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DECELERATING AND ACCELERATING BIANCHI TYPE–VI_0 VACUUM UNIVERSE IN $f(R)$ GRAVITY

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ABSTRACT

In this paper an attempt has been made to study the Bianchi type-VI_0 cosmological model in $f(R)$ gravity. We find solutions of field equations in vacuum. For this purpose we assume that expansion (θ) in the model is proportional to shear scalar (σ) and the relation between scale factor and function of Ricci scalar .physical properties of these model are discussed. Jerk and Snap parameters are also obtained.

***Keywords**: Bianchi type – VI_0 model, $f(R)$ gravity, Shear scalar, Expansion scalar.*

1. INTRODUCTION

Several recent cosmological observations such as type Ia supernovae (SNe) [1], Cosmic microwave background (CMB) [2, 3] radiation indicate that universe is under expansion To explain nature of the universe various models are proposed. Among all theories GR has its own features and it fit to observational data. According to these observations universe is filled with dark matter and dark energy .The nature of this dark matter and dark energy is still unknown. To solve this problem various extensions are suggested to GR

The $f(R)$ gravity, where $f(R)$ is a generic function of the Ricci scalar R, comes into the game as a straightforward extension of GR. It provides natural gravitational alternative to dark energy. The $f(R)$ models can cover all the domains from the cosmological to the solar system scales [4].Hence $f(R)$ gravity model is therefore very useful alternative to ΛCDM cosmology. Several eminent author have studied $f(R)$ gravity [5,6,7,8,9,10,11,12] for various space time. Recently some new exact static spherically symmetric interior solutions of metric $f(R)$ gravitational theories are obtained by Ali Shojai and Fatimah Shojai [27].

The pure Friedman-Lemaitre-Robertson-Walker (FLRW) cosmology could not explain all the properties of the universe. Naturally we consider anisotropic cosmological models that allow FLRW universe as special case. Among all space time, Bianchi type-VI_0 space time seems to be one of the most convenient for testing different cosmological models. Many authors explored Bianchi type VI_0 cosmological model in different contexts. [13, 18, 19, 20].T.Singh and R.Chaubey [14] investigated Bianchi I, V and VI_0 models in modified generalized Scalar- Tensor Theory. Shri Ram [31] studied Bianchi type VI_0 conformally invariant scalar field. S R Roy and A Prasad studied an algebraically special Bianchi type VI_h cosmological model in GR.R Venkateswarlu and DRK Reddy investigated Bianchi type VI_o in the presence of zero mass scalar field as well as in the presence of perfect fluid in Barber's second self creation theory of gravitation [15, 17].Exact Bianchi type VI_0 vacuum cosmological model in Brans-Dicke theory is evaluated by Shri. Ram and D K Singh [16] and found that the universe is anisotropic in nature.

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The objective of this paper is to investigate Bianchi type VI_0 cosmological model in $f(R)$ gravity. The field equations are solved by assuming shear scalar is proportional to expansion scalar. Jerk and Snap parameter [22] are also obtained. The cosmological Jerk parameter is the third derivative of the scale factor with respect to time where as Snap parameter is the fourth derivative of scale factor with respect to time. The cubic term of Chiba and Nakamura [23] is identical to the jerk parameter as is the state finder variable called by Sahni *et al.* [24, 25]. The other state finder variable called s, different than Snap is a particular linear combination of the jerk and deceleration parameter. Snap is also sometimes called jounce.

2. BASIC FIELD EQUATIONS

The spatially homogeneous Bianchi type – VI metric is in the form

$$ds^2 = -dt^2 + A(t)^2 dx^2 + B(t)^2 e^{-2mx} dy^2 + C(t)^2 e^{2mx} dz^2, \tag{2.1}$$

where m is the non zero constant.

The Ricci scalar R is given by

$$R = 2\left[\frac{-m^2}{A^2} + \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC}\right], \tag{2.2}$$

The action of $f(R)$ gravity is given by

$$S = \int \sqrt{-g}\left(f(R) + L_m\right) d^4 x, \tag{2.3}$$

where $f(R)$ is general function of the Ricci scalar and L_m is the matter Lagrangian.

The resulting field equations from this action are obtained as

$$F(R) R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \nabla^\mu \nabla_\nu F(R) = T_{\mu\nu}^{(m)}, \tag{2.4}$$

where $F(R) = \frac{d}{dR}(f(R)),$ ∇_μ is the covariant derivative and $T_{\mu\nu}^{(m)}$ is the standard minimally coupled stress energy tensor derived from the Lagrangian L_m.

Now contracting the field equations, it follows that

$$F(R) R - 2 f(R) + 3\nabla^\mu \nabla_\nu F(R) = T \tag{2.5}$$

In vacuum it further reduces to

$$F(R) R - 2 f(R) + 3\nabla^\mu \nabla_\nu F(R) = 0 \tag{2.6}$$

This gives

$$f(R) = \frac{3\nabla^\mu \nabla_\nu F(R) + F(R) R}{2}, \tag{2.7}$$

Using (2.7) in (2.4) we get

$$\frac{F(R) R_{\mu\nu} - \nabla_\mu \nabla_\nu F(R)}{g_{\mu\nu}} = \frac{F(R) R - \nabla^\mu \nabla_\nu F(R)}{4}, \tag{2.8}$$

The combination obtained from (2.8)

$$A_\mu = \frac{F(R)\,R_{\mu\mu} - \nabla_\mu \nabla_\mu F(R)}{g_{\mu\mu}} \,, \tag{2.9}$$

A_μ Is free from index μ hence, we have

$$A_\mu - A_\nu = 0 \,, \tag{2.10}$$

Thus $A_4 - A_1 = 0$, $A_4 - A_2 = 0$, $A_4 - A_3 = 0$ respectively gives,

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4 B_4}{AB} - \frac{A_4 C_4}{AC} - \frac{A_4 F_4}{AF} + \frac{F_{44}}{F} + \frac{2m^2}{A^2} = 0, \tag{2.11}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{B_4 F_4}{BF} + \frac{F_{44}}{F} = 0, \tag{2.12}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{A_4 C_4}{AC} - \frac{B_4 C_4}{BC} - \frac{C_4 F_4}{CF} + \frac{F_{44}}{F} = 0, \tag{2.13}$$

where subscript four (4) denotes differentiation with respect to t.

From equation (2.4) (the 41-component) we obtain the relation as

$$B = C \tag{2.14}$$

Using (2.14) we have equations (2.11)-(2.13) leads to

$$2\frac{B_{44}}{B} - 2\frac{A_4 B_4}{AB} - \frac{A_4 F_4}{AF} + \frac{F_{44}}{F} + \frac{2m^2}{A^2} = 0 \tag{2.15}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{A_4 B_4}{AB} - \frac{B_4^2}{B^2} - \frac{B_4 F_4}{BF} + \frac{F_{44}}{F} = 0 \tag{2.16}$$

We have two equations in three unknown A, B, F so that we first assume that the expansion (θ) in the model is proportional to shear (σ) [21] i.e.

$$\sigma = l\,\theta, \text{ where } l \text{ is constant.} \tag{2.17}$$

The motive behind assuming this condition is explained with reference to Throne [21].The observational of the velocity red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent. Moreover for spatially homogeneous metric the normal congruence to the homogeneous expansion satisfies that the condition $\dfrac{\sigma}{\theta}$ is constant.

We define the physical parameters as follows

Spatial volume $V = (ABC)$, (2.18)

Hubble parameter $H = \dfrac{1}{3}\left(\dfrac{A_4}{A} + \dfrac{2B_4}{B}\right)$ (2.19)

Mean Anisotropy parameter $A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H}{H}\right)^2,$ (2.20)

Expansion scalar $\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C},$ (2.21)

Shear scalar $\sigma^2 = \frac{1}{3} A_m H^2,$ (2.22)

Deceleration parameter $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$. (2.23)

Using equations (2.21), (2.22) we yield

$$A = B^n, \tag{2.24}$$

where $n = \frac{1+2l\sqrt{3}}{1-l\sqrt{3}}$

3. MODEL I

Recently Kotubuddin *et. al.* have established a result in $f(R)$ gravity which shows that $F \ \alpha \ a^u$ Where u is an arbitrary constant and a is scale factor defined by $a^3 = V$

Therefore we get $F = k B^{\frac{u(n+2)}{3}}$ (3.1)

From equations (2.15), (2.16), (2.24), (3.1) we have

$$2B_{44} + 2\alpha \frac{B_4^2}{B} = \frac{4m^2}{n-1} B^{-2n+1}, \quad n \neq 1 \tag{3.2}$$

where $\alpha = \frac{3(n+1) + u(n+2)}{3}$

Let $B_4 = f(\mathbf{B})$ so that equation (3.2) reduces to

$$2 f^1 f + 2\alpha \frac{f^2}{B} = \frac{4}{(n-1)} \frac{m^2}{B^{2n-1}}, \tag{3.3}$$

Above differential equation reduced to

$$f^2 = \left(\frac{dB}{dt}\right)^2 = \frac{4}{(n-1)(2-2n+2\alpha)} \frac{m^2}{B^{2n-2}} + NB^{-2n+2}, \tag{3.4}$$

where N is integrating constant taken to be zero.

On integrating equation (3.4) we have

$$B^n = \frac{2nm}{\sqrt{(n-1)(2-2n+2\alpha)}} t + nD, \tag{3.5}$$

where D is integrating constant. Taking D to be zero we obtain

$$A = \frac{2nm}{\sqrt{(n-1)(2-2n+2\alpha)}}t, \tag{3.6}$$

Hence

$$B = C = \left(\frac{2nm}{\sqrt{(n-1)(2-2n+2\alpha)}}\right)^{\frac{1}{n}} t^{\frac{1}{n}}, \tag{3.7}$$

These are the solution for the value of u as

$$u = \frac{3(1+n) \pm \sqrt{9(1+n)^2 + 72n}}{2(n+2)}$$

The line element takes the form

$$ds^2 = -dt^2 + b^2t^2dx^2 + b^{\frac{2}{n}}t^{\frac{2}{n}}(e^{-2mx}dy^2 + dz^2) \tag{3.8}$$

where $b = \dfrac{2nm}{\sqrt{(n-1)(2-2n+2\alpha)}}$,

SINGULARITY OF MODEL

The singularity in $f(R)$ gravity has been analyzed by many authors [11, 12] .To analyze the singularity we use Riemann tensor .If the curvature becomes infinite at a certain point then the singularity is essential. Using equations (3.5), (3.6), and (2.2) we obtain Ricci scalar as

$$R = -\left[\frac{2n + (n-1)u(n+2) - 24}{n^2t^2}\right] \tag{3.9}$$

It seems that when $t = 0$ and $n = 0$ the Ricci scalar becomes infinite hence the model has singularity at $t = 0$ and $n = 0$. Also at $n = -2$ Ricci scalar has singularity.

Physical parameters

For the line element (3.8) we have

Spatial volume $V = (ABC) = b^{\frac{n+2}{n}} t^{\frac{n+2}{n}}$, (3.10)

Hubble parameter $H = \dfrac{1}{3}\left(\dfrac{A_4}{A} + \dfrac{2B_4}{B}\right) = \dfrac{1}{3}\left(\dfrac{n+2}{nt}\right)$, (3.11)

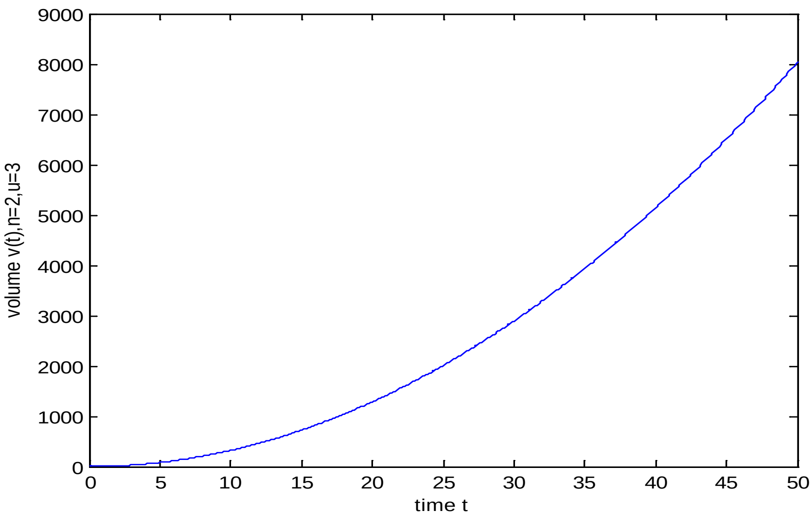
Expansion scalar $\theta = 3H = \dfrac{n+2}{nt}$, (3.12)

Shear scalar $\sigma^2 = \dfrac{2}{3}A_m H^2 = \dfrac{1}{3}\dfrac{(n-1)^2}{n^2t^2}$, (3.13)

Deceleration parameter $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{2n-2}{n+2}$, (3.14)

Mean Anisotropy parameter $A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H}{H}\right)^2 = \frac{2(n-1)^2}{(n+2)^2}$. (3.15)

For $n = 2$ from the physical parameters we find that the ratio of shear and expansion $\frac{\sigma}{\theta} = 0.144 \neq 0$ which means that the universe is anisotropic in nature. The volume of the model is increasing indefinitely with increasing time. Thus we have found that the present universe is expanding. At the epoch $t = 0, \sigma, \theta, H$ are singular which resembles to T.Singh and A K Agrawal [28].Also we find that the nature of the universe according to the parameter analogous to [11]. For value of n greater than 2 the deceleration parameter q is positive that is the universe is decelerating.



We can observe the nature of volume with increasing time in graph.

Jerk parameter $= \frac{a_{444}}{aH^3} = \frac{(2-2n)(2-5n)}{(n+2)^2}$ (3.16)

Snap parameter $= \frac{a_{4444}}{aH^4} = \frac{(2-2n)(2-5n)(2-8n)}{(n+2)^3}$ (3.17)

4. MODEL II

For the constant curvature solution say $R_0 = 0$ we have $F_4(R_0) = F_4\ (R_{40})$ and equations (2.15) and (2.16) becomes.

$$2\frac{B_{44}}{B} - 2\frac{A_4 B_4}{AB} + \frac{2m^2}{A^2} = 0, \tag{4.1}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{A_4 B_4}{AB} - \frac{B_4^2}{B^2} = 0, \tag{4.2}$$

To solve this non linear differential equation we assume that $A = k_1,\ k_1$ is constant. In literature such types of tricks used by T.Singh and R.Chaubey in the expansion of Bianchi type I,III,V VI0 and Kantowski-Sachs universe in creation field theory[29].Thus we have solutions are

$$A = k_1,\ C = B = e^{dt}\ d \neq 0 \quad . \tag{4.3}$$

These are solution of the system of equation for the condition $d^2 k_1^2 + m^2 = 0$

The line element takes the form

$$ds^2 = -dt^2 + k_1^2 dx^2 + e^{2dt}(e^{-2mx} dy^2 + e^{2mx} dz^2) \qquad (4.4)$$

We find that the model is free from singularity.

Using equation (4.3) and (2.2) we obtain Ricci scalar as

$$R = 2\left[-\frac{m^2}{k_1^2} + d^2\right], \qquad (4.5)$$

PHYSICAL PARAMETERS

For the line element (4.4) we have

Spatial volume $V = k_1 e^{2dt}$ (4.6)

Hubble paramete $H = \frac{2}{3} d$, (4.7)

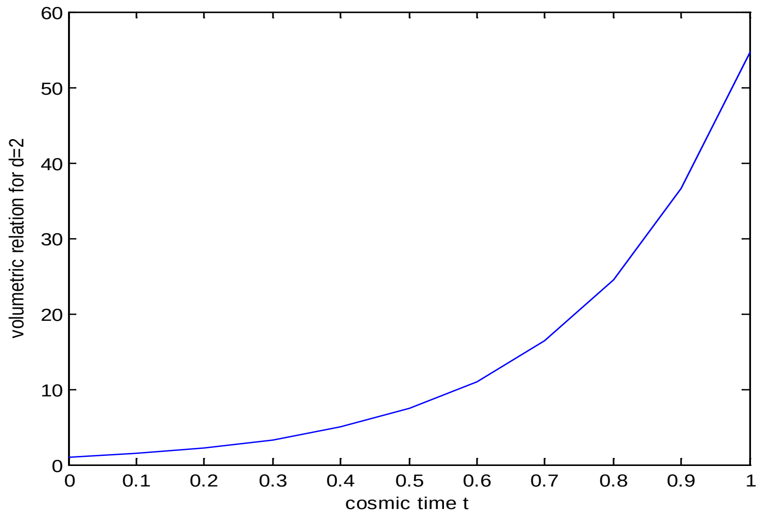
Expansion scalar $\theta = 3H = \dfrac{n+2}{nt}$, (4.8)

Shear scalar $\sigma^2 = \frac{2}{3} A_m H^2 = \frac{1}{3} \dfrac{(n-1)^2}{n^2 t^2}$, (4.9)

Deceleration parameter $q = \dfrac{d}{dt}\left(\dfrac{1}{H}\right) - 1 = \dfrac{2n-2}{n+2}$, (4.10)

Mean Anisotropy parameter $A_m = \frac{1}{2}$, (4.11)

The ratio of shear and expansion $\dfrac{\sigma}{\theta} = 1.1547 \neq 0$ that is the universe is anisotropic. The volume of the model increasing indefinitely with increasing time .The deceleration parameter is negative that is our model predicts the accelerating nature of the universe. In this model results are differ from T.Singh and A K Agrawal[28]. The graphic relation between volume and time also shows that the universe is expanding.



Jerk parameter $=27$ (4.12)

Snap parameter $=81$ (4.13)

The value of the jerk parameter is greater than 1that means that the universe is not near to the cold dark matter.

5. CONCLUSION

We have presented Bianchi type VI_0 cosmological model. We have investigated this for function of Ricci scalar is proportional to scale factor. Purpose of our investigation is to discuss the nature of the universe in $f(R)$ gravity which is expanding in present. In our investigation we pointed out that Universe has singularity at $t=0$ and $n=0$.The universe is anisotropic and expanding. The universe shows contradicting nature. Shear scalar, expansion scalar and Hubble parameter has initial singularity. The nature of these parameters is found to be similar to that of M.Farasat Shamir [11].

In the second model that is for constant curvature solution the universe is found to be anisotropic and expanding, accelerating in nature.

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