NEAR $Z_{4 p}$ - MAGIC LABELING<br>V. L. Stella Arputha Mary ${ }^{1 *}$ S. Navaneetha Krishnanan ${ }^{2}$ and A. Nagarajan ${ }^{3}$<br>${ }^{1}$ Department of Mathematics, St. Mary's College, Tuticorin-628 001, India<br>2,3Department of Mathematics, V.O.C College, Tuticorin-628 001, Tamil Nadu, India.

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#### Abstract

 function $f$ from $E(G)$ into $Z_{4}-\{0\}$ where ' 0 ' is the additive identity element of modulo $Z_{4}$, induce a mapping $f^{+}$from $V(G)$ into $Z_{4}$ such that $f^{+}(v)=\sum f(u v)$ is a constant for all vertices $v \in V$. If $f^{+}(v)=\sum f(u v)$ is a constant for almost all vertices $v \in V$ and for one or atmost two vertices of $v, f^{+}(v)$ is not the same constant where the summation is taken over all the edges $u v$ incident at $v$, then the labeling is called near $Z_{4}$ - magic labeling and the graph which admits near $Z_{4}$-magic labeling is called near $Z_{4}$-magic graph. At the end we generalize near $Z_{4}$-magic labeling into near $Z_{4 p}$ - magic labeling.


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## 1. INTRODUCTION

Throughout this paper by a graph $G(V, E)$ we mean a finite, simple, undirected graph. Magic labeling were introduced by Sedlacek in 1963 Kong, Lee and Sun[4] used the term magic labeling for the labeling of edges with non-negative integers such that for each vertex $v$, the sum of labels of all edges incident at $v$ is same for all $v$ in particular the edge labels need not be distinct. For the abelian group (modulo $Z_{4},+$ ), a graph G is said to be $Z_{4}$ magic if there exists a labeling $f$ of the edges of $G$ with non-zero elements of $Z_{4}$ which induce the vertex labeling $f^{+}$ such that $f^{+}(v)=\sum f(u v)$ where the summation is taken over all edges $u v$ incident at $v$ is a constant.

In near $Z_{4}$ magic graph $f^{+}(v)=\sum f(u v)$ where the summation is taken over all edges $u v$ incident at $v$ is a constant, for almost all vertices $v$ of G and for one or atmost two vertices of $\mathrm{G}, f^{+}(v)$ is not the same constant. It is different from the anti magic labeling which was introduced in 1990 by Hartsfield and Ringel [5].

## 2. BASIC DEFINITIONS

Definition 2.1: A walk of a graph $G$ is an alternating sequence of vertices and edges $v_{0}, e_{1}, v_{1}, e_{2}, \ldots, v_{n-1}, e_{n} v_{n}$ beginning and ending with vertices in which each edges incident with two vertices immediately proceeding and succeeding it. The number ' $n$ ' is called the length of the walk.

Definition 2.2: A Walk in which all the vertices are distinct is called a path.
Definition 2.3: If the end two vertices are same in a path it is called as a cycle. A path having ' $n$ ' vertices and $n-1$ length is denoted as $P_{n}$ here. By $C_{n}$ we mean a cycle consisting of ' $n$ ' vertices and ' $n$ ' edges.

Definition 2.4 [5]: A bipartite graph (or bigraph) $G$ is a graph whose vertex set $V$ can be partitioned into two subsets $V_{1}$ and $V_{2}$ such that every edge of $G$ joins $V_{1}$ with $V_{2}$.

Definition 2.5: If $G$ contains every line joining $V_{1}$ and $V_{2}$ then $G$ is called complete bipartite graph. If $V_{1}$ and $V_{2}$ have m and n vertices, we write $G=\mathrm{K}_{\mathrm{m}, \mathrm{n}}$.

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## 3. MAIN RESULTS

Theorem 3.1: $P_{n}$ is near $Z_{4}$-magic graph $n \geq 4$.

## Proof:

Case 1: $n$ is even
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(u_{2 i-1} u_{2 i}\right)=1, \quad 1 \leq i \leq \frac{n}{2}$ and $f\left(u_{2 i} u_{2 i+1}\right)=2,1 \leq i \leq \frac{n}{2}-1$
Now, $f^{+}: V(G) \rightarrow Z_{4}$
We get $f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right) \equiv 3(\bmod 4)=3,2 \leq i \leq n-1$
$f^{+}\left(u_{1}\right)=1=f^{+}\left(u_{n}\right)$
Here, only two vertices namely $u_{1}$ and $u_{n}$ get a different constant.
Case 2: n is odd
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(u_{2 i-1} u_{2 i}\right)=1,1 \leq i \leq \frac{n-1}{2}$ and $f\left(u_{2 i} u_{2 i+1}\right)=2,1 \leq i \leq \frac{n-1}{2}$
Now, $f^{+}: V(G) \rightarrow Z_{4}$
We get, $f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right) \equiv 3(\bmod 4)=3,2 \leq i \leq n-1$
$f^{+}\left(u_{1}\right)=1$ and $f^{+}\left(u_{n}\right)=2$
The vertices $u_{1}$ and $u_{n}$ get a different constant.
Hence $\quad P_{n}$ admits near $Z_{4}$ - magic labeling and so $P_{n}$ is near $Z_{4}$ - magic graph for $n \geq 4$.
Example 3.2: For n=5, 6 the near $Z_{4}$ - magic labeling of $P_{n}$ are given below.


Fig - 1: Near $Z_{4}$ - magic labeling of $P_{5}$


Fig - 2: Near $Z_{4}$ - magic labeling of $\mathrm{P}_{6}$
Theorem 3.3: $\mathrm{C}_{\mathrm{n}}$ is near $Z_{4}$ - magic for $\mathrm{n} \equiv 1(\bmod 2)$
Proof: Let $f: E\left(C_{n}\right) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(u_{2 i-1} u_{2 i}\right)=2,1 \leq i \leq \frac{n-1}{2}$ and $f\left(u_{2 i} u_{2 i+1}\right)=3,1 \leq i \leq \frac{n-1}{2}$
$f\left(u_{n} u_{1}\right)=2$.
Now, $f^{+}: V\left(C_{n}\right) \rightarrow Z_{4}$
We get, $\quad f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right) \equiv(2+3)(\bmod 4)=1, \quad 2 \leq i \leq n$ $f^{+}\left(u_{1}\right)=f\left(u_{1} u_{2}\right)+f\left(u_{n} u_{1}\right) \equiv(2+2)(\bmod 4)=0$

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Here only one vertex $u_{1}$ gets a different constant zero.
Therefore, $\mathrm{C}_{\mathrm{n}}$ is near $Z_{4}$ - magic for $\mathrm{n} \equiv 1(\bmod 2)$.
Note $3.4 u_{n+1} \equiv u_{1}$ (here).

## Example 3.5:



Fig - 3: Near $Z_{4}$ - magic labeling of $C_{7}$
Remark 3.6: 1. If $f\left(E\left(\mathrm{C}_{\mathrm{n}}\right)\right)$ is a constant in $Z_{4}-\{0\}$ then $\mathrm{C}_{\mathrm{n}}$ is $Z_{4}$ - magic for all $n$. Hence $Z_{4 p}-$ magic $2 . \mathrm{C}_{\mathrm{n}}, \mathrm{n}$ $\equiv 0(\bmod 2)$ is a $Z_{4}$ - magic graph for the labeling given in the theorem3.3.So it is $Z_{4 p}$ - magic

Definition3.7 [6]: The join $\mathrm{G}_{1}+\mathrm{G}_{2}$ of $\mathrm{G}_{1}$ and $\mathrm{G}_{2}$ consists of $\mathrm{G}_{1} \cup \mathrm{G}_{2}$ and all edges joining $\mathrm{V}_{1}$ with $\mathrm{V}_{2}$. The graph $P_{n}+\mathrm{K}_{1}$ is called a fan and it denoted as $\mathrm{F}_{\mathrm{n}}$.

Theorem 3.8: $\mathrm{F}_{\mathrm{n}}$ is near $Z_{4}$ - magic graph, $\mathrm{n} \geq 3$

## Proof:

Case 1: $\mathrm{n} \equiv 0,2,3(\bmod 4)$
$\mathrm{V}(\mathrm{G})=\{\mathrm{u}\} \cup\left\{v_{i} / 1 \leq i \leq n\right\}$ and $\mathrm{E}(\mathrm{G})=\left\{u v_{i} / 1 \leq i \leq n\right\} \cup\left\{v_{i} v_{i+1} / 1 \leq i \leq n-1\right\}$
Let $f: E\left(F_{n}\right) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{i} v_{i+1}\right)=1,1 \leq i \leq n-1$ and $f\left(u v_{i}\right)=1,1 \leq i \leq n-1$
$f\left(u v_{1}\right)=f\left(u v_{n}\right)=2$.
$f^{+}\left(v_{i}\right)=$ Now $f^{+}: V(F) \rightarrow Z_{4}$
$f^{+}\left(v_{1}\right)=f\left(u v_{1}\right)+f\left(v_{1} v_{2}\right)$
$f^{+}\left(v_{1}\right) \equiv(2+1)(\bmod 4)=3=f^{+}\left(v_{n}\right)$
$f^{+}\left(v_{i}\right)=f\left(v_{i} v_{i+1}\right)+f\left(v_{i-1} v_{i}\right)+f\left(u v_{i}\right)$
$f^{+}\left(v_{i}\right) \equiv(1+1+1)(\bmod 4)=3, \quad 2 \leq i \leq n-1$
$f^{+}(u)=2$, for $n \equiv 0(\bmod 4)$
$f^{+}(u)=0$, for $n \equiv 2(\bmod 4)$
$f^{+}(u)=1$, for $n \equiv 3(\bmod 4)$
Here, only one vertex $u$ gets a different constant.

Case 2: $\mathrm{n} \equiv 1(\bmod 4)$
Let $f: E\left(F_{n}\right) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{2 i-1} v_{2 i}\right)=2,1 \leq i \leq \frac{n-1}{2}$ and $f\left(v_{2 i} v_{2 i+1}\right)=1,1 \leq i \leq \frac{n-1}{2}$
$f\left(u v_{i}\right)=2,2 \leq i \leq n-1$
$f\left(u v_{1}\right)=f\left(u v_{n}\right)=3$
Now $f^{+}: V\left(F_{n}\right) \rightarrow Z_{4}$
$f^{+}\left(v_{1}\right)=f\left(u v_{1}\right)+f\left(v_{1} v_{2}\right)$
$f^{+}\left(v_{1}\right) \equiv(3+2)(\bmod 4)=1$
$f^{+}\left(v_{i}\right) \equiv(2+1+2)(\bmod 4)=1,2 \leq i \leq n-1$
$f^{+}\left(v_{n}\right) \equiv(3+1)(\bmod 4)=0$
$f^{+}(u)=\sum_{i=1}^{n} f\left(u v_{i}\right)$

$$
\equiv(3+2+2+\cdots+(n-2) \text { times } 2+3)(\bmod 4)=0
$$

Only two vertices $v_{n}$ and $u$ get a different constant.
Therefore, $\mathrm{F}_{\mathrm{n}}$ admits near $Z_{4}$ - magic labeling. for $\mathrm{n} \geq 3$.
Example 3.9: We shall verify near $Z_{4}$-magic labeling for $\mathrm{n}=3,4,5$.


Fig - 4: Near $Z_{4}$ - magic labeling of $\mathrm{F}_{3}$


Fig - 5: Near $Z_{4}$ - magic labeling of $\mathrm{F}_{4}$


Fig - 6: Near $Z_{4}$ - magic labeling of $\mathrm{F}_{5}$

Definition 3.10[2]: The graph $\mathrm{C}_{\mathrm{n}}+\mathrm{K}_{1}$ is called a wheel and it is denoted as $\mathrm{W}_{\mathrm{n}}$.
Theorem 3.11: $\mathrm{W}_{\mathrm{n}}$ is near $Z_{4}$-magic graph, $\mathrm{n} \geq 3$.

## Proof:

$\mathrm{V}\left(\mathrm{W}_{\mathrm{n}}\right)=\{u\} \cup\left[v_{i} / 1 \leq i \leq n\right]$
$\mathrm{E}\left(\mathrm{W}_{\mathrm{n}}\right)=\left\{u v_{i} / 1 \leq i \leq n\right\} \cup\left\{v_{i} v_{i+1} / 1 \leq i \leq n-1\right\} \cup\left\{v_{n} v_{1}\right\}$
Case 1: $\mathrm{n} \equiv 1(\bmod 2)$
Let $f: E\left(W_{n}\right) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{i} v_{i+1}\right)=1,1 \leq i \leq n-1$
$f\left(v_{n} v_{1}\right)=1$
$f\left(u v_{i}\right)=2,1 \leq i \leq n$
Now $f^{+}: V\left(W_{n}\right) \rightarrow Z_{4}$
$f^{+}\left(v_{i}\right)=f\left(v_{i} v_{i+1}\right)+f\left(u v_{i}\right)+f\left(v_{i-1} v_{i}\right) \quad 1 \leq i \leq n$
$f^{+}\left(v_{i}\right) \equiv(2$ times $1+2)(\bmod 4)=0,1 \leq i \leq n\left(v_{0}=v_{n}\right.$ and $\left.v_{n+1}=v_{1}\right)$
$f^{+}(u)=\sum_{i=1}^{n} f\left(u v_{i}\right)$

$$
\equiv(2+2+\cdots n \text { times } 2)(\bmod 4)=2
$$

The only vertex $u$ gets a different constant in this case.
Case 2: $\mathrm{n} \equiv 0(\bmod 2) \mathrm{n} \geq 3$.
Let $f: E\left(W_{n}\right) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{i} v_{i+1}\right)=1, \quad 1 \leq i \leq n-1$
$f\left(v_{n} v_{1}\right)=1$
$f\left(u v_{i}\right)=3,1 \leq i \leq n$
$f^{+}: V\left(W_{n}\right) \rightarrow Z_{4}$
We get $f^{+}\left(v_{i}\right)=f\left(v_{i} v_{i+1}\right)+f\left(u v_{i}\right)+f\left(v_{i-1} v_{i}\right), 1 \leq i \leq n \quad\left(v_{0}=v_{n}\right.$ and $\left.v_{n+1}=v_{1}\right)$
$f^{+}\left(v_{i}\right) \equiv(2$ times $1+3)(\bmod 4)=1,1 \leq i \leq n$
$f^{+}(u)=\sum_{i=1}^{n} f\left(u v_{i}\right)$

$$
\begin{aligned}
\equiv(3+3+\cdots n \text { times } 3)(\bmod 4)=2 \text { for } \mathrm{n} & \equiv 2(\bmod 4) \\
& =0 \text { for } \mathrm{n} \equiv 0(\bmod 4)
\end{aligned}
$$

Here vertex $u$ gets a different constant .
Therefore in both the cases $W_{n}$ admits near $Z_{4}$-magic labeling.
Hence, $W_{n}$ is near $Z_{4}$-magic graph

Example 3.12:


Fig - 7: Near $Z_{4}$ - magic labeling of $\mathrm{W}_{5}$


Fig - 8: Near $Z_{4}$ - magic labeling of $W_{6}$
Definition 3.13[6]: A Complete bigraph $\mathrm{K}_{1, \mathrm{n}}$ is called a star.
Theorem 3.14: The star graph $K_{1, \mathrm{n}}$ is near $Z_{4}$-magic for $\mathrm{n} \equiv 0,2,3(\bmod 4)$
Proof: Let $u$ be the centre vertex and $v_{1}, v_{2}, \ldots, v_{n}$ be the pendent vertices of the star.
Let Let $f: E\left(k_{1, n}\right) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(u v_{i}\right)=1,1 \leq i \leq n$
$f^{+}: V\left(k_{1, n}\right) \rightarrow Z_{4}$
We get $f\left(v_{i}\right)=1,1 \leq i \leq n$

$$
\begin{aligned}
f^{+}(u) & =\sum_{i=1}^{n} f\left(u v_{i}\right) \\
& \equiv(1+1+\cdots n \text { times } 1)(\bmod 4)=2 \text { for } \mathrm{n}
\end{aligned} \begin{aligned}
& 2(\bmod 4) \\
& =0 \text { for } \mathrm{n} \equiv 0(\bmod 4) \\
& =3 \text { for } \mathrm{n} \equiv 3(\bmod 4)
\end{aligned}
$$

The only vertex $u$ gets a different constant for different values of $n$.
Therefore the star graph $k_{1, n}$ is near $Z_{4}$-magic graph.
Remark 3.15: $k_{1, n}$ is $Z_{4}$-magic graph for $n \equiv 1(\bmod 4)$. Hence it is $Z_{4 p}$ - magic
Example 3.16: For $\mathrm{n}=4,6$, 7 the near $Z_{4}$-magic labeling of $k_{1, n}$ are given


Fig - 9: Near $\mathrm{Z}_{4}$ - magic labeling of $\mathrm{K}_{1,4}$


Fig - 10: Near $Z_{4}$ - magic labeling of $\mathrm{K}_{1,6}$


Fig - 11: Near $Z_{4}$ - magic labeling of $\mathrm{K}_{1,7}$
Definition 3.17[3]: A graph is called an $(n, t)$ - kite if a path of length $t$ attached to one vertex of the cycle $C_{n}$.
Theorem 3.18: An $(n, t)$ - kite is near $Z_{4}$-magic for $n \geq 3$ and $\mathrm{t} \geq 1$
Proof: Let G be $(n, t)$ - kite
$\mathrm{V}(\mathrm{G})=\left\{u_{i} / 1 \leq i \leq t\right\} \cup\left\{v_{i} / 1 \leq i \leq n\right\}$
$\mathrm{E}(\mathrm{G})=\left\{u_{i} u_{i+1} / 1 \leq i \leq t-1\right\} \cup\left\{v_{i} v_{i+1} / 1 \leq i \leq n-1\right\} \cup\left\{v_{n} v_{1}\right\} \cup\left\{u_{1} v_{1}\right\}$
Case 1: $n$ is odd and $t$ is odd
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{2 i-1} v_{2 i}\right)=1,1 \leq i \leq \frac{n-1}{2}$
$f\left(v_{2 i} v_{2 i+1}\right)=2,1 \leq i \leq \frac{n-1}{2}$
$f\left(v_{n} v_{1}\right)=1$
$f\left(v_{1} u_{1}\right)=1$
$f\left(u_{2 i-1} u_{2 i}\right)=2,1 \leq i \leq \frac{t-1}{2}$
$f\left(u_{2 i} u_{2 i+1}\right)=1,1 \leq i \leq \frac{t-1}{2}$
hence $f^{+}: V(G) \rightarrow Z_{4}$
We get $f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right)$

$$
\equiv(1+2)(\bmod 4)=3,2 \leq i \leq t-1
$$

$f^{+}\left(v_{i}\right)=f\left(v_{i-1} v_{i}\right)+f\left(v_{i} v_{i+1}\right)$
$\equiv(1+2)(\bmod 4)=3$, or
$\equiv(2+1)(\bmod 4)=3, \quad 2 \leq i \leq n\left(\right.$ since $\left.v_{n+1}=v_{1}\right)$
$f^{+}\left(v_{1}\right)=f\left(v_{1} v_{2}\right)+f\left(v_{n} v_{1}\right)+f\left(u_{1} v_{1}\right)$
$\equiv(1+1+1)(\bmod 4)=3$
$f^{+}\left(u_{1}\right)=f\left(u_{1} v_{1}\right)+f\left(u_{1} u_{2}\right)$

$$
\equiv(1+2)(\bmod 4)=3
$$

$f^{+}\left(u_{t}\right)=1$
$u_{t}$ alone gets a different constant 1 and all the other vertices $v \epsilon V(G)$ get the unique constant 3 .
Case 2: $n$ is odd and $t$ is even
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{2 i-1} v_{2 i}\right)=1,1 \leq i \leq \frac{n-1}{2}$
$f\left(v_{2 i} v_{2 i+1}\right)=2,1 \leq i \leq \frac{n-1}{2}$
$f\left(v_{n} v_{1}\right)=1$.
$f\left(u_{1} v_{1}\right)=1$
$f\left(u_{2 i-1} u_{2 i}\right)=2,1 \leq i \leq \frac{t}{2}$
$f\left(u_{2 i} u_{2 i+1}\right)=1,1 \leq i \leq \frac{t-2}{2}$
Then $f^{+}: V(G) \rightarrow Z_{4}$ is given by
We get $f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right)$

$$
\equiv(1+2)(\bmod 4)=3,2 \leq i \leq t-1
$$

$f^{+}\left(v_{i}\right)=f\left(v_{i-1} v_{i}\right)+f\left(v_{i} v_{i+1}\right)$
$\equiv(1+2)(\bmod 4)=3$ (or)
$\equiv(2+1)(\bmod 4)=3, \quad 2 \leq i \leq n \quad\left(\right.$ since $\left.v_{n+1}=v_{1}\right)$
$f^{+}\left(v_{1}\right)=f\left(v_{1} v_{2}\right)+f\left(v_{n} v_{1}\right)+f\left(u_{1} v_{1}\right)$

$$
\equiv(1+1+1)(\bmod 4)=3
$$

$$
\begin{aligned}
f^{+}\left(u_{1}\right) & =f\left(u_{1} v_{1}\right)+f\left(u_{1} u_{2}\right) \\
& \equiv(1+2)(\bmod 4)=3 \\
f^{+}\left(u_{t}\right) & \equiv 2(\bmod 4) \\
& =2
\end{aligned}
$$

$u_{t}$ alone gets a different constant 2 and all the other vertices $\mathrm{v} \epsilon V(G)$ get the same constant 3 .
Case 3: $n$ is even and $t$ is odd
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{2 i-1} v_{2 i}\right)=3,1 \leq i \leq \frac{n}{2}$
$f\left(v_{2 i} v_{2 i+1}\right)=2,1 \leq i \leq \frac{n}{2}\left(v_{n+1}=v_{1}\right)$
$f\left(u_{1} v_{1}\right)=3$,
$f\left(u_{2 i-1} u_{2 i}\right)=2, \quad 1 \leq i \leq \frac{t-1}{2}$
$f\left(u_{2 i} u_{2 i+1}\right)=3,1 \leq i \leq \frac{t-1}{2}$
Then $f^{+}: V(G) \rightarrow Z_{4}$ is given by
We get $f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right)$

$$
\equiv(2+3)(\bmod 4)=1,2 \leq i \leq t-1
$$

$f^{+}\left(v_{i}\right)=f\left(v_{i-1} v_{i}\right)+f\left(v_{i} v_{i+1}\right) \quad 2 \leq i \leq n$
$\equiv(3+2)(\bmod 4)=1($ or $)$
$\equiv(2+3)(\bmod 4)=1,2 \leq i \leq n \quad\left(\right.$ since $\left.v_{n+1}=v_{1}\right)$
$f^{+}\left(v_{1}\right)=f\left(v_{1} v_{2}\right)+f\left(v_{n} v_{1}\right)+f\left(u_{1} v_{1}\right)$
$\equiv(3+2+3)(\bmod 4)=0$
$f^{+}\left(u_{1}\right)=f\left(u_{1} v_{1}\right)+f\left(u_{1} u_{2}\right)$
$\equiv(3+2)(\bmod 4)=1$
$f^{+}\left(u_{t}\right)=3$
In this case, two vertices namely $v_{1}$ and $u_{t}$ get different constants 0 and 3 . All vertices except these two get the same constant 1

Case 4: $n$ is even and $t$ is even
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{2 i-1} v_{2 i}\right)=3, \quad 1 \leq i \leq \frac{n}{2}$
$f\left(v_{2 i} v_{2 i+1}\right)=2,1 \leq i \leq \frac{n}{2}\left(v_{n+1}=v_{1}\right)$
$f\left(u_{1} v_{1}\right)=3$,
$f\left(u_{2 i-1} u_{2 i}\right)=2,1 \leq i \leq \frac{t}{2}$
$f\left(u_{2 i} u_{2 i+1}\right)=3, \quad 1 \leq i \leq \frac{t-2}{2}$
Then $f^{+}: V(G) \rightarrow Z_{4}$ is given by
We get $f^{+}\left(u_{i}\right)=f\left(u_{i-1} u_{i}\right)+f\left(u_{i} u_{i+1}\right)$

$$
\equiv(2+3)(\bmod 4)=1,2 \leq i \leq t-1
$$

$$
f^{+}\left(v_{i}\right)=f\left(v_{i-1} v_{i}\right)+f\left(v_{i} v_{i+1}\right) \quad 2 \leq i \leq n
$$

$$
\equiv(3+2)(\bmod 4)=1(\text { or })
$$

$$
\equiv(2+3)(\bmod 4)=1 \quad\left(\text { since } v_{n+1}=v_{1}\right) \quad 2 \leq i \leq n
$$

$f^{+}\left(v_{1}\right)=f\left(v_{1} v_{2}\right)+f\left(v_{n} v_{1}\right)+f\left(u_{1} v_{1}\right)$

$$
\equiv(3+2+3)(\bmod 4)=0
$$

$f^{+}\left(u_{1}\right)=f\left(u_{1} v_{1}\right)+f\left(u_{1} u_{2}\right)$

$$
\equiv(3+2)(\bmod 4)=1
$$

$f^{+}\left(u_{t}\right)=2$
In this case, two vertices namely $v_{1}$ and $u_{t}$ get different constants 0 and 2 . All vertices except these two get the same constant 1

In all the cases one or atmost two vertices get different constant(s) and all other vertices get the same constant.
Therefore, ( $\mathrm{n}, \mathrm{t}$ )-kite is near $Z_{4}$-magic

## Example 3.19:



Fig - 12: Near $Z_{4}$ - magic labeling of (7, 3) - kite


Fig - 13: Near $Z_{4}$ - magic labeling of $(6,2)$ - kite
Definition 3.20[2]: The corana $G_{1} \odot G_{2}$ of two graphs $G_{1}$ and $G_{2}$ is defined as the graph $G$ obtained by taking one copy of $G_{1}$,(Which has $p_{1}$ vertices) and $p_{1}$ copies of $G_{2}$ and then joining the $i^{\text {th }}$ vertex of $G_{1}$ to every vertex in the $\mathrm{i}^{\text {th }}$ copy of $\mathrm{G}_{2}$.

The graph $P_{n} \odot K_{1}$ is called a comb.
Theorem 3.21: The comb $P_{n} \odot K_{1}$ is near $\mathrm{Z}_{4}$ - magic
Proof: Let G be $P_{n} \odot K_{1}$
Then $\mathrm{V}(\mathrm{G})=\left\{v_{i} / 1 \leq i \leq n\right\} \cup\left\{u_{i} / 1 \leq i \leq n\right\}$
and $\mathrm{E}(\mathrm{G})=\left\{v_{i} v_{i+1} / 1 \leq i \leq n-1\right\} \cup\left\{v_{i} u_{i} / 1 \leq i \leq n\right\}$
Let $f: E(G) \rightarrow Z_{4}-\{0\}$ be defined as
$f\left(v_{i} v_{i+1}\right)=2,1 \leq i \leq n-1$
$f\left(v_{i} u_{i}\right)=1,1 \leq i \leq n$
$f^{+}: V(G) \rightarrow Z_{4}$ is given by

$$
\begin{aligned}
f^{+}\left(v_{i}\right) & =f\left(v_{i-1} v_{i}\right)+f\left(v_{i} v_{i+1}\right)+f\left(v_{i} u_{i}\right), \quad 2 \leq i \leq n-1 \\
& \equiv(2+2+1)(\bmod 4)=1 \\
f^{+}\left(v_{1}\right) & =f\left(v_{1} v_{2}\right)+f\left(u_{1} v_{1}\right) \\
& \equiv(2+1)(\bmod 4)=3
\end{aligned}
$$

$$
f^{+}\left(v_{n}\right)=f\left(v_{n-1} v_{n}\right)+f\left(u_{n} v_{n}\right)
$$

$$
\equiv(2+1)(\bmod 4)=3
$$

$f^{+}\left(u_{i}\right)=1,1 \leq i \leq n$
Here only two vertices get different constant namely 3 and all other vertices of $G$ get the same constant 1
Therefore, $P_{n} \odot K_{1}$ is near $Z_{4}$-magic

## V. L. Stella Arputha Mary ${ }^{1 *}$ S. Navaneetha Krishnanan ${ }^{2}$ and A. Nagarajan ${ }^{3}$ / NEAR $Z_{4 p}$ - MAGIC LABELING/ IJMA- 4(10), Oct.-2013.

## Example 3.22:



Fig - 14: Near $Z_{4}$ - magic labeling of $P_{5} \odot K_{1}$
Observation 3.23: In all the theorems if we multiply the edge labeling by a positive integer $p$, the vertex labeling remains to be a constant and it is equal to $p$ times the constant value we obtained, to almost all vertices except one or atmost two vertices of the graph. Hence all the above graphs admit near $Z_{4 \mathrm{p}}$ magic labeling. Hence the graphs $P_{n}, C_{n}$ for $n \equiv 1(\bmod 2), F_{n}, W_{n}$, Comb, Star graphs $K_{1, n}(n \equiv 0,2,3(\bmod 4))$ and $(n, k)$-kite graph, are all near $Z_{4 p}$ -magic graphs.

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