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FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

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ABSTRACT

The focus of this paper is to propose a new notion of Fuzzy Neutrosophic soft set and to study some basic operations and results in Fuzzy Neutrosophic soft spaces. Further we develop a systematic study on Fuzzy Neutrosophic soft set and obtain various properties induced by them. Some equivalent characterization and inter-relations among them are discussed with counter example.

Keywords: *Soft sets, Neutrosophic set, Neutrosophic soft set, Fuzzy Neutrosophic soft set and Fuzzy Neutrosophic soft topological space.*

MSC 2000*: 03B99, 03E99.*

1. INTRODUCTION

In dealing with uncertainties many theories have been recently developed, including the theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets and theory of rough sets and so on. Although many new techniques have been developed as a result of these theories, yet difficulties are still there. The major difficulties arise due to inadequacy of parameters.

In 1999, Molodtsov[4] , initiated the novel concept of soft set theory, which was a completely new approach for modeling uncertainty and had a rich potential for application in several directions. This so- called soft set theory is free from the difficulties affecting existing methods. The fuzzy set was introduced by Zadeh [13] in 1965 where each element had a degree of membership. The intuitionistic fuzzy set (IFS for short) on a universe X was introduced by K.Atanaasov [1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non – membership of each element. The concept of Neutrosophic set was introduced by F. Smarandache [10] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Pabitra Kumar Maji [7] had combined the Neutrosophic set with soft sets and introduced a new mathematical model 'Neutrosophic soft set'.

In the present study, we have defined Fuzzy Neutrosophic soft set and we establish some related properties with supporting proofs and examples. We consider the topological structure of Neutrosophic soft set as a tool to develop Fuzzy Neutrosophic soft topological space and derive some of their topological properties. Here we recall the definitions that are prerequisite for this paper.

2. PRELIMINARIES

Definition 2.1: [10] A Neutrosophic set A on the universe of discourse X is defined as

$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$ where $T, I, F : X \to \left[{}^{-}0, 1^{+} \right]$ [and ${}^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}$.

Definition 2.2: [8] A neutrosophic set A is contained in another neutrosophic set B. (i.e.,)

(i) $A \subseteq B \Leftrightarrow T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$, $\forall x \in X$

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Definition 2.3: [4] Let U be the initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a non-empty set A, A $\subset$ E .A pair (F,A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U)

Definition 2.4: [7] Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, A $\subset$ E. Let P(U) denote the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A $\rightarrow$ P(U).

Definition 2.5: [7] The complement of a neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as

$(F, A)^c = (F^c, \neg A)$ where $F^c: \neg A \rightarrow P(U)$ is a mapping given by $F^c(\alpha)$=neutrosophic soft complement with $T_{F^c}(x) = F_F(x)$, $I_{F^c}(x) = I_F(x)$, $F_{F^c}(x) = T_F(x)$.

Definition 2.6:[7] Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U.(F,A) is said to be neutrosophic soft subset of (G,B) if A$\subset$B and $T_{F(e)}(x) \leq T_{G(e)}(x), I_{F(e)}(x) \leq I_{G(e)}(x), F_{F(e)}(x) \geq F_{G(e)}(x)$ $\forall$ e$\in$ A , x $\in$ U. We denote it by (F, A) $\subseteq$ (G, B).

(F, A) is said to be neutrosophic soft super set of (G, B) if (G, B) is a neutrosophic soft subset of (F, A).We denote it by (F, A) $\supseteq$ (G, B).

Definition 2.7:[7] Two neutrosophic soft sets (F, A) and (G, B) over the common universe U are said to be equal if (F, A) $\subseteq$ (G,B) and (G, B) $\subseteq$ (F, A).We denote it by (F, A) = (G, B).

Definition: 2.8 [7] Union of two Neutrosophic soft sets (F, A) and (G, B) over (U, E) is Neutrosophic soft set where C = A$\cup$B $\forall$ e$\in$ C.

$$H(e) = \begin{cases} F(e) & ; \text{ if } e \in A - B \\ G(e) & ; \text{ if } e \in B - A \\ F(e) \cup G(e) & ; \text{ if } e \in A \cap B \end{cases}$$

and is written as (F,A) $\tilde{\cup}$ (G ,B) = (H,C).

Definition: 2.9 [7] Intersection of two Neutrosophic soft sets (F,A) and (G,B) over (U, E) is Neutrosophic soft set where C = A$\cap$B $\forall$ e$\in$ C. H(e) = F(e) $\cap$ G(e) and is written as (F,A) $\tilde{\cap}$ (G,B) = (H,C).

3. FUZZY NEUTROSOPHIC SOFT SET

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subset of $]-0, 1^+[$. But in real life application in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

Definition 3.1: A Fuzzy Neutrosophic set A on the universe of discourse X is defined as

$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$ where $T, I, F : X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 3.2: Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, A $\subset$ E. Let P (U) denote the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U).

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS.

Definition 3.3: A fuzzy neutrosophic soft set A is contained in another neutrosophic set B. (i.e.) A $\subseteq$ B if $\forall$ x $\in$X, $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 3.4: The complement of a fuzzy neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as

$(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \rightarrow P(U)$ is a mapping given by

$F^c(\alpha) = \langle x, T_{F^c}(x) = F_F(x), I_{F^c}(x) = 1 - I_F(x), F_{F^c}(x) = T_F(x) \rangle$

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Definition 3.5: Let X be a non empty set, and $A=\langle x,T_A(x),I_A(x),F_A(x)\rangle, B=\langle x,T_B(x),I_B(x),F_B(x)\rangle$ are fuzzy neutrosophic soft sets. Then

$$A\tilde{\cup}B=\langle x,\max(T_A(x),T_B(x)),\max(I_A(x),I_B(x)),\min(F_A(x),F_B(x))\rangle$$

$$A\tilde{\cap}B=\langle x,\min(T_A(x),T_B(x)),\min(I_A(x),I_B(x)),\max(F_A(x),F_B(x))\rangle$$

Definition 3.6: A fuzzy neutrosophic soft set (F,A) over the universe U is said to be empty fuzzy neutrosophic soft set with respect to the parameter A if $T_{F(e)}=0, I_{F(e)}=0, F_{F(e)}=1, \forall x\in U, \forall e\in A.$ It is denoted by $\tilde{0}_N$.

Definition 3.7: A FNS set (F, A), over the universe U is said to be universe FNS set with respect to the parameter A if $T_{F(e)}=1, I_{F(e)}=1, F_{F(e)}=0, \forall x\in U, \forall e\in A$.It is denoted by $\tilde{1}_N$.

Note: $\tilde{0}_N^c=\tilde{1}_N$ and $\left(\tilde{1}_N\right)^c=\tilde{0}_N$

Definition 3.8: Let A and B be two FNS sets then A\B may be defined as

$$A\backslash B=\langle x,\min(T_A(x),F_B(x)),\min(I_A(x),1-I_B(x)),\max(F_A(x),T_B(x))\rangle.$$

Definition3.9:

(i) F_E is called absolute Fuzzy Neutrosophic soft set over U if F(e) = $\tilde{1}_N$ for any e ∈ E. We denote it by U_E

(ii) F_E is called relative null Fuzzy Neutrosophic soft set over U if F(e) = $\tilde{0}_N$ for any e ∈ E. We denote it by ϕ_E.

Obviously $\phi_E=U_E^c$ and $U_E=\phi_E^c$.

Definition 3.10: The complement of a fuzzy neutrosophic soft set (F, A) can also be defined as

$(F, A)^c = U_E \setminus F(e)$ for all e∈A

Note: We denote U_E by U in the proofs of the proposition.

Definition 3.11: If (F, A) and (G, B) be two fuzzy Neutrosophic soft set then "(F, A) AND (G, B)" is a FNSS denoted by (F, A) ∧ (G, B) and is defined by (F, A) ∧ (G, B) = (H, A x B)

where H(a, b) = F(a) ∩ G(b) ∀ a∈A and ∀ b∈B where ∩ is the operation intersection of FNSS.

Definition 3.12: If (F, A) and (G, B) be two fuzzy Neutrosophic soft set then "(F, A) OR (G, B)" is a FNSS denoted by (F, A) ∨ (G, B) and is defined by (F, A) ∨ (G, B) = (K, A x B)

where K(a, b) = F(a) ∪ G(b) ∀ a∈A and ∀ b∈B where ∪ is the operation union of FNSS.

Proposition 3.13: Let (F,A) and (G,A) be FNSS in $FNSS(U)_A$ .Then the following are true.

(i) (F,A) $\tilde{\subseteq}$ (G,A) iff (F, A) $\tilde{\cap}$ (G,A) = (F, A)

(ii) (F,A) $\tilde{\subseteq}$ (G,A) iff (F, A) $\tilde{\cup}$ (G,A) = (F, A)

Proof:

(i) Suppose that (F,A) $\tilde{\subseteq}$ (G,A) Then F(e) ⊆ G(e) for all e ∈A. Let (F, A) $\tilde{\cap}$ (G,A) = (H, A)

Since H(e) = F(e) ∩ G(e) = F(e) for all e∈A ,by definition (H, A) = (F, A). Suppose that (F, A) $\tilde{\cap}$ (G,A) = (F, A). Let (F, A) $\tilde{\cap}$ (G,A) = (H, A).Since H(e) = F(e) ∩ G(e) = F(e) for all e∈A, we know that F(e) ⊆ G(e) for all e ∈A. Hence (F, A) $\tilde{\subseteq}$ (G,A)

(ii) The proof is similar to (i)

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Proposition 3.14: Let (F, A) , (G, A), (H, A), (S, A) $\in$ FNSS$(U)_A$. Then the following are true

(i) If (F, A) $\tilde{\cap}$ (G,A) = ϕ_A then (F,A) $\tilde{\subseteq}$ $(G,A)^c$

(ii) If (F,A) $\tilde{\subseteq}$ (G,A) and (G,A) $\tilde{\subseteq}$ (H,A) then (F,A) $\tilde{\subseteq}$ (H,A)

(iii) If (F,A) $\tilde{\subseteq}$ (G,A) and (H,A) $\tilde{\subseteq}$ (S,A) then (F, A) $\tilde{\cap}$ (H,A) $\tilde{\subseteq}$ (G, A) $\tilde{\cap}$ (S,A)

(iv) (F,A) $\tilde{\subseteq}$ (G,A) iff $(G,A)^c$ $\tilde{\subseteq}$ $(F,A)^c$

Proof:

(i) Suppose that (F, A) $\tilde{\cap}$ (G,A) = ϕ_A .Then F(e) $\cap$ G(e) = ϕ . So F(e) $\subseteq$ U\G(e) =G^c(e) for all e$\in$A.

Therefore we have (F, A) $\tilde{\subseteq}$ $(G, A)^c$. Proof of (ii) and (iii) are obvious.

(iv) (F,A) $\tilde{\subseteq}$ (G,A)$\Leftrightarrow$ F(e) $\subseteq$ G(e) for all e$\in$A

$\Leftrightarrow$ $(G(e))^c \subseteq (F(e))^c$ for all e$\in$A

$\Leftrightarrow$ $G^c(e) \subseteq F^c(e)$ for all e$\in$A

$\Leftrightarrow$ $(G, A)^c$ $\tilde{\subseteq}$ $(F,A)^c$

Definition 3.15: Let I be an arbitrary index set and $\{(F_i , A)\}_{i\in I}$ be a subfamily of FNSS$(U)_A$.

(i) The union of these FNSS is the FNSS (H, A) where $H(e) = \bigcup_{i\in I} F_i(e)$ for each e$\in$A.

We write $\tilde{\bigcup}_{i\in I} (F_i , A) = (H,A)$

(ii) The intersection of these FNSS is the FNSS (M, A) where $M(e) = \bigcap_{i\in I} F_i(e)$ for each e$\in$A.

We write $\tilde{\bigcap}_{i\in I} (F_i, A) = (M,A)$.

Proposition 3.16: Let I be an arbitrary index set and $\{(F_i , A)\}_{i\in I}$ be a subfamily of FNSS$(U)_A$.

Then (i) $[\tilde{\bigcup}_{i\in I} (F_i , A)]^c = \tilde{\bigcap}_{i\in I} (F_i,A)^c$

(ii) $[\tilde{\bigcap}_{i\in I} (F_i,A)]^c = \tilde{\bigcup}_{i\in I} (F_i , A)^c$

Proof:

(i) $[\tilde{\bigcup}_{i\in I} (F_i , A)]^c = (H,A)^c$,By definition $H^c(e) = U_E \setminus H(e) = U_E \setminus \bigcup_{i\in I} F_i(e) = \bigcap_{i\in I} (U_E \setminus F_i(e))$ for all e$\in$A. On the other hand , $[\tilde{\bigcap}_{i\in I} (F_i, A)]^c$ =(K ,A) .By definition ,$K(e) = \bigcap_{i\in I} F_i^c(e) = \bigcap_{i\in I} (U - F_i(e))$ for all e$\in$A.

(ii) $[\tilde{\bigcap}_{i\in I} (F_i , A)]^c = (H,A)^c$,By definition $H^c(e) = U_E \setminus H(e) = U_E \setminus \bigcap_{i\in I} F_i(e) = \bigcup_{i\in I} (U_E \setminus F_i(e))$ for all e$\in$A. On the other hand , $[\tilde{\bigcup}_{i\in I} (F_i, A)]^c$ =(K ,A) .By definition ,$K(e) = \bigcup_{i\in I} F_i^c(e) = \bigcup_{i\in I} (U_E - F_i(e))$ for all e$\in$A.

Note: We denote ϕ_E by ϕ and U_E by U

Proposition 3.17:

i) $(\phi ,A)^c = (U, A)$

ii) $(U, A)^c = (\phi ,A)$

Proof:

i) Let $(\phi ,A) = (F,A)$

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Then $\forall e \in A$,

$F(e) = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}$

$= \{(x, 0, 0, 1): x \in U\}$

$(\phi , A)^c = (F,A)^c$

Then $\forall e \in A$,

$(F(e))^c = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}^c$

$= \{(x, F_{(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)): x \in U\}$

$= \{(x, 1, 1, 0): x \in U\} = U$

Thus $(\phi , A)^c = (U, A)$

ii) Proof is similar to (i)

Proposition 3.18:

i) $(F,A) \tilde{\cup} (\phi ,A) = (F,A)$

ii) $(F,A) \tilde{\cup} (U ,A) = (U,A)$

Proof:

i) $(F,A) = \{e, (x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$
$(\phi , A) = \{e(x, 0, 0, 1): x \in U\} \forall e \in A$

$(F,A) \tilde{\cup} (\phi , A) = \{e, (x, \max(T_{F(e)}(x), 0), \max(I_{F(e)}(x),0), \min(F_{F(e)}(x),1)): x \in U\} \forall e \in A$

$= \{e(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}\} \forall e \in A$

$= (F, A)$

ii) Proof is similar to (i)

Proposition 3.19:

i) $(F,A) \tilde{\cap} (\phi ,A) = (\phi,A)$

ii) $(F,A) \tilde{\cap} (U ,A) = (F,A)$

Proof:

i) $(F,A) = \{e, (x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

$(\phi , A) = \{e(x, 0, 0, 1): x \in U\} \forall e \in A$

$(F,A) \tilde{\cap} (\phi , A) = \{e, (x, \min(T_{F(e)}(x), 0), \min(I_{F(e)}(x),0), \max(F_{F(e)}(x),1)): x \in U\} \forall e \in A$

$= \{e(x, 0, 0, 1): x \in U\} \forall e \in A$

$= (\phi, A)$

Thus $(F, A) \tilde{\cap} (\phi , A) = (\phi, A)$

ii) Proof is similar to (i)

Proposition 3.20:

(i) $(F,A) \tilde{\cup} (\phi ,B) = (F,A)$ iff $B \subseteq A$

(ii) $(F,A) \tilde{\cup} (U ,B) = (U,A)$ iff $A \subseteq B$

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Proof:

i) We have for (F,A)

$F(e) = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

Also let $(\phi , B) = (G, B)$ then

$G(e) = \{(x, 0, 0, 1): x \in U\} \forall e \in B$

Let $(F, A) \tilde{\cup} (\phi , B) = (F, A) \tilde{\cup} (G, B) = (H, C)$ where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x)) : x \in U\} \text{if } e \in B - A \\ \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, 0,0,1) : x \in U\} \text{if } e \in B - A \\ \{(x, \max(T_{F(e)}(x),0), \max(I_{F(e)}(x),0), \min(F_{F(e)}(x),1)) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, 0,0,1) : x \in U\} \text{if } e \in B - A \\ \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

Let $B \subseteq A$

Then $H(e) = \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A \cap B \end{cases}$

$= F(e) \ \forall e \in A$

Conversely Let $(F, A) \tilde{\cup} (\phi , B) = (F, A)$

Then $A = A \cup B \Rightarrow B \subseteq A$

(ii) Proof is similar to (i)

Proposition 3.21:

(i) $(F,A) \tilde{\cap} (\phi ,B) = (\phi, A \cap B)$

(ii) $(F,A) \tilde{\cap} (U ,B) = (F, A \cap B)$

Proof:

i) We have for (F,A)

$F(e) = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

let $(\phi ,B) = (G,B)$ then

$G(e) = \{(x, 0, 0, 1): x \in U\} \forall e \in B$

Let $(F, A) \tilde{\cap} (\phi , B) = (F, A) \tilde{\cap} (G, B) = (H, C)$ where $C = A \cap B$ and $\forall e \in C$

$H(e) = \{(x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))) : x \in U\}$

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$= \{(x, \min(T_{F(e)}(x),0), \min(I_{F(e)}(x),0), \max(F_{F(e)}(x),1)) : x \in U\}$

$= \{(x, 0,0,1) : x \in U\}$

$= (G, B) = (\phi, B)$

Thus $(F, A) \tilde{\cap} (\phi, B) = (\phi, B)$.

ii) Proof is similar to (i).

Proposition 3.22:

i) $((F,A) \tilde{\cup} (G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G,B)^c$

ii) $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\cap} (G,B))^c$

Proof: Let $(F,A) \tilde{\cup} (G,B) = (H,C)$ where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{ if } e \in A\text{-}B \\ \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x)) : x \in U\} \text{ if } e \in B\text{-}A \\ \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))) : x \in U\} \text{ if } e \in A \cap B \end{cases}$$

Thus $((F, A) \tilde{\cup} (G, B))^c = (H,C)^c$ where $C = A \cup B$ and $\forall e \in C$

$$(H(e))^c = \begin{cases} (F(e))^c \text{ if } e \in A - B \\ (G(e))^c \ e \in B - A \\ ((F(e) \cup G(e))^c \text{ if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)) : x \in U\} \text{ if } e \in A\text{-}B \\ \{(x, F_{G(e)}(x), 1 - I_{G(e)}(x), T_{G(e)}(x)) : x \in U\} \text{ if } e \in B\text{-}A \\ \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), 1 - \max(I_{F(e)}(x), I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))) : x \in U\} \text{ if } e \in A \cap B \end{cases}$$

Again, $(F, A)^c \tilde{\cup} (G, B)^c = (I, J)$ say $J = A \cup B$ and $\forall e \in J$.

$$I(e) = \begin{cases} (F(e))^c \text{ if } e \in A - B \\ (G(e))^c \ e \in B - A \\ (F(e))^c \cup (G(e))^c \text{ if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)) : x \in U\} \text{ if } e \in A - B \\ \{(x, F_{G(e)}(x), 1 - I_{G(e)}(x), T_{G(e)}(x)) : x \in U\} \text{ if } e \in B - A \\ \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), \max(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))) : x \in U\} \text{ if } e \in A \cap B \end{cases}$$

$C \subseteq J \quad \forall e \in J$, $(H(e))^c \subseteq I(e)$

Thus $((F, A) \tilde{\cup} (G, B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G, B)^c$

ii) Let $(F, A) \tilde{\cap} (G, B) = (H, C)$ where $C = A \cap B$ and $\forall e \in C$

$H(e) = F(e) \cap G(e)$

$= \{(x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))\}$

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Thus $((F,A) \tilde{\cap} (G,B))^c = (H, C)^c$ where $C = A \cap B$ and $\forall e \in C$

$(H(e))^c = \{(x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))\}^c$

$= \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), 1 - \min(I_{F(e)}(x), I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))\}$

Again $(F, A)^c \tilde{\cap} (G,B)^c = (I, J)$ say where $J = A \cap B$ and $\forall e \in J$

$I(e) = (F(e))^c \cap (G(e))^c$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), \min(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

We see that $C = J$ and $\forall e \in J$, $I(e) \subseteq (H(e))^c$

Thus $(F, A)^c \tilde{\cap} (G, B)^c \tilde{\subseteq} ((F,A) \tilde{\cap} (G,B))^c$

Proposition 3.23: (De Morgan's Laws) For Fuzzy Neutrosophic soft sets (F, A) and (G, A) over the same universe U we have the following

i) $((F,A) \tilde{\cup} (G,A))^c = (F,A)^c \tilde{\cap} (G,A)^c$

ii) $((F,A) \tilde{\cap} (G,A))^c = (F,A)^c \tilde{\cup} (G,A)^c$

Proof: (i) Let $(F,A) \tilde{\cup} (G,A) = (H,A)$ where $\forall e \in A$

$H(e) = F(e) \cup G(e)$

$= \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))\}$

Thus $((F, A) \tilde{\cup} (G,A))^c = (H,A)^c$ where $\forall e \in A$

$(H(e))^c = (F(e) \cup G(e))^c$

$= \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))\}^c$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), 1 - \max(I_{F(e)}(x), I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

Again $(F, A)^c \tilde{\cap} (G, A)^c = (I, A)$ say where $\forall e \in A$

$I(e) = (F(e))^c \cap (G(e))^c$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), \min(1 - I_{F(e)}(x), 1 - I_{G(e)}(x), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), 1 - \max(I_{F(e)}(x), I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

Thus $((F, A) \tilde{\cup} (G, A))^c = (F,A)^c \tilde{\cap} (G,A)^c$

(ii) Let $(F,A) \tilde{\cap} (G,A) = (H, A)$ where $\forall e \in A$

$H(e) = F(e) \cap G(e)$

$= \{x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))\}$ where $\forall e \in A$

$((F,A) \tilde{\cap} (G, A))^c = (H, A)^c$

$(H(e))^c = (F(e) \cap G(e))^c$

$= \{x, \max(F_{F(e)}(x), F_{G(e)}(x)), 1 - \min(I_{F(e)}(x), I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))\}$ where $\forall e \in A$

Again $(F, A)^c \tilde{\cup} (G,A)^c = (I, A)$ say where $\forall e \in A$

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$I(e) = (F(e))^c \cup G(e))^c$

$= \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), \max(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))\}$

$= \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), 1 - \min(I_{F(e)}(x), I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))\}$

Thus $((F,A) \tilde{\cap} (G,A))^c = (F,A)^c \tilde{\cup} (G,A)^c$.

Proposition 3.24: For FNSS (F,A) and (G, B) over the same universe U, we have the following.
(i) $((F, A) \wedge (G, B))^c = (F,A)^c \vee (G,B)^c$
(ii) $((F, A) \vee (G, B))^c = (F,A)^c \wedge (G,B)^c$

Proof:

(i) Let $(F,A) \wedge (G,B) = (H, A \times B)$ where $H(a,b) = F(a) \cap G(b)$ $\forall$ $a \in A$ and $\forall$ $b \in B$ where $\cap$ is the operation intersection of FNSS.

Thus $H(a, b) = F(a) \cap G(b)$

$= \{x, \min(T_{F(a)}(x), T_{G(b)}(x)), \min(I_{F(a)}(x), I_{G(b)}(x)), \max(F_{F(a)}(x), F_{G(b)}(x))\}$

$((F,A) \wedge (G,B))^c = (H, A \times B)^c \ \forall (a, b) \in A \times B$

$(H(a,b))^c = \{x, \max(F_{F(a)}(x), F_{G(b)}(x)), 1 - \min(I_{F(a)}(x), I_{G(b)}(x)), \min(T_{F(a)}(x), T_{G(b)}(x))\}$

Let $(F, A)^c \vee (G,B)^c = (R, A \times B)$

where $R(a, b) = (F(a))^c \cup (G(b))^c \ \forall$ $a \in A$ and $\forall$ $b \in B$ where $\cap$ is the operation intersection of FNSS.

$R(a, b) = \{x, \max(F_{F(a)}(x), F_{G(b)}(x)), \max(1 - I_{F(a)}(x), 1 - I_{G(b)}(x)), \min(T_{F(a)}(x), T_{G(b)}(x))\}$

$= \{x, \max(F_{F(a)}(x), F_{G(b)}(x)), 1 - \min(I_{F(a)}(x), I_{G(b)}(x)), \min(T_{F(a)}(x), T_{G(b)}(x))\}$

Thus $((F, A) \wedge (G, B))^c = (F,A)^c \vee (G,B)^c$

(ii) Similarly we can prove (ii).

4. FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

Definition 4.1: Let (F_A, E) be FNS set on (U, E) and τ be a collection of Fuzzy Neutrosophic soft subsets of (F_A, E). (F_A, E) is called Fuzzy neutrosophic soft topology (FNST) if the following conditions hold.
(i) $\phi_E, U_E \in \tau$
(ii) $F_E, G_E \in \tau$ implies $F_E \tilde{\cap} G_E \in \tau$
(iii) $\{(F_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau$ implies $\tilde{\cup} \{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau$

The triplet (U, τ, E) is called an Fuzzy Neutrosophic soft topological space (FNSTS)over U.

Every member of τ is called an Fuzzy Neutrosophic soft open set in U.

F_E is called an Fuzzy Neutrosophic soft closed set in U if $F_E \in \tau^c$, where $\tau^c = \{F_E^C : F_E \in \tau\}$

Example 4.2: Let $U = \{b_1, b_2, b_3\}$ and $E = \{e_1, e_2\}$. Let F_E, G_E, H_E, L_E be Neutrosophic soft set where

$H(e_1) = \{< b_1, 0.8, 0.4, 0.5>, < b_2, 0.7, 0.7, 0.3>, < b_3, 0.7, 0.5, 0.4>\}$

$H(e_2) = \{< b_1, 0.9, 0.5, 0.6>, < b_2, 0.8, 0.8, 0.4>, < b_3, 0.8, 0.6, 0.5>\}$

$F(e_1) = \{< b_1, 0.5, 0.4, 0.5>, < b_2, 0.6, 0.7, 0.3>, < b_3, 0.6, 0.5, 0.4>\}$

$F(e_2) = \{< b_1, 0.6, 0.5, 0.6>, < b_2, 0.7, 0.8, 0.4>, < b_3, 0.7, 0.6, 0.5>\}$

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$G(e_1)$ = {< b_1,0.8,0.4,0.7>, < b_2, 0.7,0.3,0.4>, < b_3, 0.7,0.5,0.6>}

$G(e_2)$ = {< b_1,0.9,0.5,0.8>, < b_2, 0.8,0.4,0.5>, < b_3, 0.8,0.6,0.7>}

$L(e_1)$ = {< b_1,0.5,0.4,0.7>, < b_2, 0.6,0.3,0.4>, < b_3, 0.6,0.5,0.6>}

$L(e_2)$ = {< b_1,0.6,0.5,0.8>, < b_2, 0.7,0.4,0.5>, < b_3, 0.7,0.6,0.7>}

$\tau = \{F_E, G_E, H_E, L_E, \phi_E, U_E\}$ is an Fuzzy Neutrosophic soft topology on U.

Proposition 4.3: Let (U, τ_1, E) and (U, τ_2, E) be two Fuzzy Neutrosophic soft topological spaces.

Denote $\tau_1 \cap \tau_2 = \{F_E: F_E \in \tau_1 \text{ and } F_E \in \tau_2\}$ .Then $\tau_1 \cap \tau_2$ is an FNST on U.

Proof: Obviously $\phi_E, U_E \in \tau_1 \cap \tau_2$. Let $F_E, G_E \in \tau_1 \cap \tau_2$ .Then $F_E, G_E \in \tau_1$ and $F_E, G_E \in \tau_2$. τ_1 and τ_2 are two FNST's on U. Then $F_E \cap G_E \in \tau_1$ and $F_E \cap G_E \in \tau_2$. Hence $F_E \cap G_E \in \tau_1 \cap \tau_2$.

Let $\{(F_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau_1 \cap \tau_2$. Then $(F_\alpha)_E \in \tau_1$ and $(F_\alpha)_E \in \tau_2$ for any $\alpha \in \Gamma$.

Since τ_1 and τ_2 are two FNST's on U, $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_1$ and $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_2$.

Thus $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_1 \cap \tau_2$.

Let τ_1 and τ_2 are two FNSTs on U. Denote

$\tau_1 \vee \tau_2 = \{F_E \tilde{\cup} G_E: F_E \in \tau_1 \text{ and } G \in \tau_2\}$.

$\tau_1 \wedge \tau_2 = \{F_E \tilde{\cap} G_E: F_E \in \tau_1 \text{ and } G \in \tau_2\}$.

Example 4.2: Let F_E and G_E be FNST as in example 4.3.

Define $\tau_1 = \{\phi_E, U_E, F_E\}$, $\tau_2 = \{\phi_E, U_E, G_E\}$

Then $\tau_1 \cap \tau_2 = \{\phi_E, U_E\}$ is FNSTs on U.

But $\tau_1 \cup \tau_2 = \{\phi_E, U_E, F_E, G_E\}$, $\tau_1 \vee \tau_2 = \{\phi_E, U_E, F_E, G_E, F_E \tilde{\cup} G_E\}$ and

$\tau_1 \wedge \tau_2 = \{\phi_E, U_E, F_E, G_E, F_E \tilde{\cap} G_E\}$ are not FNSTs.

Theorem 4.5: Let (U,τ, E) be a FNSTS & Let $e \in E$, $\tau(e) = \{F(e): F_E \in \tau\}$ is an FNST on U.

Proof: Let $e \in E$.

(i) $\phi_E, U_E \in \tau$, $\tilde{0}_N = \phi(e)$ and $\tilde{1}_N = U(e)$, we have $\tilde{0}_N, \tilde{1}_N \in \tau(e)$.

(ii) Let $V, W \in \tau(e)$. Then there exist $F_E, G_E \in \tau$, such that $V = F(e)$ and $W = G(e)$.
By τ be an FNST on U, $F_E \tilde{\cap} G_E \in \tau$.
Put $H_E = F_E \tilde{\cap} G_E$. Then $H_E \in \tau$.
Note that $V \cap W = F(e) \cap G(e) = H(e)$ and $\tau(e) = \{F(e): F_E \in \tau\}$ Then $V \cap W \in \tau(e)$.

(iii) Let $\{(V_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau(e)$. Then for every $\alpha \in \Gamma$, there exist $\{(F_\alpha)_E \in \tau$ such that $V_\alpha = F_\alpha(e)$. By τ be an FNST on U, $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau$.

Put $F_E = \tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\}$, then $F_E \in \tau$. Note that $\bigcup_{\alpha \in \Gamma} V_\alpha = \{F_\alpha(e) : \alpha \in \Gamma\} = F(e)$ and $\tau(e) = \{F(e) : F_E \in \tau\}$.Then $\bigcup_{\alpha \in \Gamma} V_\alpha \in \tau(e)$.

Therefore $\tau(e) = \{F(e): F_E \in \tau\}$ is an FNST on U.

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Definition 4.6: Let (U, τ, E) be a FNSTS and $\mathcal{B} \subseteq \tau$. $\mathcal{B}$ is a basis on τ if for each $G_E \in \tau$, there exist $\mathcal{B}' \subseteq \mathcal{B}$ such that $G_E = \tilde{\cup}\ \mathcal{B}'$.

Example 4.2: Let τ be a FNST as in example 4.3. Then

$\mathcal{B} = \{F_E, G_E, L_E, \phi_E, U_E\}$ is a basis for τ.

Theorem 4.8: Let $\mathcal{B}$ be a basis for FNST onτ. Denote $\mathcal{B}_e = \{F(e): F_E \in \mathcal{B}\}$and $\tau(e) = \{F(e): F_E \in \tau\}$for and e ∈ E. Then $\mathcal{B}_e$ is a basis for fuzzy neutrosophic topology τ(e).

Proof: Let e∈E. For any V ∈ τ(e), V = G(e) for $G_E \in \tau$. Here $\mathcal{B}$ is a basis for τ. Then there exists $\mathcal{B}' \subseteq \mathcal{B}$ such that $G_E = \tilde{\cup}\ \mathcal{B}'$. So $V = \tilde{\cup}\ \mathcal{B}'$ where $\mathcal{B}_e' = \{F(e) : F_E \in \mathcal{B}'\} \subseteq \mathcal{B}_e$. Thus $\mathcal{B}_e$ is a basis for Fuzzy Neutrosophic topology τ(e) for and e ∈ E.

5. SOME PROPERTIES OF FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

In this section, we give some properties of Fuzzy Neutrosophic soft topological space.

Definition 5.1: Let (U,τ, E) be a FNSTS & Let $F_E \in FNSS(U)_E$. Then interior and closure of F_E denoted respectively by FNSInt(F_E) and FNSCl(F_E) are defined as follows.

$FNSInt(F_E) = \tilde{\cup}\{G_E \in \tau: G_E \ \tilde{\subset}\ F_E\}$

$FNSCl(F_E) = \tilde{\cap}\{G_E \in \tau^c: F_E \ \tilde{\subset}\ G_E\}$.

Example 4.2: We consider the FNST given in example 4.3.

Let $M(e_1)= \{< b_1, 0.6,0.5,0.4>, < b_2, 0.7,0.8,0.3>, < b_3, 0.8,0.6,0.3>\}$

$M(e_2)= \{ < b_1, 0.7,0.6,0.4>, < b_2, 0.8,0.9,0.4>, < b_3, 0.8,0.7,0.3>\}$

$FNSInt(M_E) = F_E$

$FNSCl(M_E) = F_E^c$.

Theorem 5.3: Let (U,τ, E) be FNST over U. Then the following properties hold.
(i) U_E and ϕ_E are FNS closed sets over U.
(ii) The intersection of any number of FNS closed sets is a FNS closed set over U.
(iii) The union of any two FNS closed sets is an FNS closed set over U.

Proof: It is obvious from the proposition 3.12

Theorem 5.4: Let (U,τ, E) be a FNST & Let $F_E \in FNSS(U)_E$ .Then the following properties hold.
(i) $FNSInt(F_E) \ \tilde{\subset}\ F_E$
(ii) $F_E \ \tilde{\subset}\ G_E \Rightarrow FNSInt(F_E) \ \tilde{\subset}\ FNSInt(G_E)$.
(iii) $FNSInt(F_E) \in \tau$.
(iv) F_E is a FNS open set $\Leftrightarrow FNSInt(F_E) = F_E$.
(v) $FNSInt(FNSInt(F_E)) = FNSInt(F_E)$
(vi) $FNSInt(\phi_E) = \phi_E$, $FNSInt(U_E) = U_E$.

Proof:
(i) and (ii) are obvious.

(iii) Obviously $\tilde{\cup}\{G_E \in \tau : G_E \ \tilde{\subset}\ F_E\} \in \tau$

Note that $\tilde{\cup}\{G_E \in \tau : G_E \ \tilde{\subset}\ F_E\} = FNSInt(F_E)$

$\Rightarrow FNSInt(F_E) \in \tau$.

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(iv) Necessity: Let F_E be a FNS open set. ie., $F_E \in \tau$.

By (i) and (ii) FNSInt $(F_E) \tilde{\subseteq} F_E$.

Since $F_E \in \tau$ and $F_E \tilde{\subseteq} F_E$

Then $F_E \tilde{\subseteq} \tilde{\cup} \{G_E \in \tau: G_E \tilde{\subseteq} F_E\}$= FNSInt (F_E).

ie ., $F_E \tilde{\subseteq}$ FNSInt(F_E).

Thus FNSInt = F_E.

Sufficiency: Let FNSInt$(F_E) = F_E$

By (iii) FNSInt$(F_E) \in \tau$, ie., F_E is a FNS open set.

(v) To prove FNSInt (FNSInt (F_E)) = FNSInt (F_E)

By (iii) FNSInt $(F_E) \in \tau$.

By (iv) FNSInt (FNSInt (F_E)) = FNSInt (F_E).

(vi) We know that U_E and $\phi_E \in \tau$.

By (iv) FNSInt $(\phi_E) = \phi_E$, FNSInt $(U_E) = U_E$.

Theorem 5.5: Let (U, τ, E) be a FNST & Let $F_E \in$ FNSS $(U)_E$. Then the following properties hold.
(i) $(F_E) \tilde{\subseteq}$ FNSCl(F_E)
(ii) $F_E \tilde{\subseteq} G_E \Rightarrow$ FNSCl $(F_E) \tilde{\subseteq}$ FNSCl (G_E).
(iii) $(\text{FNSCl}(F_E))^c \in \tau$.
(iv) F_E is a FNS closed set $\Leftrightarrow$ FNSCl$(F_E) = F_E$.
(v) FNSCl (FNSCl (F_E)) = FNSCl (F_E)
(vi) FNSCl $(\phi_E) = \phi_E$, FNSCl$(U_E) = U_E$.

Proof:
(i) and (ii) are obvious.

(iii) By theorem 5.4 (iii) FNSInt(F_E^C)$\in \tau$.

Therefore $[(\text{FNSCl}(F_E)]^c = (\tilde{\cap} \{G_E \in \tau^c : F_E \tilde{\subseteq} G_E\})^c$

$= \tilde{\cup} \{G_E \in \tau : G_E \tilde{\subseteq} F_E^C\}$

$= \text{FNSInt } F_E^C$

Then $[(\text{FNSCl}(F_E)]^c \in \tau$.

(iv) Necessity: By theorem 5.5 (i) $F_E \tilde{\subseteq}$ FNSCl (F_E)

Let F_E be a FNS closed set ie., $F_E \in \tau^c$ and $F_E \tilde{\subseteq} F_E$.

FNSCl$(F_E) = \tilde{\cap} \{G_E \in \tau^c : F_E \tilde{\subseteq} G_E\} \tilde{\subseteq} \{F_E \in \tau^c : F_E \tilde{\subseteq} F_E\}$.

Then FNSCl$(F_E) \tilde{\subseteq} F_E$. Thus F_E = FNSCl(F_E).

Sufficiency: This holds by (iii).

(v) and (vi) hold by (iii) and (iv).

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Theorem 5.6: Let (U, τ, E) be a FNST & Let $F_E, G_E \in FNSS(U)_E$ .Then the following properties hold.

(i) $FNSInt(F_E) \tilde{\cap} FNSInt(F_E) = FNSInt(F_E \tilde{\cap} G_E)$

(ii) $FNSInt(F_E) \tilde{\cup} FNSInt(F_E) \tilde{\subseteq} FNSInt(F_E \tilde{\cup} G_E)$

(iii) $FNSCl(F_E) \tilde{\cup} FNSCl(G_E) = FNSCl(F_E \tilde{\cup} G_E)$

(iv) $FNSCl(F_E \tilde{\cup} G_E) \tilde{\subseteq} FNSCl(F_E) \tilde{\cap} FNSCl(G_E)$

(v) $(FNSInt(F_E))^c = FNSCl(F_E^c)$

(vi) $(FNSCl(F_E))^c = FNSInt(F_E^c)$

Proof:

(i) Since $F(e) \tilde{\cap} G(e) \tilde{\subseteq} F(e)$ for any $e \in E$,

We have $F_E \tilde{\cap} G_E \tilde{\subseteq} F_E$

By theorem 5.4 (ii) $FNSInt(F_E \tilde{\cap} G_E) \tilde{\subseteq} FNSInt(F_E)$.

Similarly $FNSInt(F_E \tilde{\cap} G_E) \tilde{\subseteq} FNSInt(G_E)$.

Thus $FNSInt(F_E \tilde{\cap} G_E) \tilde{\subseteq} FNSInt(F_E) \tilde{\cap} FNSInt(G_E)$

By theorem 5.4 (i), $FNSInt(F_E) \tilde{\subseteq} (F_E)$ and $FNSInt(G_E) \tilde{\subseteq} (G_E)$.

Then $FNSInt(F_E \tilde{\cap} G_E) \tilde{\subseteq} F_E \tilde{\cap} G_E$

So $FNSInt(F_E) \tilde{\cap} FNSInt(F_E) \tilde{\subseteq} FNSInt(F_E \tilde{\cap} G_E)$.

Similarly we can prove (ii), (iii) & (iv).

(v) $(FNSInt(F_E))^c = \tilde{\cup}(\{G_E \in \tau : G_E \tilde{\subseteq} F_E\})^c = \tilde{\cap}\{G_E \in \tau^c : F_E^C \tilde{\subseteq} G_E\}$

$= FNSCl(F_E^C)$

(vi) The proof is similar to (v).

Example 5.7: Let $U = \{b_1, b_2\}$ and $E=\{e_1, e_2\}$. Let F_E be Fuzzy Neutrosophic soft set where

$F(e_1) = \{< b_1, 0.2,0.6,0.8>, < b_2, 0.6,0.5,0.3>\}$

$F(e_2) = \{< b_1, 0.2,0.4,0.5>, < b_2, 0.9,0.7,0.1>\}$

Obviously $\tau = \{F_E, \phi_E, U_E\}$ is an Fuzzy Neutrosophic soft topology on U.

G_E, H_E are

$G(e_1)= \{< b_1, 0.1,0.2,0.8> , < b_2, 0.6,0.5,0.3 >\}$

$G(e_2)= \{< b_1, 0.2,0.4,0.5>, < b_2, 0.9,0.7,0.1>\}$

$H(e_1)= \{< b_1, 0.2,0.6,0.8>, < b_2, 0.6,0.5,0.3>\}$

$H(e_2)= \{< b_1, 0.2,0.4,0.5>, < b_2, 0.7,0.6,0.1>\}$

(i) $FNSInt(G_E) = \phi_E = FNSInt(H_E)$

$G_E \tilde{\cup} H_E = F_E$

$FNSInt(G_E) \tilde{\cup} FNSInt(H_E) = \phi_E \tilde{\cup} \phi_E = \phi_E$ and $FNSInt(G_E \tilde{\cup} H_E) = FNSInt(F_E) = F_E$.

Therefore $FNSInt(G_E) \tilde{\cup} FNSInt(H_E) \neq FNSInt(G_E \tilde{\cup} H_E)$.

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(ii) By theorem 5.6 (v)

$$FNSCl\ (G_E^c) = (FNSInt\ (G_E))^c = \phi_E^c = U_E$$

Similarly $FNSCl\ (H_E^c) = U_E$.

$$FNSCl\ (G_E^c) \tilde{\cap} FNSCl\ (H_E^c) = U_E \tilde{\cap} U_E = U_E.$$

Similarly $FNSCl(G_E^c \tilde{\cap} H_E^c) = FNSCl(G_E \tilde{\cup} H_E)^c$

$$= [FNSInt\ (G_E \tilde{\cup} H_E)]^c$$

$$= F_E^c$$

Thus $FNSCl(G_E^c \tilde{\cap} H_E^c) \neq FNSCl\ (G_E^c) \tilde{\cap} FNSCl\ (H_E^c)$.

6. CONCLUSION

We have introduced the concept of fuzzy Neutrosophic soft set and studied some of its properties. We have put forward some proposition based on this new notion. We have introduced topological structure on fuzzy Neutrosophic soft set and characterized some of its properties. We hope that this paper will promote the future study on FNSS and FNSTS to carry out a general framework for their application in practical life.

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