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EFFECT OF POROUS MEDIUM AND MAGNETIC FIELD ON THE FLOW OF DUSTY VISCO-ELASTIC SECOND ORDER OLDROYD FLUID THROUGH A LONG RECTANGULAR CHANNEL

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ABSTRACT

The aim of the present study is to analyse the effect of porous medium and a magnetic field of uniform strength applied perpendicularly to the flow of fluid on the unsteady flow of dusty incompressible visco-elastic second order Oldroyd fluid with transient pressure gradient through a long rectangular channel with uniform cross-section. The magnetic Reynolds number is assumed to be small enough so that the induced magneticfield is negligible. The visco-elastic fluid is electrically conducting where as dust particles are non-conducting and small in size. The analytical expression for the velocities of viscoelastic fluid and the dust particles are obtained in elegent form. The particular cases corresponding to Oldroyd dusty fluid of first order, Maxwell dusty fluid, Rivlin-Ericksen dusty fluid and dusty viscous fluid are derived for velocity field. The results for velocities of visco-elastic fluid and the dust particles are also deduced in the absence of porous medium and in the absence of magnetic field by taking limits $K_1 \to \infty$ and $B_0 \to 0$ respectively.

INTRODUCTION

The problems of laminar flow of dusty visco-elastic fluid through channels has become very important in recent years particularly in the field of industrial and chemical engineering such as lalex particles in emulsion paints, reinforcing particles in polymer melts and rock crystals in molten lava etc.

Saffman (1962) has considered the stability of the laminar flow of a dusty gas with uniform distribution of small dust particles. The basic theory of flow of multiphase fluid is given by Soo (1967). Using the Saffman's model equations several researchers such as Dube and Srivastava (1972), Bagchi and Maiti (1980), Mukherjee etal (1984), Maudel etal (1986), Sharma and Singh (1987), Shrivastav, Chaudhary and Singh (1987), Shrivastav, and Singh(1988), Singh and Singh (1989), Garg, Shrivastav and Singh (1994), Johri and Gupta (1999), Das (2004,2005), Singh (2010), Prasad, Nagaich and Varshney (2012) etc. have studied the unsteady flow of dusty visco-elastic fluids through channels of various cross-sections with arbitrary time dependent pressure gradient.

Bhatanagar and Bhardwaj (1998), Mal and Sengupta (2003), Mishra and Bhola (2005), Kumar (2005), Varshney and Singh (2006), Kumar, Jha and Shrivastav (2006), Singh, Singh and Jha (2009), Agrawal, Agrawal and Varshney (2012) and Agrawal and Singh (2012), Mishra, Singh and Kumar (2013) etc. have discussed the unsteady flow of dusty fluid through porous medium in the channel of various cross-section with time dependent pressure gradient.

BASIC THEORY FOR SECOND ORDER OLDROYD FLUID

For slow motion, the rheological equations for second order Oldroyd visco-elastic fluid are:

$$\tau_{ij} = p\delta_{ij} + \tau'_{ij}$$

$$(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2})\tau'_{ij} = 2\mu(1 + \mu_1 \frac{\partial}{\partial t} + \mu_2 \frac{\partial^2}{\partial t^2})e_{ij}$$

$$e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$$

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where τ_{ij} is the stress tensor, τ'_{ij} the deviatoric stress tensor, e_{ij} the rate of strain tensor , p the pressure , λ_1 the stress relaxation time parameter, μ_1 the strain rate relaxation time parameter, λ_2 the additional material constant , μ_2 the additional material constant, δ_{ij} the metric tensor in cartesian co-ordinates, μ the coefficient of viscosity and v_i is the velocity components.

FORMULATION OF THE PROBLEM

Let us consider a rectangular cartesion coordinate system (x,y,z) such that z- axis is along the axis of the channel and the walls of the rectangular channel are taken to be the planes $x = \pm a$ and $y = \pm b$. u(x,y,z) and w(x,y,z) are the axial velocities of visco-elastic fluid and the dust particle respectively i.e. in the direction of z-axis. A transient pressure gradient -$Pe^{-\omega t}$ varying with time is applied to the fluid. Taking the number density of particles to be constant throughout motion.

Following Saffman (1962) with the stress-strain relations, the equations of motion of a dusty visco-elastic second order Oldroyd fluid through porous medium under the influence of transverse uniform magnetic field are:

$$\begin{aligned}(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial u}{\partial t} &= -\frac{1}{\rho}(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial p}{\partial z}+\upsilon(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2})\nabla^2 \mathrm{u} \\ &+\frac{KN_0}{\rho}(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})(w-u) - \left(\frac{\sigma B_0^2}{\rho}+\frac{\upsilon}{K_1}\right)(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})u \end{aligned} \quad (1)$$

$$M\frac{\partial w}{\partial t} = K(u-w) \quad (2)$$

where ρ is the density, υ the kinematic coefficient of viscosity, M the mass of particle, K the stokes resistance coefficient , N_0 the number of density of the particle, σ the conductivity of fluid, B_0 the electro- magnetic induction and K_1 is the coefficient of permeability.

Introducing the following non-dimensional quantities:

$$u^* = \frac{a}{\nu}u,\ \ w^* = \frac{a}{\nu}w,\ \ (x^*,y^*,z^*) = \frac{1}{a}(x,y,z),\ \ p^* = \frac{a^2}{\rho\nu^2}p,\ \ t^* = \frac{\nu}{a^2}t,$$

$$w^* = \frac{a^2}{\nu}w,\ \ \lambda_1^* = \frac{\nu}{a^2}\lambda_1,\ \ \mu_1^* = \frac{\nu}{a^2}\mu_1,\ \ \lambda_2^* = \frac{\nu^2}{a^4}\lambda_2,\ \ \mu_2^* = \frac{\nu^2}{a^4}\mu_2,\ \ K_1^* = \frac{1}{a^2}K_1$$

in equation (1) and (2), we get (after dropping stars)

$$\begin{aligned}(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial u}{\partial t} &= -(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial p}{\partial z}+(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2})\nabla^2 \mathrm{u} \\ &+\beta(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})(w-u) - \left(H^2+\frac{1}{K_1}\right)(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})u \end{aligned} \quad (3)$$

$$\frac{\partial w}{\partial t} = \frac{1}{\gamma}(u-w) \quad (4)$$

where

$$\beta = \frac{f_0}{\gamma} = \frac{N_0 K a^2}{\rho}, \qquad f_0 = \frac{MN_0}{\rho}, \qquad \gamma = \frac{M\upsilon^2}{Ka^2}\ \ and\ \ H = aB_0\sqrt{\frac{\sigma}{\mu}} \qquad (Hartmann\ number)$$

The boundary conditions are:

$$u = 0, \qquad w = 0, \qquad when\ x = \pm 1, \qquad -\frac{b}{a} \le y \le \frac{b}{a} \quad (5)$$

$$u = 0, \qquad w = 0, \qquad when\ y = \pm\frac{b}{a}, \qquad -1 \le x \le 1 \quad (6)$$

SOLUTION OF THE PROBLEM

We have considered those type of situations of the flow which is transient in nature with respect to time and periodic in nature with respect to y.

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From the nature of boundary conditions, we may choose the solution of equations (3) and (4) as

$$u = U(x)\cos my.\, e^{-\omega t} \tag{7}$$

$$w = W(x)\cos my.\, e^{-\omega t} \tag{8}$$

Conditions (6) will be satisfied if

$$\cos m\frac{b}{a} = 0$$

or

$$m = (2n+1)\frac{\pi a}{2b}, \quad n = 0,1,2,3,\ldots\ldots \tag{9}$$

Now the boundary conditions (5) become

$$U = 0, \qquad W = 0, \qquad when\ x = \pm 1 \tag{10}$$

We construct the solution as the sum of all possible solutions for each value of n

$$u = \sum_{n=0}^{\infty} U(x)\cos my.\, e^{-\omega t} \tag{11}$$

$$w = \sum_{n=0}^{\infty} \mathrm{W(x)}\cos my.\, e^{-\omega t} \tag{12}$$

Using equations (11) and (12) in the equation (4), we get

$$\sum_{n=0}^{\infty} \mathrm{W(x)}\cos my. = \frac{1}{(1-\gamma\omega)}\sum_{n=0}^{\infty} U(x)\cos my \tag{13}$$

By putting

$$\frac{\partial p}{\partial z} = -Pe^{-\omega t} \quad (\omega > 0)$$

in equation (3) and using equations (11), (12) and (13), we obtain

$$\sum_{n=0}^{\infty}\frac{d^2U(x)}{dx^2}\cos my - \sum_{n=0}^{\infty} m^2 U(x)\cos my + \frac{P(1-\lambda_1\omega+\lambda_2\omega^2)}{(1-\mu_1\omega+\mu_2\omega^2)} + \sum_{n=0}^{\infty}\frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega - \frac{\beta\omega}{1-\gamma\omega} - \left(H^2 + \frac{1}{K_1}\right)\right\}}{(1-\mu_1\omega+\mu_2\omega^2)} U(x)\cos my = 0$$

Or

$$\sum_{n=0}^{\infty}\frac{d^2U(x)}{dx^2}\cos my + \frac{4P}{\pi}\frac{(1-\lambda_1\omega+\lambda_2\omega^2)}{(1-\mu_1\omega+\mu_2\omega^2)}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos my - \sum_{n=0}^{\infty}\left[m^2 - \frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega - \frac{\beta\omega}{1-\gamma\omega} - \left(H^2+\frac{1}{K_1}\right)\right\}}{(1-\mu_1\omega+\mu_2\omega^2)}\right] U(x)\cos my = 0$$

$$\because \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos my = \frac{\pi}{4} \quad (by\ Fourier\ cosine\ series)$$

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Or

$$\sum_{n=0}^{\infty}\left[\frac{d^2U(x)}{dx^2}+\frac{4P(-1)^n(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)} - \left\{m^2-\frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega-\frac{\beta\omega}{1-\gamma\omega}-H^2-\frac{1}{K_1}\right\}}{(1-\mu_1\omega+\mu_2\omega^2)}\right\}\right] U(x)\cos my=0 \tag{14}$$

Equating the coefficient of $\cos my$ *equal to zero, we get*

$$\frac{d^2U(x)}{dx^2}-\frac{K_2^2}{a^2}U(x)+A_n=0 \tag{15}$$

where

$$K_2^2=a^2\left\{m^2-\frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega-\frac{\beta\omega}{1-\gamma\omega}-H^2-\frac{1}{K_1}\right\}}{(1-\mu_1\omega+\mu_2\omega^2)}\right\},$$

$$\text{and } A_n=\frac{4P(-1)^n(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)} \tag{16}$$

Solution of equation (15) subject to boundary condition (10) is given by

$$U(x)=\frac{(-1)^{n+1}4P(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)K_2^2}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\} \tag{17}$$

Thus the velocity of visco-elastic liquid is

$$u(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}4P(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}\right].\cos(2n+1)\frac{\pi a}{2b}y\,.\,e^{-\omega t} \tag{18}$$

and the velocity of dust particles is

$$w(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}4P(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)(1-\gamma\omega)K_2^2}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}\right].\cos(2n+1)\frac{\pi a}{2b}y.\,e^{-\omega t} \tag{19}$$

PARTICULAR CASES

Case I: *If we put* $\lambda_2=0$, $\mu_2=0$ *in the equations (18) and (19), we obtain the velocities of first order Oldroyd fluid and the dust particles.*

Case II: *If we put* $\mu_1=0,\mu_2=0$ *in the equations (18) and (19), we obtain the velocities of second order Maxwell fluid and the dust particles.*

Case III: *If we put* $\lambda_1=0,\lambda_2=0$ *in the equations (18) and (19), we obtain the velocities of second order Rivlin Ericksen fluid and dust particles.*

Case IV: *If we put* $\lambda_1=0,\lambda_2=0$ *and* $\mu_1=0,\mu_2=0$ *in the equations (18), and (19), we obtain the velocities of purely viscous fluid and the dust particles.*

Case V: *If we put* $\lambda_1=0,\lambda_2=0$ *and* $\mu_1=0,\mu_2=0$ *in the equation (18), we obtain the velocity of peurly viscous fluid through rectangular channel.*

DEDUCTIONS

I. *If magnetic field is withdrawn i.e.* $B_0=0$, *then all the results of Prasad, Nagaich and Varshney (2012) can be obtained with slight change of notations.*

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II. *If porous medium is withdrawn i.e.* $K_1 = \infty$, *then all the results of Mishra, Singh and Kumar (2013) can be obtained with slight change of notations.*

III. *If magnetic field and porous medium both are withdrawn i.e.* $B_0 = 0, K_1 = \infty$, *simultaneously, then all the results of Kumar (2009) can be obtained with slight change of notations.*

DISCUSSION

From the velocity expressions (18) and (19) it is clear that the effect of application of magnetic field applied perpendicularly to the flow of fluid and the porous medium is to reduce the velocity of dusty visco-elastic second order Oldroyd fluid. It is also clear that the presence of dust particles reduce the velocity of second order Oldroyd visco-elastic fluid.

REFERENCES

1. *Agrawal, S. and Singh, M. P. (2012): Acta Ciencia Indica Vol. XXXVIII M, No. 4, p. 793.*
2. *Agrawal, V. P., Agrawal, N. K. and Varshney, N. K. (2012): Acta Ciencia Indica Vol. XXXVIII M, No. 1, p. 47.*
3. *Bagchi and Maiti, M. K. (1980): Acta Ciencia Indica, Vol. VI, No. 3, p.-130.*
4. *Bhatnagar, B. R. and Bhardwaj, R. (1998): Jour. Pure Appl. Math., Vol. 29, No. 2, p. 139.*
5. *Das, K.K. (2004): Ind. Jour. Theo. Phy., Vol. 52, No. 1, p. 31.*
6. *Das, K.K. (2005): Ind. Jour. Theo. Phy., Vol. 55, No. 2, p. 145.*
7. *Dube, S.N. and Srivastava, L.P. (1972): Def. Sci. Jour., p.22.*
8. *Garg, A., Shrivastav, R. K. and Singh, K. K. (1994): Proc. Nat. Acad. Sci. India, Vol. 64(A), part III, p. 355.*
9. *Johari, R. and Gupta, G. D. (1999): Acta Ciencia Indica, Vol. XXXII, No. 4, p.-1713.*
10. *Kumar, S. (2005): Acta Ciencia Indica, Vol.XXXIM, No. 1, p. 77.*
11. *Kumar, D., Jha, R. and Shrivastav R. K. (2006): Acta Ciencia Indica, Vol. XXXIIM, No. 1, p. 135.*
12. *Mal, R. K. and Sengupta, P. R. (2003): Jour. Proc. Ind. Math. Sci. Acad., Vol. 59(A) No. 2, p. 157.*
13. *Maudal, G. C., Mukherjee, S. and Mukherjee, S. (1986): Jour. Indeian Sci.Vol. 66, p. 77.*
14. *Mishra, B. K. and Bhola, J. (2005): Acta Ciencia Indica, Vol. XXXIM, No. 1, p. 33.*
15. *Mishra, G., Singh, K. K. and Kumar, R. (2013): International Journal of Mathematical Archive, Vol. 4(7), p.318.*
16. *Mukherjee, S., Maiti, M. K. and Mukherjee, S. (1984): Ind. Jour. Technol., Vol. 22, p. 41.*
17. *Oldroyd, J. G. (1958): Proc. Roy. Soc.(London) A, Vol. 245,p. 278.*
18. *Prasad, V. Nagaich, M. and Varshney, N. K. (2012): Ultra Scientist Vol. 24(3) B, p.511.*
19. *Saffman, P.G. (1962): Jour. Fluid Mechanics, Vol. 13, p. 120.*
20. *Sharma, J.P.and Singh, K.K. (1987): Jour. MACT, Vol. 20, p. 13.*
21. *Shrivastav, R. K. and Singh, K. K., (1988): Bull. Cal. Math. Soc., Vol. 80, p. 286.*
22. *Shrivastav, R.K., Chaudhary, R.K.S. and Singh, K.K. (1978):Ind. Jour. Theo. Phy., Vol. 35, p. 4.*
23. *Singh, K. and Singh, K. K. (1989): Bull. Cal. Math. Soc., Vol. 81, p. 436.*
24. *Singh, P. (2010): Ind. Jour. Theo. Phy. Vol. 58, No. 1, p.27.*
25. *Singh, P. Singh, U. R. and Jha, R. (2009): Gunita, Vol. 60, No. 2, p. 45.*
26. *Soo, S. L. (1967): Fluid Dynamics of Multiphase Systems, I[st] Ed. Blaisdell Pub. Co. Waltham Mass, U.S.A..*
27. *Varshney, N. K. and Singh M. B. (2006): Acta Ciencia Indica, Vol. XXXII, M, No. 3, p. 1013.*

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