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RADIATION EFFECTS FOR BOUNDARY LAYER FLOW OVER A FLATE WITH A CONVECTIVE THERMAL SURFACE BOUNDARY CONDITION

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ABSTRACT

The boundary layer flow over a flat plate with convective thermal bounday condition in the presence of MHD and temperature dependent heat source with effects of radiation is studied.
A similarity transformation is used to reduce the governing nonlinear partial differential equations into a system of nonlinear ordinary differential equations. These equations are solved numerically by shooting iteration technique together with fourth order Runge-kutta integration scheme. Solutions showing the effects of Prandtl numbers $(\Pr)$ *,local Grashof numbers* (G_x) *,Hartmann number* (M) *,Biot numbers* (a_x) *,heat source parameters* (α) *and radiation parameter* (ω)*. The solutions are discussed with aid of tables and graphs.*

Keywords: *flat plate, boundary layer,convective thermal condition and radiation.*

1. INTRODUCTION

Boundary layer flow over a flate has attracted attention of researchers because of its application in many areas of studies such as aerodynamics,hydrodyanmics,transportation,wind and ocean enginering.The continuity and momentum equations of hydrodynamics boundary layer was first solved in 1908 by Blasius.This was done by transforming the two partial equation into a single ordinary differential equation by introducing a new indepenent variables,called the similarity variables.The similarity variables have been applyiing for the thermal boundary layer for the case of constant surface temperature at the plate is also well established and quoted in heat transfer textbooks such as [2].

A convective thermal surface boundary condition was first considered by Aziz[3], applied similarity variables to laminar thermal boundary layer flow over a flat with a convective surface boundary conditions and He considered only Biot number on the flow characteristics. Makinde and Olanrewaju [4] have presented buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition. Okedayo T et al. [5] presented analysis of convective plane stagnation point flow with convective boundary conditions. Recently, Disu and Ajibola [6] have extended the work of Mankinde and Olanrewaju to include the effect of magnetic field and temperature dependent source. However,the effects of thermal radiation on the boundary layer flow over a flat with a convective boundary condition has been not considered,which is the focus of this work.

In this paper, we present radiation effects for boundary layer flow over a flate with a convective thermal surface boundary condition.The numerical solutions of the resulting momentum and energy equations are reported for representative values of thermophysical parameters characterizing the fluid convection process.

2. MATHEMATICAL ANALYSIS

We consider two-dimensional steady incompressible fluid flow with heat transfer by convection over a flat plate. A stream of cold fluid at temperature T_∞ moving over the right surface of the plate with a uniform velocity U_∞ while the left surface of the plate is heated by convection from a hot fluid at temperature T_f, which provides a heat transfer coefficient h_f under the influence of magnetic field, temperature dependent heat source and effect of radiation.

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The continuity, momentum and energy equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\mu_\infty H_0^2 u}{\rho} \tag{2}$$

$$\rho Cp\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q}{\partial y}+Q(T-T_\infty) \tag{3}$$

where u, v are the velocity components, p the pressure, μ_∞ the magnetic permeability, k the thermal conductivity, ρ the density of the fluid, β the coefficient of volume expansion, Cp the specific heat at constant pressure. T_∞ is the equilibrium temperature and other symbols have their usual meanings.

The velocity boundary conditions relevant to the problem are taken as

$$u(x,0)=v(x,0)=0 \tag{4}$$

$$u(x,\infty)=T_\infty \tag{5}$$

We assume the bottom surface is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f .The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right] \tag{6}$$

$$T(x,\infty)=T_\infty \tag{7}$$

We assume the Rosseland approximation (Brewster 1992) for radiative heat flux, which leads to

$$q=-\frac{4\sigma}{3R}\frac{\partial T^4}{\partial y} \tag{8}$$

where is σ the Stefan-Boltzmann constant and R is the mean absorption coefficient.

If the temperature differences within the flow are such that T^4 may be expressed as a linear function of the temperature T. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, is given by

$$T^4=4T_\infty^3 T-3T_\infty^4 \tag{9}$$

Using Eqs. (8) and (9),Eq. (3) gives

$$\rho Cp\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\frac{16T_\infty^3}{3R}\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty) \tag{10}$$

A similarity solution of Eqs. (1), (2) (4),(5) and (10) is obtained by defining an independent variable η and a dependent variable variable f in terms of the stream function ψ as

$$\eta=y\sqrt{\frac{U_\infty}{\upsilon x}}\quad,\ f(\eta)=\frac{\psi}{U_\infty\sqrt{\frac{\upsilon x}{U_\infty}}}\quad,u=U_\infty\frac{df}{d\eta}\ ,v=\frac{1}{2}\sqrt{\frac{U_\infty\upsilon}{x}}\left(\eta\frac{df}{d\eta}-f\right) \tag{11}$$

and the dimensionless temperature θ, Prandtl number $\Pr$, magnetic parameter M,Grashof number Gr and heat source parameter γ

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}\ ,\ \Pr = \frac{\mu Cp}{k},\ M = \frac{\mu_\infty H_0^2 x}{\rho U_\infty}\ ,\ G_x = \frac{g\beta x^3 \left(T_f - T_\infty\right)}{\upsilon^3}, \alpha = \frac{Qx}{kU_\infty} \tag{12}$$

the continuity equation is identically satisfied and the momentum and heat transfer equations reduced to

$$f''' + \frac{1}{2} ff'' - Mf' + G_x \theta = 0 \tag{13}$$

$$\omega\theta'' + \frac{1}{2}\Pr f\theta' - \alpha\theta = 0 \tag{14}$$

where $\omega = 1 + \dfrac{4}{3R}$, it may also be noted here that when radiation parameter $R \to \infty$, the set of Eqs. (1) - (3) represents boundary layer flow in absence of radiation.

The transformed boundary conditions are

$$f(0) = 0,\ f'(0) = 0,\ \theta'(0) = -a_x\left[1 - \theta(0)\right] \tag{15}$$

$$f'(\infty) = 1, \theta(\infty) = 0 \tag{16}$$

where

$$a_x = \frac{hf}{k}\sqrt{\frac{\upsilon x}{U_\infty}}. \text{ is the Biot number} \tag{17}$$

For the momentum and the energy equations to have a similarity solution, the parameter Gr_x and a_x must be constants and not functions of x as in Eqs. (12) and (15).This condition can be met if the thermal expansion coefficient β is proportional to x^{-1} [11] and the heat transfer coefficient h_f is proportional to $x^{-\frac{1}{2}}$ [6] .We therefore assume

$$h_f = cx^{-\frac{1}{2}},\ \beta = mx^{-1} \tag{18}$$

where c and m are constants . Using Eq. (15) in (14), we obtained

$$G_x = \frac{\upsilon mg(T_f - T_\infty)}{U_\infty^2},\ a_x = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}} \tag{16}$$

With a_x and G_x by Eq. (16), the solutions of Eqs. (13)–(16) yield the similarity solutions. However, the solutions generated are the local similarity solutions whenever a_x and Gr_x are defined as in Eqs. (12) and (17).

3. NUMERICAL RESULTS AND DISCUSSIONS

The governing partial differential equations are transformed into non-linear ordinary differential equations with appropriate boundary conditions by similarity variable. Finally, the systems of similarity equations with boundary conditions are solved numerically by employing using the shooting iteration technique together with fourth order Runge-kutta integration scheme built in Maple 17.

The computations have been carried out for various values of Hartmann M, Grashof number G_x, Prandtl number $\Pr$, heat source parameter α, Biot number a_x and the Radiation parameter R. The edge of the boundary layer depending on the values of $\eta = 5$.

The velocity and temperature distributions are discussed with reference to variations in Grashof number G_x, Hartmann number M, heat source parameter α, Biot number a_x and the Radiation parameter R. The velocity and temperature profiles are drawn for different sets of values.

Figure 1(a) shows the numerical results for different values of Hartmann number M. It is observed that as the magnetic strength increases, the velocity decreases. Furthermore, the velocity boundary layer thickness becomes thinner. This shows that the rate of transport is considerably reduced by the presence of the magnetic field. This is due to the fact that the variation of M leads to the variation of Lorentz force due to the magnetic field and Lorentz force produces more resistance to the transport phenomena. In Figure(b) ,magnetic field has no effect on the thermal boundary layer thickness.

Figure 2(a) shows the numerical results for different values of Grashof number, an increase in Grashof number, also increase the velocity and thicken velocity boundary layer. This shows that the rate of transport increases by the presence of Grashof number but it has no effect on thermal boundary layer thickness in Figure 2(b).

Figure 3(a) shows that for different values of Prandtl number, an increase in Prandtl number has no effect on the velocity boundary layer thickness and has no effect on the velocity boundary layer thickness in Figure 3(b).

Figure 4(a) shows the numerical result for different values of radiation parameter, an increases in heat source parameter, the velocity increases and the velocity boundary layer thickness becomes thicken. This shows that the rate of transport is increased by the presence of the radiation . In Figure 4(b) the thermal boundary layer thickness becomes thicken and also increase the wall temperature.

Figure 5(a) shows the numerical result for different values of heat source parameter,an increases in heat source parameter, the velocity decreases and the velocity boundary layer thickness becomes thinner. This shows that the rate of transport is reduced by the presence of the heat source parameter . In Figure 5(b) the thermal boundary layer thickness becomes thinner and also decreases the wall temperature.

Figure 6(a) the numerical results for different values of, increase in the Biot number thickens the thermal boundary layer thickness and also increases the velocity and thickens the thermal boundary layer thickness and also increases the wall temperature in Figure 6(b).

From table1, we see that the local Skin Friction Coefficient at the surface decreases by increasing of Magnetic field strength but has no effect on the Nusselt Number. The results presented demonstrate quite clearly that M,which is an indicator of the viscosity with temperature, has a substantial effect on the drag and heat transfer characteristics. From Table 2, we see that the local skin friction coefficient at the surface increases by increasing of the Grashof number but has no local Nusselt number.

Table 3 shows that an increase in Prandtl number has no significant on both local skin friction coefficient at the surface and the local Nusselt number.Table 4 shows that the local Skin Friction Coefficient at the surface decreases by increasing of heat source but the Nusselt Number decreases.

Table 5, show that an increase in Biot number has significant increseases on both local skin friction coefficient at the surface and the local Nusselt number.Table 6, show that an increase in Biot number has significant increseases on both local skin friction coefficient at the surface and the local Nusselt number.

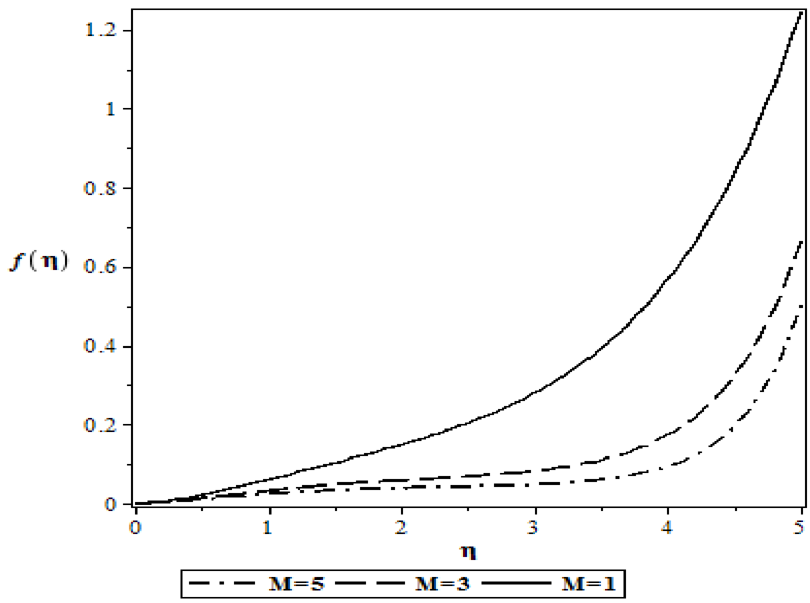

Figure 1(a): Effects of magnetic parameter on the velocity profiles for G = 10,Pr =0.72, $\omega = 2$, $\alpha = 5$, and a=0.1

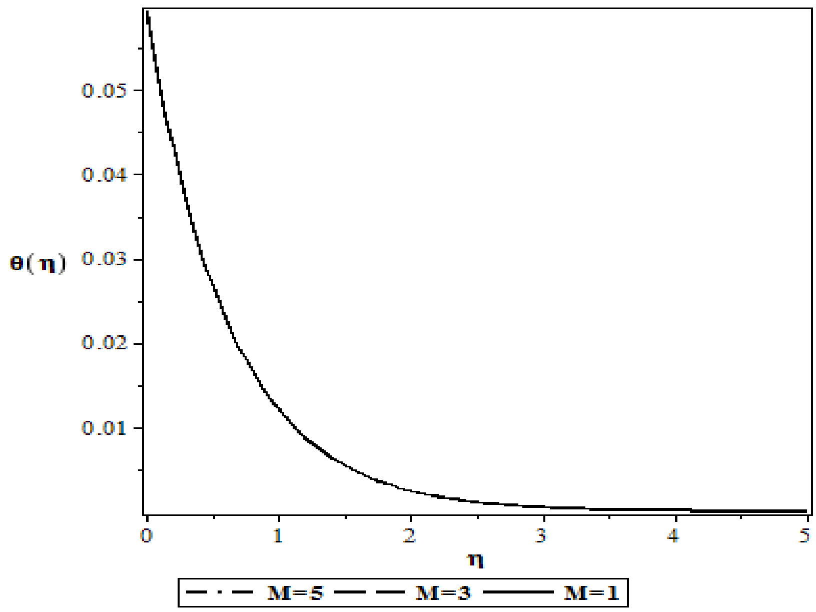

Figure 1(b): Effects of magnetic parameter on the temperature profiles for G = 10,Pr =0.72, $\omega = 2$, $\alpha = 5$, and a=0.1

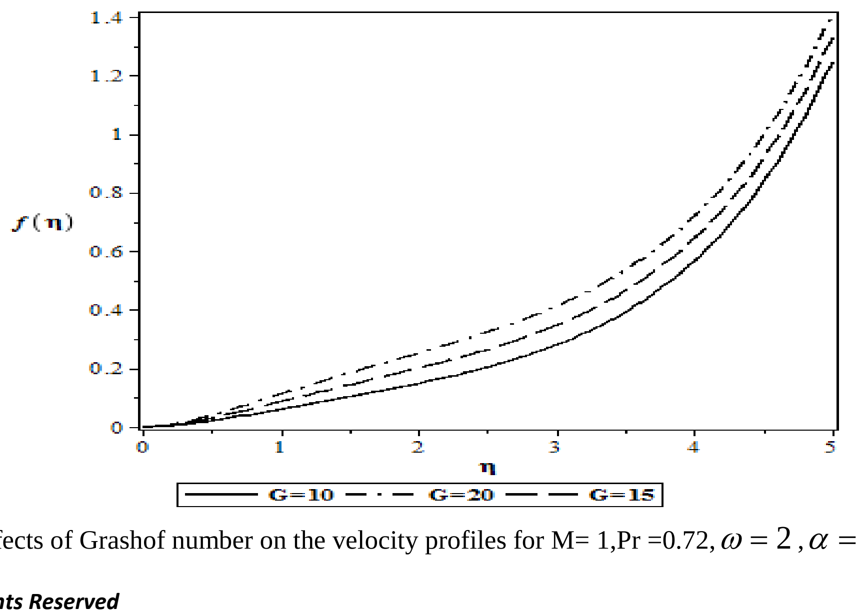

Figure 2(a): Effects of Grashof number on the velocity profiles for M= 1,Pr =0.72, $\omega = 2$, $\alpha = 5$, and a=0.1

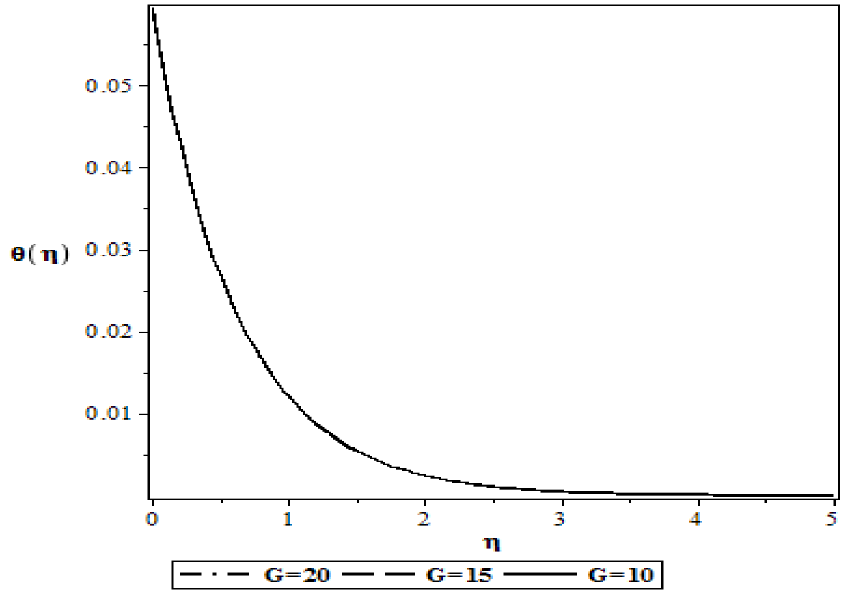

Figure 2(b): Effects of Grashof number on the temperature profiles for M= 1,Pr =0.72, $\omega = 2$, $\alpha = 5$, and a=0.1

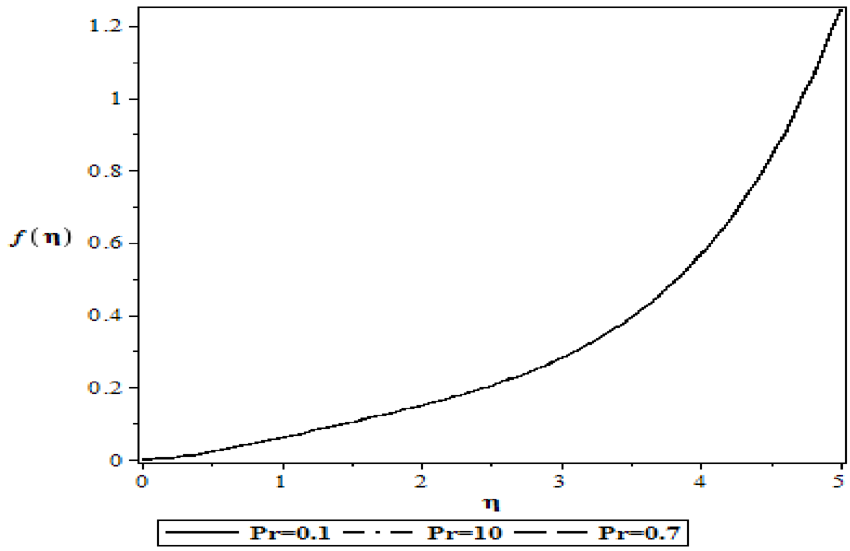

Figure 3(a): Effects of Prandtl number on the velocity profiles for M= 1,Gr=10, $\omega = 2$, $\alpha = 5$, and a=0.1

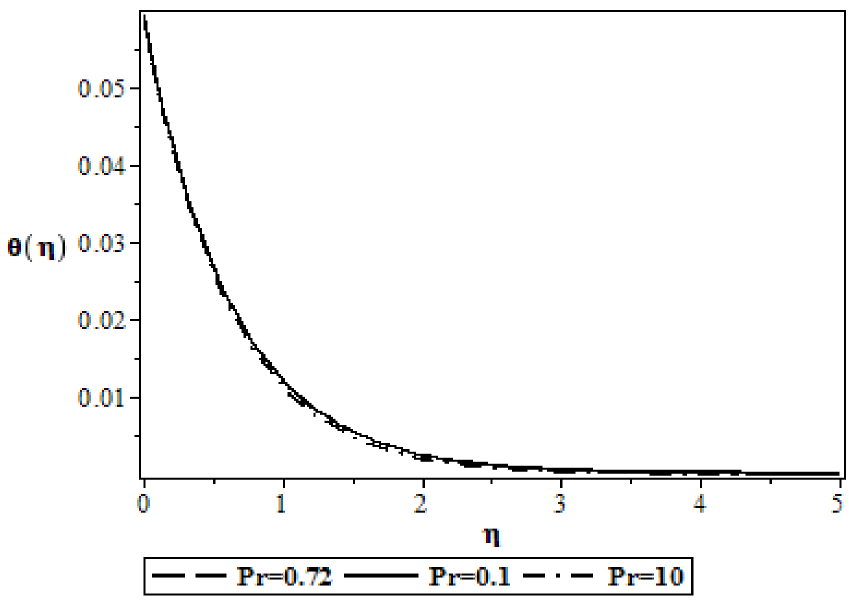

Figure 3(b): Effects of Prandtl number on the temperature profiles for M= 1,Gr=10, $\omega = 2$, $\alpha = 5$, and a=0.1

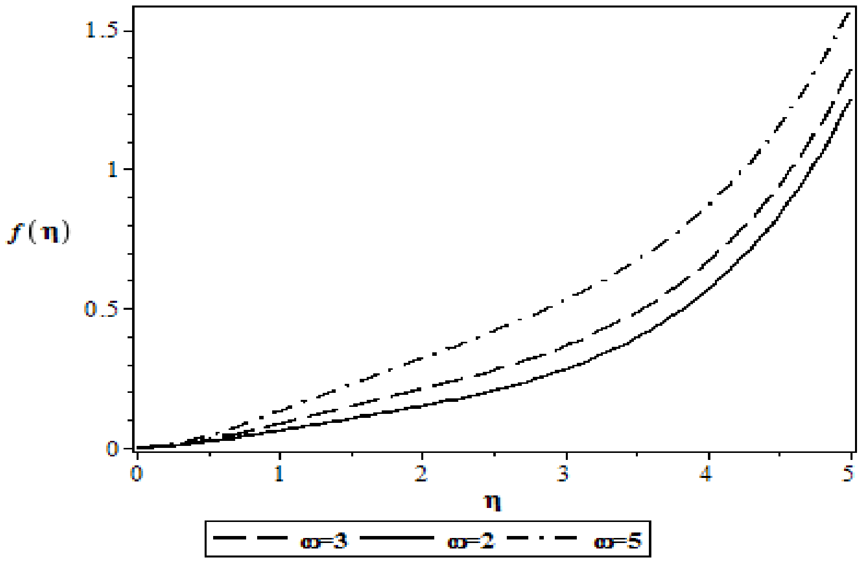

Figure 4(a): Effects of Radiation parameter on the velocity profiles for M= 1,Gr=10,Pr=0.72, $\alpha = 5$, and a=0.1

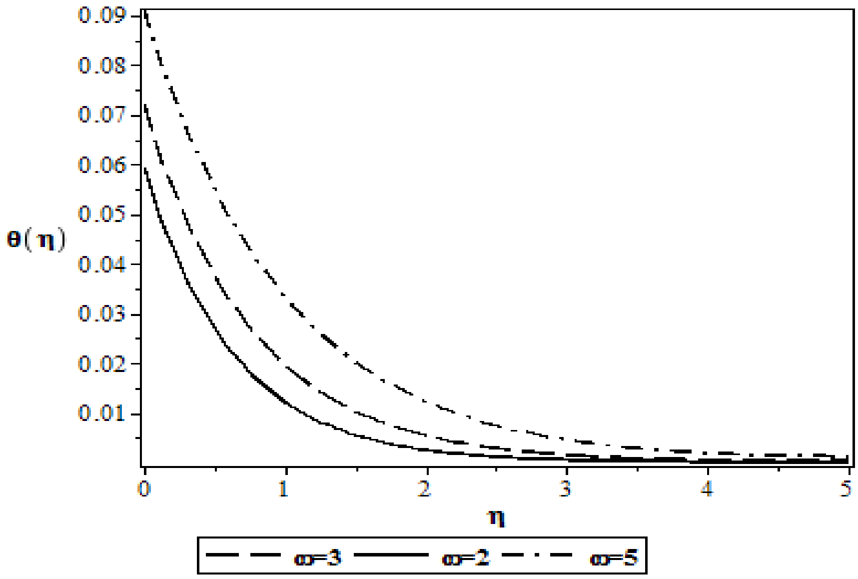

Figure 4(b): Effects of Radiation parameter on the temperature profiles for M= 1,Gr=10,Pr=0.72, $\alpha = 5$, and a=0.1

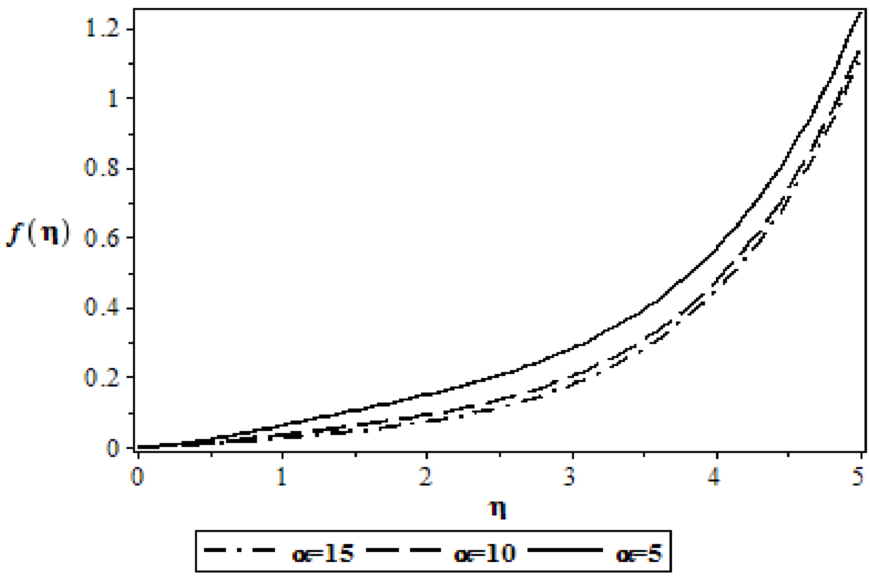

Figure 5(a): Effects of Heat source on the velocity profiles for M= 1,Gr=10,Pr=0.72, $\omega = 2$ and a=0.1

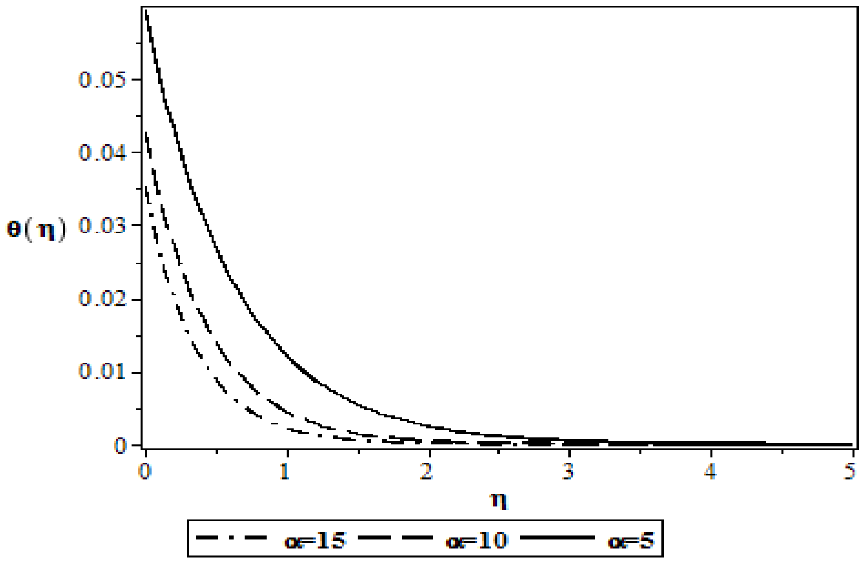

Figure 5(b): Effects of Heat source on the temperature profiles for M= 1,Gr=10,Pr=0.72, $\omega = 2$ and a=0.1

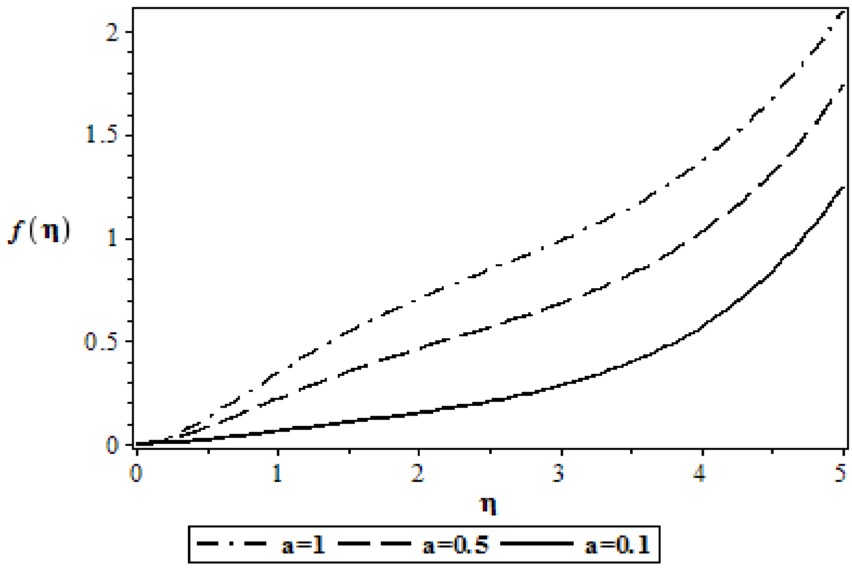

Figure 6(a): Effects of Biot number on the velocity profiles for M= 1,Gr=10,Pr=0.72, $\omega = 2$ and $\alpha = 5$

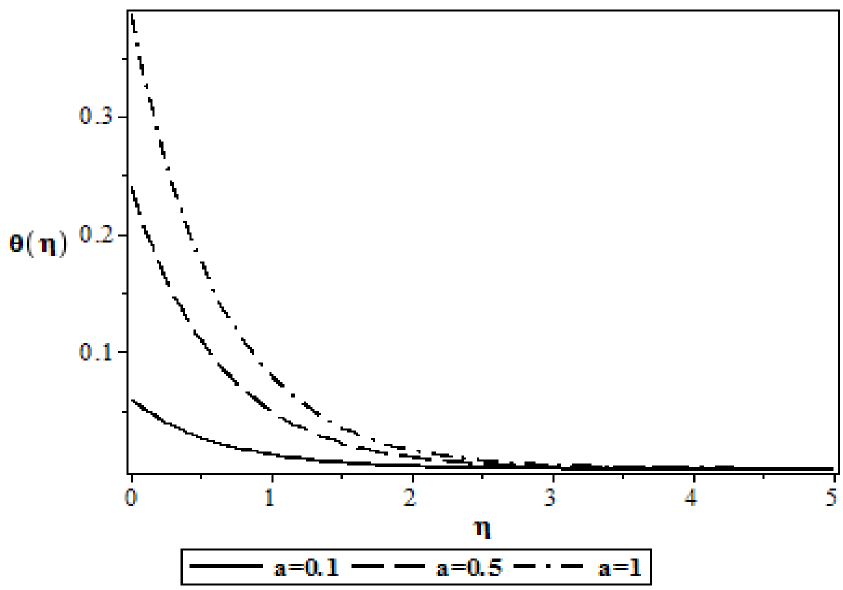

Figure 6(b): Effects of Biot number on the temperature profiles for M= 1,Gr=10,Pr=0.72, $\omega = 2$ and $\alpha = 5$

Table 1: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for G=10, Pr=0.72, $\omega = 2$, $\alpha = 5$ and $a_x = 0.1$

M	$f''(0)$	$\theta(0)$	$-\theta'(0)$
1	0.247240	0.059440	0.094055
2	0.180019	0.059586	0.094054
3	0.155793	0.059464	0.094053

Table 2: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Pr=0.72, $\omega = 2$, $\alpha = 5$ and $a_x = 0.1$

Gr	$f''(0)$	$\theta(0)$	$-\theta'(0)$
10	0.2472438	0.0594404	0.0940560
15	0.3624294	0.0594218	0.0940578
20	0.4773061	0.0594033	0.0940597

Table 3: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1,Gr=10, $\omega = 2$, $\alpha = 5$ and $a_x = 0.1$

Pr	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0.1	0.2476665	0.0594775	0.0940522
0.72	0.2472389	0.0594404	0.0940560
1	0.2412338	0.0589080	0.0941092

Table 4: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Gr=10,Pr=0.72 and $a_x = 0.1$

ω	$f''(0)$	$\theta(0)$	$-\theta'(0)$
2	0.2476665	0.0594775	0.0940522
3	0.3316948	0.0718803	0.0928120
5	0.4739065	0.0908933	0.0909107

Table 5: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Pr=0.72, $\omega = 2$ and $a_x = 0.1$

α	$f''(0)$	$\theta(0)$	$-\theta'(0)$
5	0.2476665	0.0597749	0.0940522
10	0.1488100	0.0402806	0.0957194
15	0.11052434	0.0352281	0.0964772

Table 6: Values of $f''(0)$, $\theta(0)$ and $-\theta'(0)$ for M=1, Pr=0.72 and $\alpha = 5$

a_x	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0.1	0.2476665	0.0594775	0.0940522
0.5	0.9485332	0.2401824	0.3799088
1	1.5152720	0.3872806	0.6127193

4. CONCLUSION

Radiation effects for boundary layer flow over a flate with a convective thermal surface boundary condition is studied. It is observed that the magnetic field strength increases, the velocity decreases and velocity boundary becomes thinner. An increase in Grashof number , increases the velocity and thicken velocity boundary layer.Both have no effect on thermal boundary layer thickness. However,Prandtl number has no effect on the velocity boundary and thermal boundary layer thickness. The heat source parameter increases, the velocity decreases and the velocity boundary thickness becomes thinner and thermal boundary layer thickness becomes thinner and also decreases the wall temperature. Radiation parameter and Biot number increases, thickens the thermal boundary layer thickness , increases the velocity, thickens the thermal boundary layer thickness and also increases the wall temperature. Moreso, This result qualitatively agrees with the expectations, since an increment in radiation parameter and Biot number exerts accelerating force on the flow.

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