# EVEN GRACEFUL LABELING FOR DIFFERENT PATHS USING PADAVON SEQUENCE 

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#### Abstract

The even graceful labeling of a graph $G$ with $q$ edges means that there is an injection $f$ : $V(G)$ to $\{0,2,4, \ldots, 2 q+2 i / i=1$ to $n\}$ such that when each edge $u v$ is assigned the label $|f(u)-f(v)|$ the resulting edge labels are $\{2,4,6, \ldots, 2 q\}$. A graph which admits an even graceful labeling is called an even graceful graph. In this paper, the even gracefulness of paths $p_{1,}, p_{2}, p_{3, \ldots,}, p_{11}$ is obtained.


Keywords: Padavon sequence, vertex labeling, edge labeling, graceful labeling, even graceful labeling.

## 1. INTRODUCTION

By a graph, we mean a finite undirected graph without loops or multiple edges. A path on n vertices is denoted by $\mathrm{P}_{\mathrm{n}}$. Graph labeling, where the vertices are assigned certain values subject to some conditions, have often motivated by practical problems. In the last five decades, enormous work has been done on this subject [2].

A study of graceful graphs and graceful labeling methods was introduced by Rosa [4]. Rosa defined a $\beta$-valuation of a graph $G$ with $q$ edges an injection from the vertices of $G$ to the set $\{0,1 \ldots \mathrm{q}\}$ such that when each edge uv is assigned the label $|\mathrm{f}(\mathrm{u})-\mathrm{f}(\mathrm{v})|$, the resulting edges are distinct. $\beta$ - valuation is a function that produces graceful labeling. However the term graceful labeling was not used until Golomb studied such labeling several years later [3].

## 2. DEFINITIONS

Definition 2.1: Graceful graph: A function $f$ of a graph $G$ is called a graceful labeling with $m$ edges, if $f$ is an injection from the vertex set of $G$ to the set $\{0,1 \ldots \mathrm{~m}\}$ such that when each edge uv is assigned the label $|f(\mathrm{u})-\mathrm{f}(\mathrm{v})|$ and the resulting labels are distinct. Then the graph G is graceful.

Definition 2.2: Path: A path in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. The first vertex is called the start vertex and the last vertex is called the end or terminal vertices of the path and the other vertices in the path are internal vertices.

Definition 2.3: Even graceful graph: The even graceful labeling of a graph G with q edges means that there is an injection $f: V(G)$ to $\{0,2,4, \ldots, 2 q+2 i / i=1$ to $n\}$ such that when each edge uv is assigned the label $|f(\mathrm{u})-\mathrm{f}(\mathrm{v})|$ the resulting edge labels are $\{2,4,6, \ldots, 2 \mathrm{q}\}$. A graph which admits an even graceful labeling is called an even graceful graph.

## Definition 2.4: Padavon sequence:

The padavon sequence is the sequence of integers $\mathrm{P}(\mathrm{n})$ defined by the initial values $\mathrm{P}(0)=\mathrm{P}(1)=\mathrm{P}(2)=1$, and the recurrence relation
$\mathrm{P}(\mathrm{n})=\mathrm{P}(\mathrm{n}-2)+\mathrm{P}(\mathrm{n}-3)$.
The first few values of $\mathrm{P}(\mathrm{n})$ are $1,1,1,2,2,3,4,5,7,9,12,16,21,28,37,49,65,86,114,151,200,265,351,465$, 616,...

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## 3. VARIOUS EVEN GRACEFUL LABELING

### 3.1 Even gracefulness of path $\mathbf{P}_{\mathbf{2}}$ :

Theorem 1: $\mathrm{P}_{\mathrm{n}}$ is even graceful for every integer $\mathrm{n} \geq 2$.
Proof: Let $V\left(P_{2}\right)=\left\{u_{1}, u_{2}\right\}$ where $V\left(P_{2}\right)$ is the vertex set of the path $P_{2}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$f\left(u_{1}\right)=0 ; f\left(u_{2}\right)=q+1$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=\mathrm{q}+1$
Figure 1 shows the method of even graceful labeling of the path $\mathrm{P}_{2}$.
This completes the proof.
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Fig. - 1
Theorem 2: $P_{n}$ is even graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{3}\right)=\left\{u_{1}, u_{2}, u_{3}\right\}$ where $V\left(P_{3}\right)$ is the vertex set of the path $P_{3}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(\mathrm{u}_{\mathrm{i}}\right)$ is defined as follows.
$\mathrm{f}\left(\mathrm{u}_{1}\right)=0 ; \mathrm{f}\left(\mathrm{u}_{2}\right)=\mathrm{q} ; \mathrm{f}\left(\mathrm{u}_{3}\right)=\mathrm{q}+2$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=\mathrm{q} ; \mathrm{f}^{*}\left(\mathrm{u}_{2} \mathrm{u}_{3}\right)=\mathrm{q}$
Figure 2 shows the method of even graceful labeling of the path $\mathrm{P}_{3}$.
This completes the proof.


Fig. - 2
Theorem 3: $P_{n}$ is even graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{4}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}\right\}$ where $V\left(P_{4}\right)$ is the vertex set of the path $P_{4}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$\mathrm{f}\left(\mathrm{u}_{1}\right)=0 ; \mathrm{f}\left(\mathrm{u}_{2}\right)=\mathrm{q}-1 ; \mathrm{f}\left(\mathrm{u}_{3}\right)=\mathrm{q}+3 ; \mathrm{f}\left(\mathrm{u}_{4}\right)=\mathrm{q}+1$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=\mathrm{f}^{*}\left(\mathrm{u}_{3} \mathrm{u}_{4}\right)=\mathrm{q}-1 ; \mathrm{f}^{*}\left(\mathrm{u}_{2} \mathrm{u}_{3}\right)=\mathrm{q}+1$
Figure 3 shows the method of even graceful labeling of the path $P_{4}$.
This completes the proof.

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Fig. - 3
Theorem 4: $\mathrm{P}_{\mathrm{n}}$ is even graceful for every integer $\mathrm{n} \geq 2$.
Proof: Let $V\left(P_{5}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}\right\}$ where $V\left(P_{5}\right)$ is the vertex set of the path $P_{5}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$f\left(u_{1}\right)=0 ; f\left(u_{2}\right)=3 q ; f\left(u_{3}\right)=2 q ; f\left(u_{4}\right)=q+2 ; f\left(u_{5}\right)=q ;$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=3 \mathrm{q} ; \mathrm{f}^{*}\left(\mathrm{u}_{2} \mathrm{u}_{3}\right)=\mathrm{q} ; \mathrm{f}^{*}\left(\mathrm{u}_{3} \mathrm{u}_{4}\right)=\mathrm{q}-2=\mathrm{f}^{*}\left(\mathrm{u}_{4} \mathrm{u}_{5}\right)$
Figure 4 shows the method of even graceful labeling of the graph $\mathrm{P}_{5}$
This completes the proof.


Fig. - 4
Theorem 5: $P_{n}$ is even graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{6}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}\right\}$ where $V\left(P_{6}\right)$ is the vertex set of the path $P_{6}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$\mathrm{f}\left(\mathrm{u}_{1}\right)=0 ; \mathrm{f}\left(\mathrm{u}_{2}\right)=3 \mathrm{q}+1 ; \mathrm{f}\left(\mathrm{u}_{3}\right)=\mathrm{q}-\mathrm{q} ; \mathrm{f}\left(\mathrm{u}_{4}\right)=\mathrm{q}-3 ;$
$\mathrm{f}\left(\mathrm{u}_{5}\right)=\mathrm{q}+1 ; \mathrm{f}\left(\mathrm{u}_{6}\right)=\mathrm{q}+3$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=3 \mathrm{q}+1 ; \mathrm{f}^{*}\left(\mathrm{u}_{2} \mathrm{u}_{3}\right)=2 \mathrm{q}+2 ; \mathrm{f}^{*}\left(\mathrm{u}_{3} \mathrm{u}_{4}\right)=\mathrm{f}^{*}\left(\mathrm{u}_{5} \mathrm{u}_{6}\right)=\mathrm{q}-3 ;$
$\mathrm{f}^{*}\left(\mathrm{u}_{4} \mathrm{u}_{5}\right)=\mathrm{q}-1$
Figure 5 shows the method of even graceful labeling of the path $\mathrm{P}_{6}$.
This completes the proof.


Fig. - 5

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Theorem 6: $P_{n}$ is padavon graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{7}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}, u_{7}\right\}$ where $V\left(P_{7}\right)$ is the vertex set of the path $P_{7}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$f\left(u_{1}\right)=0 ; f\left(u_{2}\right)=5 q-2 ; f\left(u_{3}\right)=2 q ; f\left(u_{4}\right)=3 q-2 ;$
$\mathrm{f}\left(\mathrm{u}_{5}\right)=\mathrm{q}+2 ; \mathrm{f}\left(\mathrm{u}_{6}\right)=\mathrm{q} ; \mathrm{f}\left(\mathrm{u}_{7}\right)=\mathrm{q}+2$
The edge labeling function $f$ * is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=5 \mathrm{q}-2 ; \mathrm{f}^{*}\left(\mathrm{u}_{2} \mathrm{u}_{3}\right)=3 \mathrm{q}-2: \mathrm{f}^{*}\left(\mathrm{u}_{3} \mathrm{u}_{4}\right)=\mathrm{q}-2 ;$
$\mathrm{f}^{*}\left(\mathrm{u}_{4} \mathrm{u}_{5}\right)=2 \mathrm{q} ; \mathrm{f}^{*}\left(\mathrm{u}_{5} \mathrm{u}_{6}\right)=\mathrm{q}-4=\mathrm{f}^{*}\left(\mathrm{u}_{6} \mathrm{u}_{7}\right)$
Figure 6 shows the method of even graceful labeling of the path $P_{7}$.
This completes the proof.


Fig. - 6
Theorem 7: $P_{n}$ is padavon graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{8}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}, u_{7}, u_{8}\right\}$ where $V\left(P_{8}\right)$ is the vertex set of the path $P_{8}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$f\left(u_{1}\right)=0 ; f\left(u_{2}\right)=12 q+2 ; f\left(u_{3}\right)=8 q+2 ; f\left(u_{4}\right)=5 q-3 ;$
$f\left(u_{5}\right)=4 q+2 ; f\left(u_{6}\right)=4 q-2 ; f\left(u_{7}\right)=3 q+3 ; f\left(u_{8}\right)=3 q+1$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$\mathrm{f}^{*}\left(\mathrm{u}_{1} \mathrm{u}_{2}\right)=12 \mathrm{q}+2 ; \mathrm{f}^{*}\left(\mathrm{u}_{2} \mathrm{u}_{3}\right)=4 \mathrm{q} ; \mathrm{f}^{*}\left(\mathrm{u}_{3} \mathrm{u}_{4}\right)=2 \mathrm{q}+2 ;$
$\mathrm{f}^{*}\left(\mathrm{u}_{4} \mathrm{u}_{5}\right)=2 \mathrm{q}-2 ; \mathrm{f}^{*}\left(\mathrm{u}_{5} \mathrm{u}_{6}\right)=\mathrm{q}-3 ; \mathrm{f}^{*}\left(\mathrm{u}_{6} \mathrm{u}_{7}\right)=\mathrm{q}-5=\mathrm{f}^{*}\left(\mathrm{u}_{7} \mathrm{u}_{8}\right)$
Figure 7 shows the method of even graceful labeling of the path $P_{8}$.
This completes the proof.


Fig. -7

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Theorem 8: $P_{n}$ is even graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{9}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}, u_{7}, u_{8}, u_{9}\right\}$ where $V\left(P_{9}\right)$ is the vertex set of the path $P_{9}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$\mathrm{f}\left(\mathrm{u}_{1}\right)=0 ; \mathrm{f}\left(\mathrm{u}_{2}\right)=14 \mathrm{q}+2 ; \mathrm{f}\left(\mathrm{u}_{3}\right)=3 \mathrm{q}+4 ; \mathrm{f}\left(\mathrm{u}_{4}\right)=\mathrm{q}+4 ;$
$f\left(u_{5}\right)=5 q ; f\left(u_{6}\right)=4 q+4 ; f\left(u_{7}\right)=3 q ; f\left(u_{8}\right)=3 q-2 ; f\left(u_{9}\right)=2 q+4$
The edge labeling function $\mathrm{f}^{*}$ is defined as follows.
$f^{*}\left(u_{1} u_{2}\right)=14 q+2 ; f^{*}\left(u_{2} u_{3}\right)=11 q-2: f^{*}\left(u_{3} u_{4}\right)=2 q ;$
$f^{*}\left(u_{4} u_{5}\right)=3 q+4 ; f^{*}\left(u_{5} u_{6}\right)=q-4 ; f^{*}\left(u_{6} u_{7}\right)=q+4 ; f^{*}\left(u_{7} u_{8}\right)=q-6=f^{*}\left(u_{8} u_{9}\right)$
Figure 8 shows the method of even graceful labeling of the path $\mathrm{P}_{9}$.
This completes the proof.


Fig. - 8
Theorem 9: $P_{n}$ is even graceful for every integer $n \geq 2$.
Proof: Let $\mathrm{V}\left(\mathrm{P}_{10}\right)=\left\{\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{u}_{4}, \mathrm{u}_{5}, \mathrm{u}_{6}, \mathrm{u}_{7}, \mathrm{u}_{8}, \mathrm{u}_{9}, \mathrm{u}_{10}\right\}$ where $\mathrm{V}\left(\mathrm{P}_{10}\right)$ is the vertex set of the path $\mathrm{P}_{10}$. For every vertex $\mathrm{u}_{\mathrm{i}}$, the even graceful labeling functions $\mathrm{f}\left(\mathrm{u}_{\mathrm{i}}\right)$ is defined as follows.
$f\left(u_{1}\right)=0 ; f\left(u_{2}\right)=22 q+2 ; f\left(u_{3}\right)=9 q+5 ; f\left(u_{4}\right)=8 q-2 ;$
$\mathrm{f}\left(\mathrm{u}_{5}\right)=17 \mathrm{q}+3 ; \mathrm{f}\left(\mathrm{u}_{6}\right)=14 \mathrm{q}+2 ; \mathrm{f}\left(\mathrm{u}_{7}\right)=13 \mathrm{q}+1 ; \mathrm{f}\left(\mathrm{u}_{8}\right)=13 \mathrm{q}-5 ;$
$f\left(u_{9}\right)=12 q+2 ; f\left(u_{10}\right)=12 q-2 ;$
The edge labeling function $f^{*}$ is defined as follows.
$f^{*}\left(u_{1} u_{2}\right)=22 q+2 ; f^{*}\left(u_{2} u_{3}\right)=13 q-3 ; f^{*}\left(u_{3} u_{4}\right)=2 q-2 ;$
$\mathrm{f}^{*}\left(\mathrm{u}_{4} \mathrm{u}_{5}\right)=9 \mathrm{q}+5 ; \mathrm{f}^{*}\left(\mathrm{u}_{5} \mathrm{u}_{6}\right)=3 \mathrm{q}+1 ; \mathrm{f}^{*}\left(\mathrm{u}_{6} \mathrm{u}_{7}\right)=\mathrm{q}+3 ;$
$\mathrm{f}^{*}\left(\mathrm{u}_{7} \mathrm{u}_{8}\right)=\mathrm{q}-5 ; \mathrm{f}^{*}\left(\mathrm{u}_{8} \mathrm{u}_{9}\right)=\mathrm{q}-7=\mathrm{f}^{*}\left(\mathrm{u}_{9} \mathrm{u}_{10}\right)$
Figure 9 shows the method of even graceful labeling of the path $\mathrm{P}_{10}$.
This completes the proof.


Fig. - 9

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Theorem 10: $P_{n}$ is even graceful for every integer $n \geq 2$.
Proof: Let $V\left(P_{11}\right)=\left\{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}, u_{7}, u_{8}, u_{9}, u_{10}, u_{11}\right\}$ where $V\left(P_{11}\right)$ is the vertex set of the path $P_{11}$. For every vertex $u_{i}$, the even graceful labeling functions $f\left(u_{i}\right)$ is defined as follows.
$f\left(u_{1}\right)=0 ; f\left(u_{2}\right)=62 q-4 ; f\left(u_{3}\right)=42 q-4 ; f\left(u_{4}\right)=30 q+2 ;$
$f\left(u_{5}\right)=21 q+6 ; f\left(u_{6}\right)=19 q-2 ; f\left(u_{7}\right)=17 q+2 ; f\left(u_{8}\right)=16 q ;$
$f\left(u_{9}\right)=16 q-4 ; f\left(u_{10}\right)=15 q+4 ; f\left(u_{11}\right)=15 q+2$.
The edge labeling function $f^{*}$ is defined as follows.
$f^{*}\left(u_{1} u_{2}\right)=62 q-4 ; f^{*}\left(u_{2} u_{3}\right)=20 q ; f^{*}\left(u_{3} u_{4}\right)=11 q+4 ;$
$f^{*}\left(u_{4} u_{5}\right)=8 q+6 ; f^{*}\left(u_{5} u_{6}\right)=3 q-2 ; f^{*}\left(u_{6} u_{7}\right)=2 q-4 ;$
$\mathrm{f}^{*}\left(\mathrm{u}_{7} \mathrm{u}_{8}\right)=\mathrm{q}+2 ; \mathrm{f}^{*}\left(\mathrm{u}_{8} \mathrm{u}_{9}\right)=\mathrm{q}-6 ; \mathrm{f}^{*}\left(\mathrm{u}_{9} \mathrm{u}_{10}\right)=\mathrm{q}-8=\mathrm{f}^{*}\left(\mathrm{u}_{10} \mathrm{u}_{11}\right)$
Figure 10 shows the method of even graceful labeling of the pathgraph $\mathrm{P}_{11}$.
This completes the proof.


Fig. - 10

## CONCLUSION

In all the above paths, we can see that both the edges and vertices are labeled with even numbers and hence even graceful graph.

## REFERENCES

1. R.B. Gnanajothi, Topics in Graph Theory, Ph.D. Thesis, Madurai Kamaraj University, 1991.
2. G. J. Gallian, A dynamic survey of graph labeling, The electronic journal of combinatorics, 16 (2009), \#DS6.
3. W. Golomb, How to number a graph in graph theory and computing, R.C. Read, ed., Academic Press, New York (1972), 23-37.
4. A. Rosa, On certain valuations of the vertices of a graph, Theory of graphs(International Symposium, Rome), July (1966).
5. Uma Maheswari, Graceful Labeling of the paths using padavon sequence, International Journal of Mathematical Archive-4(4), 013, 66-71.

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