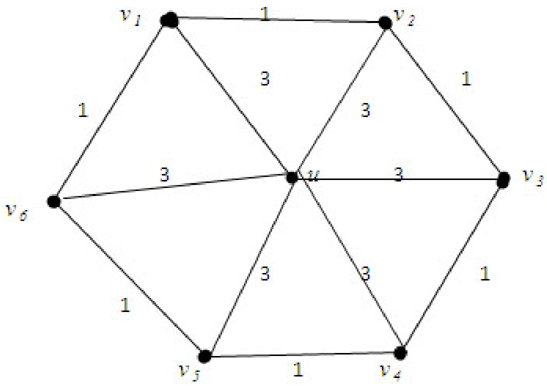

Fig – 8: *Near* Z_4 – magic labeling of W_6

Definition 3.13[6]: A Complete bigraph $K_{1,n}$ is called a star.

Theorem 3.14: The star graph $K_{1,n}$ is near Z_4 –magic for n$\equiv$ $0,2,3(mod4)$

Proof: Let *u* be the centre vertex and $v_1, v_2, \ldots, v_n$ be the pendent vertices of the star.

Let Let $f: E(k_{1,n}) \to Z_4 - \{0\}$ be defined as

$f(uv_i)$=1, 1$\leq i \leq n$

$f^+: V(k_{1,n}) \to Z_4$

We get $f(v_i)$=1, 1$\leq i \leq n$

$$f^+(u) = \sum_{i=1}^{n} f(uv_i)$$

$\equiv (1 + 1 + \cdots n \; times \; 1)(mod4)$ =2 for n$\equiv 2(mod4)$
= 0 for n$\equiv 0(mod4)$
= 3 for n$\equiv 3(mod4)$

The only vertex *u* gets a different constant for different values of n.

Therefore the star graph $k_{1,n}$ is near Z_4 –magic graph.

Remark 3.15: $k_{1,n}$ is Z_4 –magic graph for n$\equiv$ $1(mod4)$. Hence it is Z_{4p} - magic

Example 3.16: For n=4, 6, 7 the near Z_4 –magic labeling of $k_{1,n}$ are given

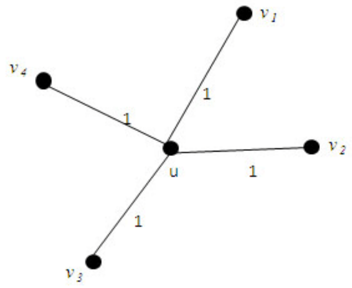

Fig – 9: *Near Z_4* – magic labeling of $K_{1,4}$

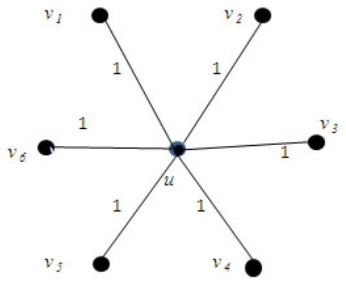

Fig – 10: *Near Z_4* – magic labeling of $K_{1,6}$

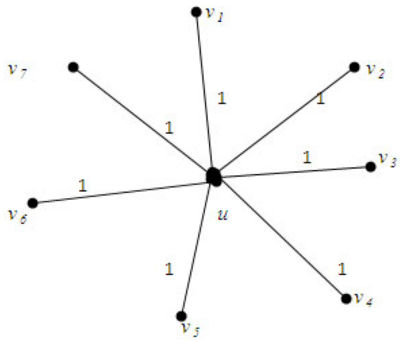

Fig – 11: *Near Z_4* – magic labeling of $K_{1,7}$

Definition 3.17[3]: A graph is called an *(n, t)*- kite if a path of length t attached to one vertex of the cycle C_n.

Theorem 3.18: An *(n, t)* - kite is near Z_4 –magic for n≥ 3 and t≥ 1

Proof: Let G be (*n*, *t)* - kite

V(G)=$\{u_i/1 \leq i \leq t\} \cup \{v_i/1 \leq i \leq n\}$

E(G)= $\{u_iu_{i+1}/1 \leq i \leq t-1\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{v_nv_1\} \cup \{u_1v_1\}$

Case 1: *n* is odd and *t* is odd

Let $f: E(G) \rightarrow Z_4 - \{0\}$ be defined as

$f(v_{2i-1}v_{2i})$=1, 1≤ $i \leq \frac{n-1}{2}$

$f(v_{2i}v_{2i+1})$= 2, 1≤ $i \leq \frac{n-1}{2}$

$f(v_nv_1)$= 1

$f(v_1u_1)$=1

$f(u_{2i-1}u_{2i})$ =2, 1≤ $i \leq \frac{t-1}{2}$

$f(u_{2i}u_{2i+1})$= 1, 1≤ $i \leq \frac{t-1}{2}$

hence $f^+: V(G) \rightarrow Z_4$

We get $f^+(u_i)$ = $f(u_{i-1}u_i) + f(u_iu_{i+1})$

$\equiv (1+2)$ $(mod4)$ = 3, 2≤ $i \leq t-1$

$f^+(v_i)$ = $f(v_{i-1}v_i) + f(v_iv_{i+1})$

$\equiv (1+2)(mod4)$ = 3, or

$\equiv (2+1)$ $(mod4)$ = 3, 2≤ $i \leq n$ (since v_{n+1}= v_1)

$f^+(v_1)$ = $f(v_1v_2) + f(v_nv_1) + f(u_1v_1)$

$\equiv (1+1+1)(mod4)$ = 3

$f^+(u_1)$ = $f(u_1v_1) + f(u_1u_2)$

$\equiv (1+2)(mod4)$ = 3

$f^+(u_t)$ =1

u_t alone gets a different constant 1 and all the other vertices v$\epsilon V(G)$ get the unique constant 3.

Case 2: *n* is odd and *t* is even

Let $f: E(G) \rightarrow Z_4 - \{0\}$ be defined as

$f(v_{2i-1}v_{2i})$=1, 1≤ $i \leq \frac{n-1}{2}$

$f(v_{2i}v_{2i+1})$= 2, 1≤ $i \leq \frac{n-1}{2}$

$f(v_nv_1)$= 1.

$f(u_1v_1)$ =1

$f(u_{2i-1}u_{2i})$ =2, 1≤ $i \leq \frac{t}{2}$

$f(u_{2i}u_{2i+1})$ = 1, 1≤ $i \leq \frac{t-2}{2}$

Then $f^+: V(G) \rightarrow Z_4$ is given by

We get $f^+(u_i)$ = $f(u_{i-1}u_i) + f(u_iu_{i+1})$

$\equiv (1+2)$ $(mod4)$ = 3 , 2≤ $i \leq t-1$

$f^+(v_i)$ = $f(v_{i-1}v_i) + f(v_iv_{i+1})$

$\equiv (1+2)(mod4)$ = 3 (or)

$\equiv (2+1)$ $(mod4)$ = 3, 2≤ $i \leq n$ (since v_{n+1}=v_1)

$f^+(v_1)$ = $f(v_1v_2) + f(v_nv_1) + f(u_1v_1)$

$\equiv (1+1+1)(mod4)$ = 3

$f^+(u_1) = f(u_1v_1) + f(u_1u_2)$

$\equiv (1+2)(mod4) = 3$

$f^+(u_t) \equiv 2(\text{mod}4)$

$= 2$

u_t alone gets a different constant 2 and all the other vertices v ϵ $V(G)$ get the same constant 3.

Case 3: *n* is even and *t* is odd

Let $f: E(G) \to Z_4 - \{0\}$ be defined as

$f(v_{2i-1}v_{2i})$=3, 1≤ $i \le \frac{n}{2}$

$f(v_{2i}v_{2i+1})$= 2, 1≤ $i \le \frac{n}{2}$ (v_{n+1}=v_1)

$f(u_1v_1)$= 3,

$f(u_{2i-1}u_{2i})$ =2, 1≤ $i \le \frac{t-1}{2}$

$f(u_{2i}u_{2i+1})$ = 3, 1≤ $i \le \frac{t-1}{2}$

Then $f^+: V(G) \to Z_4$ is given by

We get $f^+(u_i) = f(u_{i-1}u_i) + f(u_iu_{i+1})$

$\equiv (2+3)(mod4) = 1$, 2≤ $i \le t-1$

$f^+(v_i) = f(v_{i-1}v_i) + f(v_iv_{i+1})$ 2≤ $i \le n$

$\equiv (3+2)(mod4) = 1$ (or)

$\equiv (2+3)(mod4) = 1$, 2≤ $i \le n$ (since v_{n+1}=v_1)

$f^+(v_1) = f(v_1v_2) + f(v_nv_1) + f(u_1v_1)$

$\equiv (3+2+3)(mod4) = 0$

$f^+(u_1) = f(u_1v_1) + f(u_1u_2)$

$\equiv (3+2)(mod4) = 1$

$f^+(u_t)$ =3

In this case, two vertices namely v_1 and u_t get different constants 0 and 3. All vertices except these two get the same constant 1

Case 4: *n* is even and *t* is even

Let $f: E(G) \to Z_4 - \{0\}$ be defined as

$f(v_{2i-1}v_{2i})$=3, 1≤ $i \le \frac{n}{2}$

$f(v_{2i}v_{2i+1})$= 2, 1≤ $i \le \frac{n}{2}$ (v_{n+1}=v_1)

$f(u_1v_1)$ = 3,

$f(u_{2i-1}u_{2i})$=2, 1≤ $i \leq \frac{t}{2}$

$f(u_{2i}u_{2i+1})$= 3, 1≤ $i \leq \frac{t-2}{2}$

Then $f^+: V(G) \rightarrow Z_4$ is given by

We get $f^+(u_i) = f(u_{i-1}u_i) + f(u_iu_{i+1})$

$\equiv (2+3)(mod4) = 1$, 2≤ $i \leq t-1$

$f^+(v_i) = f(v_{i-1}v_i) + f(v_iv_{i+1})$ 2≤ $i \leq n$

$\equiv (3+2)(mod4) = 1$ (or)

$\equiv (2+3)(mod4) = 1$ (since v_{n+1}=v_1) 2≤ $i \leq n$

$f^+(v_1) = f(v_1v_2) + f(v_nv_1) + f(u_1v_1)$

$\equiv (3+2+3)(mod4) = 0$

$f^+(u_1) = f(u_1v_1) + f(u_1u_2)$

$\equiv (3+2)(mod4) = 1$

$f^+(u_t) = 2$

In this case, two vertices namely v_1 and u_t get different constants 0 and 2. All vertices except these two get the same constant 1

In all the cases one or atmost two vertices get different constant(s) and all other vertices get the same constant.

Therefore, (n, t)-kite is near Z_4 –magic

Example 3.19:

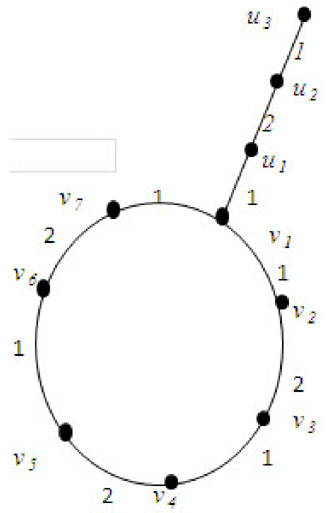

Fig – 12: *Near Z_4* – magic labeling of (7, 3) – kite

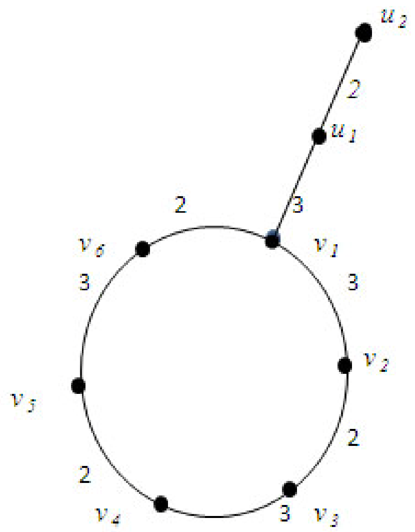

Fig – 13: *Near Z_4* – magic labeling of (6, 2) - kite

Definition 3.20[2]: The corana $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1,(Which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

The graph $P_n \odot K_1$ is called a comb.

Theorem 3.21: The comb $P_n \odot K_1$ is near Z_4- magic

Proof: Let G be $P_n \odot K_1$

Then V(G) $= \{v_i/1 \leq i \leq n\} \cup \{u_i/1 \leq i \leq n\}$

and E(G) $= \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_i u_i/1 \leq i \leq n\}$

Let $f: E(G) \rightarrow Z_4 - \{0\}$ be defined as

$f(v_i v_{i+1})$ =2, $1 \leq i \leq n-1$

$f(v_i u_i)$ = 1, $1 \leq i \leq n$

$f^+: V(G) \rightarrow Z_4$ is given by

$f^+(v_i) = f(v_{i-1} v_i) + f(v_i v_{i+1}) + f(v_i u_i)$, $2 \leq i \leq n-1$

$\equiv (2+2+1)(mod 4) = 1$

$f^+(v_1) = f(v_1 v_2) + f(u_1 v_1)$

$\equiv (2+1)(mod 4) = 3$

$f^+(v_n) = f(v_{n-1} v_n) + f(u_n v_n)$

$\equiv (2+1)(mod 4) = 3$

$f^+(u_i)$ = 1, $1 \leq i \leq n$

Here only two vertices get different constant namely 3 and all other vertices of *G* get the same constant 1

Therefore, $P_n \odot K_1$ is near Z_4 –magic

Example 3.22:

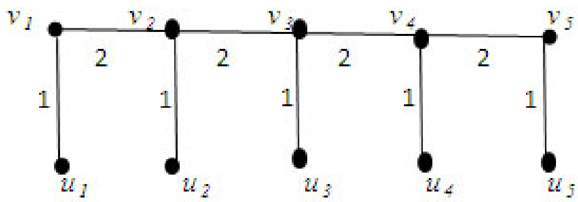

Fig – 14: *Near Z_4* – magic labeling of $P_5 \odot K_1$

Observation 3.23: In all the theorems if we multiply the edge labeling by a positive integer p, the vertex labeling remains to be a constant and it is equal to p times the constant value we obtained, to almost all vertices except one or atmost two vertices of the graph. Hence all the above graphs admit near $Z_{4\text{p}}$ magic labeling. Hence the graphs $P_n, C_n \; for \; n \equiv 1(mod 2), F_n, W_n, Comb, Star \; graphs \; K_{1,n}$ $(n \equiv 0,2,3(mod 4))$ and *(n,k)-kite* graph, are all near $Z_{4\text{p}}$ –magic graphs.

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