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AMENDMENT IN THE CONCEPT OF PHEROMONE EVAPORATION–AN INNOVATIVE APPROACH IN ANT COLONY OPTIMIZATION

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ABSTRACT

In this paper, a new concept is introduced in the field of pheromone evaporation for Ant Colony Optimization (ACO). Traditionally ACO is applicable over Single Criterion Decision Making problems (SCDM), and evaporation parameter is a unique value that is associated within an iteration of this method. But in this paper, we are going to enhance the concept of evaporation parameter, by considering the optimization problem as Multi Criteria Decision Making problem (MCDM). This enhancement will lead us towards a different pheromone evaporation technique, and it will definitely affect the convergence of the solution for combinatorial problems.

Keywords: *Ant Colony Optimization, Multi- Criteria Decision Making, Pheromone Evaporation.*

1. INTRODUCTION

Marco Dorigo (University of Paderborn, Germany), has given an approach for solving several combinatorial optimization problems. This approach is globally known as *Ant Colony Optimization.* He has considered many factors like Probabilistic Approach for edge decision making, Ants Behavior of Pheromone Laying, Natural Pheromone Evaporation tendency, Importance of Pheromone Intensity and Importance of Pheromone Visibility. Concentration point in this paper is the concept of *Pheromone Evaporation*, which will be enhanced here, provided that the problem domain is MCDM domain.

Ant Colony Optimization- Dorigo's Definition:

Ant colony optimization is a meta heuristic in which a colony of artificial ants cooperate in finding good solutions to difficult discrete optimization problems. While travelling on a graph (path), decision for next edge to travel is taken in probabilistic way using the relation:

$$P_{ij}^{k}(t) = \begin{cases} \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l\in J_k(i)}[\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta}}, & if\ j \in J_k(i) \\ 0, & if\ j \notin J_k(i) \end{cases} \tag{2.1}$$

Here,
$P^{k}{}_{ij}(t)$ = Probability that an ant k will travel from i to j node in graph at time t.

This probability is dependent on several factors like,

τ_{ij}(t) = Pheromone intensity at time t while travelling from node i to node j.
$\eta_{ij}(t)$= Pheromone visibility at time t while travelling from node i to node j.
α= Importance of pheromone intensity
β= Importance of pheromone visibility
$J_k(i)$= Neighborhood set of node i for ant k.

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Pheromone updates are dependent on relation,

$$\Delta\tau_{ij}^{k} = \begin{cases} \frac{Q}{L_k}, & if\ (i,j) \in tour \\ 0, & otherwise \end{cases} \tag{2.2}$$

Here
$\Delta\tau_{i,j}^{k}$= Amount of pheromone updated by ant k while traveling from node i to j.
Q= Constant.
L_k= Total path length covered by ant k.

Pheromone updates finally happens with this equation,

$$\tau^{k}{}_{ij} = \tau^{k}{}_{ij} + \Delta\tau^{k}{}_{ij} \tag{2.3}$$

And pheromone decay is dependent on the equation,

$$\tau_{ij} = (1-\rho)\tau_{ij} \tag{2.4}$$

Here
τ_{ij}= Pheromone intensity on edge joining node i and node j.
$\rho(t)$= Evaporation parameter of the graph.

3. PHEROMONE EVAPORATION

It has been observed that a colony of ants is able to find the shortest path to a food source. As an ant moves and searches for food, it lays down a chemical substance called pheromone along its path. As the ant travels, it looks for pheromone trails on its path and prefers to follow trails with higher levels of pheromone deposits. If there are multiple paths to reach a food source, an ant will lay the same amount of pheromone at each step regardless of the path chosen. However, it will return to its starting point quicker when it takes the shorter path, which contains more pheromone. It is then able to return to the food source to collect more food. Thus, in an equal amount of time, the ant would lay a higher concentration of pheromone over its path if it takes the shorter path, since it would complete more trips in the given time. The pheromone is then used by other ants to determine the shortest path to find food as described in [number] During the process, another factor affects on the amount of pheromone deposition namely, evaporation of pheromone, which can be seen as an exploration mechanism that delays faster convergence of all ants towards a suboptimal path. The decrease in pheromone intensity favors the exploration of different paths during the whole search process. In real ant colonies, pheromone trail also evaporate, but evaporation does not play an important role in real ant's shortest path finding. But on the contrary, the importance of pheromone evaporation in artificial ants is probably die to fact that the optimization problems tackled by artificial ants are much more complex than those real ants can solve. A mechanism like evaporation that by favoring the forgetting of errors or of poor choices done in the past plays the important function of bounding the maximum value achievable by pheromone trails. In ACO, the pheromone evaporation is interleaved with pheromone deposit of ants. After each ant has moved to a next node according to ant's search behavior, pheromone trails are evaporated by applying equation 2.4 to all arcs. For further details one can refer [3].

4. NEW PARADIGM FOR EVAPORATION PARAMETER

We are starting our proposal with a specific assumption that the ants are travelling on an insulated bar of length l, which is initially maintained at temperature of $3\sin\frac{\pi x}{l}$, and than change in temperature on both the ends of the rod is applied after the steady state condition prevails. Both ends are cooled to $0^{\circ}C$ and are kept at that temperature.

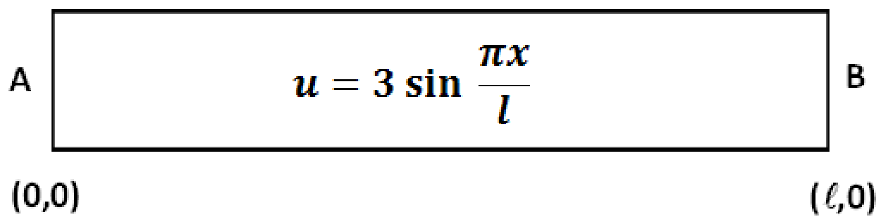

Figure 1: Insulated Path with two ends A & B.

The initial setup that we have created here is a case of one dimensional heat flow condition, and we know partial differential equation for the same is:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{4.1}$$

Here: $u =$ Temperature, $x =$Distance from the origin, and, $t =$Time.

According to the case considered here, the boundary conditions are:
$u(0, t) = 0$
$u(l, t) = 0$

And initial condition is:

$$u(x, 0) = 3 \sin \frac{\pi x}{l}$$

Assuming u as multiple of two functions,

$$u(x, t) = X.T \tag{4.2}$$

Here, X is a function of x, and T is a function of t.

Substitute this assumed value of u in equation 4.1

$$\frac{\partial(XT)}{\partial t} = c^2 \frac{\partial^2(XT)}{\partial x^2}$$

$X \frac{\partial T}{\partial t} = c^2 T \frac{\partial^2 X}{\partial x^2}$ [$\because$ X is a function of x alone and T is a function of t alone]

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{c^2 T} \frac{\partial T}{\partial t}$$

Equating this equation with a negative constant,

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{c^2 T} \frac{\partial T}{\partial t} = -p^2$$

$\frac{1}{X} \frac{d^2 X}{dx^2} = \frac{1}{c^2 T} \frac{dT}{dt} = -p^2$ [$\because$ X is a function of x alone and T is a function of t alone]

From the above equation, we will get two ordinary differential equations,

$$\frac{1}{X} \frac{d^2 X}{dx^2} = -p^2 \tag{4.3}$$

$$\frac{1}{c^2 T} \frac{dT}{dt} = -p^2 \tag{4.4}$$

Solving equation 4.3

$$\frac{d^2 X}{dx^2} + p^2 X = 0$$

$$m^2 + p^2 = 0$$

$$m^2 = -p^2$$

$$m = \pm pi$$

$$X = (c_1 cospx + c_2 sinpx)$$

Now solving equation 4.4

$$\frac{dT}{T} = -p^2c^2dt$$

$$\int \frac{dT}{T} = \int -p^2c^2dt$$

$$logT = -p^2c^2t + logc_3$$

$$log\frac{T}{c_3} = -p^2c^2t$$

$$T = c_3e^{-p^2c^2t}$$

Putting obtained values of X and T in equation 4.2,

$$u(x,t) = (c_1\cos px + c_2\sin px)c_3e^{-p^2c^2t} \tag{4.5}$$

Put x = 0 in this obtained value,

$$u(0,t) = c_1c_3e^{-p^2c^2t}$$

$c_1c_3e^{-p^2c^2t} = 0$ [$\because u(0,t)$=0, as first boundary condition states]

From here it has been concluded that $c_1 = 0$, since $c_3\ and\ e^{-p^2c^2t}$ are not suiting the practical aspect of the problem. i.e. if we conclude that either of these two entities is zero, than its impact on equation …… makes $u(x,t) = 0$, which is practically not possible.

Substitute this value of c_1 in equation 4.5,

$$u(x,t) = c_2\,c_3\sin px\,e^{-p^2c^2t} \tag{4.6}$$

Now put $x = l$ in this equation,

$$u(l,t) = c_2\,c_3\sin pl\,e^{-p^2c^2t}$$

$c_2\,c_3\sin pl\,e^{-p^2c^2t} = 0$ [$\because u(l,t)$=0, as second boundary condition states]

From here, we'll conclude that, $\sin pl = 0$

Or, $\sin pl = sinn\pi$ [$\because sinn\pi = 0, \forall n$]

$$\therefore\ p = \frac{n\pi}{l}$$

Put this value of p in equation 4.6,

$$u(x,t) = c_2\,c_3\sin\frac{n\pi x}{l}e^{\frac{-n^2\pi^2c^2t}{l^2}} \tag{4.7}$$

Now put t = 0 in this equation,

$$u(x,0) = c_2\,c_3\sin\frac{n\pi x}{l}$$

$$c_2\,c_3\sin\frac{n\pi x}{l} = 3\sin\frac{\pi x}{l}$$

[$\because u(x,0)$= 3 $\sin\frac{\pi x}{l}$, as initial condition states]

$$\therefore c_2\,c_3 = 3, n = 1$$

Putting these values in equation 4.7, it will get transformed into:

$$u(x,t) = 3e^{\frac{-\pi^2 c^2 t}{l^2}} sin\frac{\pi x}{l} \tag{4.8}$$

Now coming to the concept of pheromone evaporation, which is a chemical substance that the ants lay while travelling. This is a world wide known natural phenomenon that this chemical evaporates with respect to time, and this evaporation of pheromone is directly dependent on the temperature of the path where the ants are travelling. Higher the value of temperature is, larger is the intensity of evaporation. So if this concept is applied on the path shown in figure 1, then we will get the relation, $\rho \propto u(x,t)$ i.e. pheromone evaporation parameter is directly proportional to the temperature distribution function of the bar. (For better understanding towards this concept, refer [5])

Or we can say, $\rho = k.u(x,t)$

Putting value of $u(x,t)$ from equation 4.8 in above relation,

$$\rho(x,t) = 3ke^{\frac{-\pi^2 c^2 t}{l^2}} sin\frac{\pi x}{l} \tag{4.9}$$

So, for the initial and boundary conditions taken, we can say that evaporation parameter is now an entity which is different at each point of the bar/path (unlike the traditional approach of ACO, in which only a single criterion *distance* was always considered). This entity, evaporation parameter, is now dependent on distance of the specific point from origin (first end of rod), and time at which measurement of evaporation is performed.

Generally we can say that evaporation parameter $\rho(x,t)$ can be formulated according to the initial temperature distribution of the path, on which ants are travelling.

5. CONCLUSION

The specific formula (equation 4.9) for evaporation parameter and the general concept stated after that is very fruitful in those cases, when we are considering an optimization problem as multi criteria based problem. Traditionally in shortest path problems, vehicle routing problems etc, whenever ACO is applied, researchers have always worked with a single criterion *distance.* If more real and practical results are desired, then we have to solve these optimization problems under multiple criteria. And specifically if the problem domain is involving criteria *distance* and *temperature*, then evaporation of pheromone needs to be calculated using an alternate strong approach.

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