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UNSTEADY CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A VERTICAL CHANNEL WITH CHEMICAL REACTION, RADIATION ABSORPTION AND DISSIPATION

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ABSTRACT

We consider the unsteady thermal convection due to the imposed traveling thermal wave boundary through a vertical channel bounded by flat walls. The effects of free convective heat and mass transfer flow has been discussed by solving the governing unsteady non-linear equations under perturbation scheme. The velocity, the temperature and the concentration have been analysed for different variations of the governing parameters. The shear stress, the rate of heat transfer and the rate of mass transfer have been evaluated and tabulated for these sets of parameters.

Keywords: *Heat and Mass transfer, Chemical Reaction, Radiation Absorption & Dissipation.*

1. INTRODUCTION

There are many transport processes in nature and in many industries where flows with free convection currents caused by the temperature differences are affected by the differences in concentration or material constitutions. In a number of engineering applications foreign gases are injected to attain more efficiency, the advantage being the reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer. Gases such as H_2, H_2O, CO_2, etc., are usually used as foreign gases in air flowing past bodies. So the problems of heat and mass transfer past vertical bodies in boundary layer flows have been studied by many of whom the names of Somers[18],Gil *et al* [5], Adeams and Lowell[1] and Gebhart and Peera[3] are worth mentioning. The mass transfer phenomenon in unsteady free convective flow past infinite vertical porous plate was also studied by Soudalgekar and Wavre[19] and Hossain and Begum[7].

The combined effects of thermal and mass diffusion in channel flows has been studied in the recent times by a few authors notably. Nelson and wood [12, 13]. Lee *et al* [8], Miyatake and Fujii [9, 10], Sparrow *et al* [21] and others [16,20,23,27]. Nelson and Wood [13] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. For along channel (low Rayleigh numbers) the numerical solutions approach the fully developed flow analytical solutions. At intermediate Rayleigh numbers it is observed that the parallel plate heat and mass transfer is higher than that for a single plate. Yan and Lin [30] have examined the effects of the latent heat transfer associated with the liquid film vapourization on the heat transfer in the laminar forced convection channel flows. Results are presented for an air-water system under various conditions. The effects of system temperature on heat and mass transfer are investigated. Recently Atul Kumar Singh *et al* [2] investigated the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Convection fluid flows generated by traveling thermal waves have also received attention due to applications in physical problems. The linearised analysis of these flows has shown that a traveling thermal wave can generate a mean shear flow within a layer of fluid, and the induced mean flow is proportional to the square of the amplitude of the wave. From a physical point of view, the motion induced by traveling thermal waves is quite interesting as a purely fluid-dynamical problem and can be used as a possible explanation for the observed four – day retrograde zonal motion of the upper atmosphere of venus. Also, the heat transfer results will have a definite bearing on the design of oil or gas –fired boilers.

Vajravelu and Debnath [25] have made an interesting and a detailed study of non-linear convection heat transfer and fluid flows, induced by traveling thermal waves. The traveling thermal wave problem was investigated both analytically and experimentally by Whitehead [28] by postulating series expansion in the square of the aspect ratio (assumed small) for both the temperature and flow fields. Whitehead [28] obtained an analytical solution for the mean flow produced by a moving source. Theoretical predictions regarding the ratio of the mean flow velocity to the source speed were found to be good agreement with experimental observations in Mercury which therefore justified the

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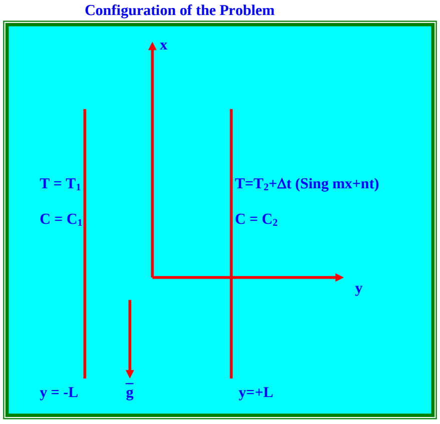
validity of the asymptotic expansion a posteriori. Ravindra [15] has analysed the mixed convection flow of a viscous fluid through a porous medium in a vertical channel. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundaries. Purushothama Reddy [14] has analysed the unsteady mixed convective effects on the flow induced by imposing traveling thermal waves on the boundaries. Nagaraja [11] has investigated the combined heat and mass transfer effects on the flow of a viscous fluid through a porous medium in a vertical channel, with the traveling thermal waves imposed on the boundaries while the concentration is maintained uniform on the boundaries Sivanjaneya Prasad[17] has analysed heat and mass transfer effects on the flow of an incompressible viscous fluid through a porous medium in vertical channel. Sulochana *et al* [23] have considered the unsteady convective heat and mass transfer through a porous medium due to the imposed traveling thermal wave boundary through a horizontal channel bounded by non –uniform walls. Tanmay Basak *et al* [24] have analysed the natural convection flows in a square cavity filled with a porous matrix for uniformly and non-uniformly heated bottom wall and adiabatic top wall maintaining crust temperature of cold vertical walls Darcy – Forchheimer model is used to simulate the momentum transfer in the porous medium. Guria and Jana [6] have discussed the two dimensional free and forced convection flow and heat transfer in a vertical wavy channel with traveling thermal waves embedded in a porous medium. The set of non-linear ordinary differential equations are solved analytically. The velocity and temperature fields have been obtained using perturbation technique.

In this chapter we consider the unsteady thermal convection due to the imposed traveling thermal wave boundary through a vertical channel bounded by flat walls. The effects of free convective heat and mass transfer flow has been discussed by solving the governing unsteady non-linear equations under perturbation scheme. The velocity, the temperature and the concentration have been analysed for different variations of the governing parameters. The shear stress, the rate of heat transfer and the rate of mass transfer have been evaluated and tabulated for these sets of parameters.

2. FORMULATION OF THE PROBLEM

Configuration of the Problem



We consider the motion of viscous, incompressible electrically conducting fluid in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at y=L while the boundary at y = -L is maintained at constant temperature T_1. The walls are maintained at constant concentrations. A uniform magnetic field of strength H0 is applied transverse to the walls. Assuming the magnetic Reynolds to be small we neglect the induced magnetic field in comparison to the applied magnetic field. Assuming that the flow takes place at low concentration we neglect the Duffor effect .The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are taken into account to the transport of heat by conduction and convection in the energy equation. Also the kinematic viscosity ν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0 (x, y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y = ± L.

The equations governing the unsteady flow and heat transfer are

Equation of linear momentum

$$\rho_e\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g-\left(\sigma\mu_e^2 H_o^2\right)u \tag{1}$$

$$\rho_e\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

Equation of energy

$$\rho_e C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q+\mu\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right)+(\sigma\mu_e^2 H_o^2)(u^2+v^2)+Q_1^{'}(C-C_e) \tag{4}$$

Equation of Diffusion

$$\left(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1(C-C_e) \tag{5}$$

Equation of state

$$\rho-\rho_e=-\beta\rho_e(T-T_e)-\beta^*\rho_e(C-C_e) \tag{6}$$

where ρ_e is the density of the fluid in the equilibrium state, Te, Ce are the temperature and Concentration in the equilibrium state, (u, v)are the velocity components along O(x, y) directions, p is the pressure, T ,C are the temperature and Concentration in the flow region,ρis the density of the fluid,μ is the constant coefficient of viscosity, Cp is the specific heat at constant pressure, λis the coefficient of thermal conductivity, D_1 is the molecular diffusivity, Q_1' is the radiation absorption coefficient,β is the coefficient of thermal expansion, β^* is the volume expansion with mass fraction ,k1 is the chemical reaction coefficient, Q is the strength of the constant internal heat source, σ is electrical conductivity of the medium magnetic permeability, μe is magnetic permeability .

In the equilibrium state

$$0=-\frac{\partial p_e}{\partial x}-\rho_e g \tag{7}$$

where $p=p_e+p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q=\frac{1}{L}\int_{-L}^{L}u\,dy. \tag{8}$$

The boundary conditions for the velocity and temperature fields are

u = 0, v = 0, $T=T_1$, $C = C_1$ on y = -L

$$u=0,\ v=0, T=T_2+\Delta T_e\,Sin(mx+nt), C=C_2 \text{ on } y=L \tag{9}$$

where $\Delta T_e=T_2-T_1$ and $Sin(mx+nt)$ is the imposed traveling thermal wave

In view of the continuity equation we define the stream function ψ as

$$u=-\psi_y,\ v=\psi_x \tag{10}$$

Eliminating pressure p from equations (2)&(3)and using the equations governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t+\psi_x(\nabla^2\psi)_y-\psi_y(\nabla^2\psi)_x]=\nu\nabla^4\psi-\beta g(T-T_0)_y-\beta^\bullet g(C-C_0)_y-\left(\frac{\sigma\mu_e^2H_o^2}{\rho_e}\right)\frac{\partial^2\psi}{\partial y^2} \tag{11}$$

$$\rho_e C_p\left(\frac{\partial T}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right)=\lambda\nabla^2 T+Q+\mu\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right)$$
$$+\left(\left(\frac{\sigma\mu_e^2H_o^2}{\rho_e}\right)\right)\left(\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right)+Q_1'(C-C_e) \tag{12}$$

$$\left(\frac{\partial C}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right)=D_1\nabla^2 C-k_1 C \tag{13}$$

Introducing the non-dimensional variables in (11) - (13) as

$$x' = mx,\ y' = y/L, t' = t\nu m^2, \Psi' = \Psi/\nu, \theta = \frac{T - T_e}{\Delta T_e}, C' = \frac{C - C_1}{C_2 - C_1} \tag{14}$$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R\left(\delta(\nabla_1^2\psi)_t + \frac{\partial(\psi, \nabla_1^2\psi)}{\partial(x,y)}\right) = \nabla_1^4\psi - \left(\frac{G}{R}\right)(\theta_y + NC_y) - M^2\frac{\partial^2\psi}{\partial y^2} \tag{15}$$

The energy equation in the non-dimensional form is

$$\delta P\left(\delta\frac{\partial\psi}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \nabla_1^2\theta + \alpha + \left(\frac{PR^2E_c}{G}\right)\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \delta^2\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right)$$
$$+ \left(M^2\right)\left(\delta^2\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2\right) + Q_1 C \tag{16}$$

The Diffusion equation is

$$\delta Sc\left(\delta\frac{\partial C}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = \nabla_1^2 C - kC \tag{17}$$

where

$R = \dfrac{qL}{\nu}$ (Reynolds number) $G = \dfrac{\beta g \Delta T_e L^3}{\nu^2}$ (Grashof number)

$P = \dfrac{\mu c_p}{k_1}$ (Prandtl number), $E_c = \dfrac{\beta g L^3}{C_p}$ (Eckert number)

$\delta = mL$ (Aspect ratio) $\gamma = \dfrac{n}{\nu m^2}$ (non-dimensional thermal wave velocity)

$Sc = \dfrac{\nu}{D_1}$ (Schimdt Number) $N = \dfrac{\beta^*\Delta C}{\beta\Delta T}$ (Buoyancy ratio)

$Q_1 = \dfrac{Q_1'\Delta C L^2}{\Delta T k_1}$ (**R**adiation absorption parameter)

$M^2 = \dfrac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartmann Number) $k = \dfrac{k_1 L^2}{D_1}$ (Chemical reaction parameter)

$\nabla_1^2 = \delta^2\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = -1$$

$$\frac{\partial\psi}{\partial x} = 0,\ \ \frac{\partial\psi}{\partial y} = 0\ \ at\ y = \pm 1 \tag{18}$$

$\theta(x,y) = 1,\ C(x,y) = 0$ on y = -1

$\theta(x,y) = Sin(x + \gamma t)$, C(x, y) = 1 on y = 1

$$\frac{\partial\theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0\ at\ y = 0 \tag{19}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hyphothesis (8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

We adopt the perturbation scheme and write

$$\psi(x,y,t)=\psi_0(x,y,t)+\delta\psi_1(x,y,t)+\delta^2\psi_2(x,y,t)+...............$$
$$\theta(x,y,t)=\theta_0(x,y,t)+\delta\,\theta_1(x,y,t)+\delta^2\,\theta_2(x,y,t)+.............$$
$$C(x,y,t)=C_0(x,y,t)+\delta C_1(x,y,t)+\delta^2\,C_2(x,y,t)+............. \tag{20}$$

On substituting (20) in (16) - (17) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,\,yyyyy}-M_1^2\psi_{0,\,yy}=\frac{G}{R}(\theta_{0,y}+NC_{O,Y}) \tag{21}$$

$$\theta_{o,yy}+\alpha+\frac{PE_cR^2}{G}(\psi_{o,yy})^2+\frac{PE_cM_1^2}{G}(\psi_{o,y}^2)+Q_1C_0=0 \tag{22}$$

$$C_{O,YY}-(kSc)C_0=0 \tag{23}$$

With $\psi_0(+1)-\Psi(-1)=-1$,
$\psi_{0,y}=0$, $\psi_{0,x}=0$ at $y=\pm1$ (24)

$$\theta_o=1, C_0=0 \; on \; y=-1$$
$$\theta_o=Sin(x+\gamma t), C_0=1 \; on \; y=1 \tag{25}$$

and to the first order are

$$\psi_{1,\,yyyyy}-M_1^2\psi_{1,\,yy}=\frac{G}{R}(\theta_{1,Yy}+NC_{1,Y})+(\psi_{0,\,y}\,\psi_{0,\,xyy}-\psi_{0,\,x}\psi_{0,yyy}) \tag{26}$$

$$\theta_{1,yy}=(\psi_{0,x}\theta_{o,y}-\psi_{0,y}\theta_{ox})+\frac{2PE_cR^2}{G}(\psi_{0,yy}.\psi_{1,yy})+\frac{2PE_cM_1^2}{G}(\psi_{0,y}.\psi_{1,y})-Q_1C_1 \tag{27}$$

$$C_{1,yy}-(kSc)\,C_1=(\psi_{0,x}C_{o,y}-\psi_{0,y}C_{ox}) \tag{28}$$

with $\psi_{1(+1)}-\psi_{1(-1)}=0$
$\psi_{1,y}=0$, $\psi_{1,x}=0$ at $y=\pm1$ (29)
$\theta_1(\pm1)=0$, $C_1(\pm1)=0$ at $y=\pm1$ (30)

Assuming Ec<<1 to be small we take the asymptotic expansions as

$$\psi_0(x,y,t)=\psi_{00}(x,y,t)+Ec\psi_{01}(x,y,t)+...........$$
$$\psi_1(x,y,t)=\psi_{10}(x,y,t)+Ec\,\psi_{11}(x,y,t)+...........$$
$$\theta_0(x,y,t)=\theta_{00}(x,y,t)+\theta_{01}(x,y,t)+...............$$
$$\theta_1(x,y,t)=\theta_{10}(x,y,t)+\theta_{11}(x,y,t)+.............$$
$$C_0(x,y,t)=C_{00}(x,y,t)+C_{01}(x,y,t)+.............$$
$$C_1(x,y,t)=C_{10}(x,y,t)+C_{11}(x,y,t)+............ \tag{31}$$

Substituting the expansions (31) in equations (21)-(23) and separating the like powers-of Ec we get t1he following

$$\theta_{00,yy} = -\alpha - Q_1 C_{00}, \quad \theta_{00}(-1) = 1, \theta_{00}(+1) = SinD_1 \tag{32}$$

$$C_{00,yy} - (kSc)C_{00} = 0, \quad C_{00}(-1) = 0, C_{00}(+1) = 1 \tag{33}$$

$$\begin{gathered}\psi_{00,yyyy} - M_1^2 \psi_{00,yy} = \frac{G}{R}(\theta_{00,y} + NC_{00,y}), \\ \psi_{00}(+1) - \psi_{00}(-1) = 1, \psi_{00,y} = 0, \psi_{00,x} = 0 \;\; at \;\; y = \pm 1\end{gathered} \tag{34}$$

$$\theta_{01,yy} = -\frac{PR}{G}\psi^2{}_{00,yy} - \frac{PM_1^2}{G}\psi_{00,y}^2 - Q_1 C_{01}, \quad \theta_{01}(\pm 1) = 0 \tag{35}$$

$$C_{01,yy} \;\; - (kSc)C_{01} = 0, \quad C_{01}(-1) = 0, C_{01}(+1) = 0 \tag{36}$$

$$\psi_{01,yyyy} - M_1^2 \psi_{01,yy} = \frac{G}{R}(\theta_{01,y} + NC_{01,y})$$

$$\psi_{01}(+1) - \psi_{01}(-1) = 0, \psi_{01,y} = 0, \psi_{01,x} = 0 \;\; at \;\; y = \pm 1 \tag{37}$$

$$\theta_{10,yy} = GP_1(\psi_{00,y}\theta_{00,x} - \psi_{00,x}\theta_{00,y}) - Q_1 C_{10}, \; \theta_{10}(\pm 1) = 0 \tag{38}$$

$$C_{10,yy} - (kSc)C_{10} = Sc(\psi_{00,y}C_{00,x} - \psi_{00,x}C_{00,y}) \quad C_{10}(\pm 1) = 0 \tag{39}$$

$$\psi_{10,yyyy} - M_1^2 \psi_{10,yy} = \frac{G}{R}(\theta_{10,y} + NC_{10,Y}) + (\psi_{00,y}\psi_{00,xyy} - \psi_{00,x}\psi_{00,yyy}) \tag{40}$$

$$\psi_{10}(+1) - \psi_{10}(-1) = 0, \psi_{10,y} = 0, \psi_{10,x} = 0 \;\; at \;\; y = \pm 1$$

$$\begin{aligned}\theta_{11,yy} = {} & P(\psi_{00,y}\theta_{01,x} - \psi_{01,x}\theta_{00,y} + \theta_{00,x}\psi_{01,y} - Q_1 C_{11} - \theta_{01,y}\psi_{0,x}) \\ & -\frac{2PR^2}{G}\psi_{00,yy}\psi_{10,yy} - \frac{2PM_1^2}{G}\psi_{00,y}\psi_{10,y} \;, \; \theta_1(\pm 1) = 0\end{aligned} \tag{41}$$

$$C_{11,yy} - (kSc)C_{11} = Sc(\psi_{00,y}C_{01,x} - \psi_{01,x}C_{00,y} + C_{00,x}\psi_{01,y} - C_{01,y}\psi_{0,x}) \tag{42}$$

$$\begin{gathered}\psi_{11,yyyy} - M_1^2 \psi_{1,yy} = \frac{G}{R}(\theta_{11,y} + NC_{11,Y}) + (\psi_{00,y}\psi_{11,xyy} - \psi_{00,x}\psi_{01,yyy} + \psi_{01,y}\psi_{00,xyy} - \psi_{01.x}\psi_{00,yyy}) \\ \psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,y} = 0, \psi_{11,x} = 0 \;\; at \;\; y = \pm 1\end{gathered} \tag{43}$$

4. SOLUTION OF THE PROBLEM

Solving the equations (32) - (43) subject to the relevant boundary conditions we obtain

$$C_{00} = 0.5\left(\frac{Sinh(\beta_1 y)}{Sinh(\beta_1)} + \frac{Cosh(\beta_1 y)}{Cosh(\beta_1)}\right)$$

$$\theta_{oo}(y,t) = \left(\frac{\alpha}{2}\right)(1-y^2) + \frac{Q_1}{2\beta_1^2}\left(1 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)}\right) + \frac{Q_1}{2\beta_1^2}\left(y - \frac{Sh(\beta_1 y)}{Sh(\beta_1)}\right) + \frac{Sin(D_1)}{2}(y+1) + 0.5\,(1-y)$$

$$\psi_{oo}(y,t) = a_{13}Cosh(M_1 y) + a_{14}Sinh(M_1 y) + a_{15}y + a_{16} + \varphi_1(y)$$

$$\varphi_1(y) = a_9 + a_{10}y^3 + a_{11}Ch(\beta_1 y) + a_{12}Sh\left(\beta_1 y\right)$$

$$C_{01}(y,t) = 0$$

$$\begin{aligned}\theta_{01}(y,t) = {} & a_{31}(y^2-1) + a_{32}(y^4-1) + a_{33}(Ch(2\beta_1 y) - Ch(2\beta_1)) + a_{34}(Sh(2\beta_1 y) \\ & -ySh(2\beta_1)) + a_{35}(Sh(2M_1 y) - ySh(2M_1)) + a_{36}(ySh(M_1 y) \\ & -Sh(M_1)) + a_{37}(ySh(\beta_1 y) - Sh(\beta_1)) + a_{38}(Ch(\beta_1 y) - Ch(\beta_1)) + a_{39}y(Ch(\beta_1 y) \\ & -Ch(\beta_1)) + a_{40}(Sh(\beta_1 y) - ySh(\beta_1)) + a_{41}(Ch(M_1 y) - Ch(M_1)) + a_{42}(Ch(\beta_2 y) \\ & -Ch(\beta_2)) + a_{43}(Ch(\beta_3 y) - Ch(\beta_3)) + + a_{44}(Sh(\beta_2 y) - ySh(\beta_2)) + a_{45}(Sh(\beta_3 y) - ySh(\beta_3))\end{aligned}$$

$$\psi_{01}(y,t) = a_{68} + a_{67}y + a_{65}Ch(M_1 y) + a_{66}Sh(M_1 y) + \varphi_2(y)$$

$$\begin{aligned}\varphi_2(y) = {} & -a_{47}y^2 + a_{48}y^3 + a_{49}y^5 - a_{50}Sh(2\beta_1 yt) - a_{51}Ch(2\beta_1 y) - a_{52}yCh(M_1 y) \\ & +a_3 y^3 Sh(M_1 y) + a_{54}y^2 Sh(\beta_1 y) + a_{55}yCh(M_1 y) + a_{56}ySCh(\beta_1 y) + a_{57}Sh(\beta_1 y) \\ & -a_{58}Ch(\beta_1 y) - a_{59}ySh(M_1 y) - a_{60}y^2 Ch(\beta_1 y) - a_{61}Sh(\beta_2 y) - a_{62}Sh(\beta_3 y) \\ & -a_{63}Ch(\beta_2 y) - a_{64}Ch(\beta_3 y)\end{aligned}$$

$$C_{10}(y,t) = b_1 Ch(\beta_1 y) + b_2 Sh(\beta_1 y) + \varphi_3(y)$$

$$\begin{aligned}\varphi_3(y) = {} & a_{85} + a_{86}Sh(\beta_2 y) + a_{85}Sh(\beta_3 y) + a_{88}Ch(\beta_2 y) + a_{89}Ch(\beta_3 y) \\ & +(a_{90}y + a_{92}y^2 + a_{95}y^3 + a_{96}y^4)Ch(\beta_1 y) + (a_{91}y + a_{93}y^2 + a_{94}y^3 \\ & +a_{97}y^4)Sh(\beta_1 y) + a_{98}Ch(2\beta_1 y) + a_{99}Sh(2\beta_1 y)\end{aligned}$$

$$\begin{aligned}\theta_{10}(y,t) = {} & b_{42}y^2 + b_{43}y^3 + b_{44}y^5 + b_{46}y^6 + (b_{47} + b_{49}y)Ch((M_1 y) + (b_{48} + b_{50}y)Sh(M_1 y) \\ & +(b_{51} + b_{54}y + b_{55}y^2 + b_{58}y^3 + b_{60}y^4)Ch(\beta_1 y) + (b_{52} + b_{53}y + b_{56}y^2 + b_{57}y^3 \\ & +b_{59}y^4)Sh(\beta_1 y) + b_{61}Sh(2\beta_1 y) + b_{62}Ch(2\beta_1 y) + b_{63}Sh(\beta_2 y) + b_{64}Sh(\beta_3 y) \\ & +b_{65}Ch(\beta_2 y) + b_{66}Ch(\beta_3 y) + b_{67}y + b_{68}\end{aligned}$$

$$\psi_{10} = d_{59}Ch(M_1 y) + d_{61}y + d_{62} + \varphi_4(y) + d_{60}Sh(M_1 y)$$

$$\begin{aligned}\varphi_4(y) = {} & d_{27}y + d_{28}y^2 + d_{29}y^3 + d_{30}y^4 + d_{31}y^5 + d_{32}y^6 + (d_{34} + d_{35}y + d_{40}y^2 + d_{44}y^3 \\ & +d_{47}y^4 + d_{49}y^6)Sh(\beta_1 y) + (d_{33} + d_{36}y + d_{39}y^2 + d_{43}y^3 + d_{48}y^4)Ch(\beta_1 y) \\ & +(d_{38}y + d_{41}y^2 + d_{46}y^3)Ch(M_1 y) + (d_{37}y + d_{42}y^2 + d_{45}y^5)\ Sh(M_1 y) \\ & +d_{53}Ch(2M_1 y) + d_{545}Sh(2M_1 y) + d_{51}Sh(2\beta_1 y) + d_{52}Ch(2\beta_1 y) + d_{55}Sh(\beta_2 y) \\ & +d_{56}Sh(\beta_3 y) + d_{57}Ch(\beta_2 y) + d_{58}Ch(\beta_3 y)\end{aligned}$$

where $a_1, a_2, ...a_{105}, b_1, b_2, ...b_{79}$, $d_1,\ldots,d_{58}$ are constants

5. NUSSELT NUMBER and SHERWOOD NUMBER

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$Nu = \dfrac{1}{\theta_m - \theta_w}\left(\dfrac{\partial\theta}{\partial y}\right)_{y=\pm 1}$ and the corresponding expressions are

$$(Nu)_{y=+1} = \frac{(d_{65} + Ecd_{67} + \delta\, d_{69})}{(\theta_m - Sin(D_1)}$$

$$(Nu)_{y=-1} = \frac{(d_{66} + Ecd_{68} + \delta\, d_{70})}{(\theta_m - 1)}$$

$$\theta_m = d_{71} + Ecd_{72} + \delta\, d_{73}$$

The local rate of mass transfer coefficient(Sherwood number)(Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm 1}$$ and the corresponding expressions are

$$(Sh)_{y=+1} = \frac{(d_{74} + \delta\, d_{76})}{(C_m - 1)}$$

$$(Sh)_{y=-1} = \frac{(d_{75} + \delta\, d_{77})}{(C_m)}$$

$$C_m = d_{78} + \delta\, d_{79}$$

where d_1,………..d_{77} constants.

6. DISCUSSION OF THE RESULTS

In this analysis we discuss the combined influence of radiation absorption, chemical reaction and dissipation on unsteady convective heat and mass transfer flow of a viscous dissipative fluid in a vertical channel under the influence of the uniform magnetic field. The unsteady flow is due to the travel in thermal waves imposed on the boundaries. The non-linear coupled equations governing the flow heat and mass transfer are solved by a perturbation technique with aspect ratio δ and Eckert number Ec as perturbation parameters.

The axial velocity (u) is shown in figs (1 – 3) for different values of Sc, γ, Q_1, Ec. The actual axial flow is in the vertically upward direction and hence u < 0 represents a reversal flow. Fig (1) represents u with Schmidt number Sc. It is found that the flow exhibits a reversal flow in the entire flow region with Sc = 0.24 and for higher Sc = 0.6 it is confined to the regions adjacent to y = ±1 and for still higher Sc ≥ 1.3 it disappears in the entire flow region. |u| depreciates with increase in Sc ≤ 0.6 and enhances with higher Sc ≥ 1.3. The variation of u with chemical reaction parameter γ and radiation absorption parameter Q_1 is shown in fig (2). An increase in γ or Q_1 leads to an enhancement in u in the entire flow region. Fig (3) represents u with Eckert number Ec. It is found that higher the dissipative heat smaller the axial velocity in the flow region.

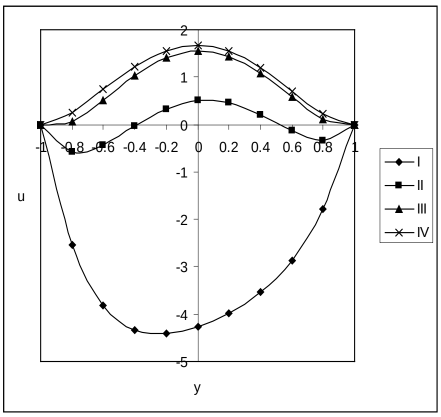


Fig. 1 : Variation of u with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

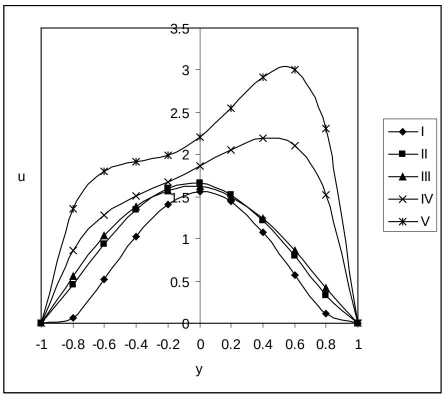


Fig. 2 : Variation of u with γ & Q_1

	I	II	III	IV	V
γ	0.5	1.5	2.5	0.5	0.5
Q_1	1.5	1.5	1.5	2.5	3.5

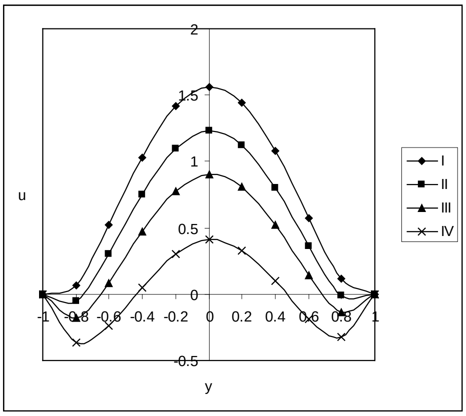


Fig. 3 : Variation of u with Ec

	I	II	III	IV
Ec	0.02	0.04	0.06	0.09

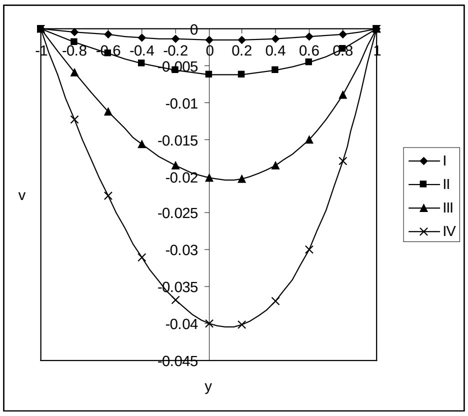


Fig. 4 : Variation of v with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

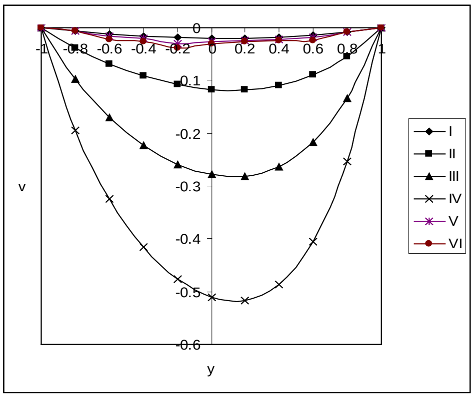


Fig. 5 : Variation of v with γ & Q_1

	I	II	III	IV	V
γ	0.5	1.5	2.5	0.5	0.5
Q_1	1.5	1.5	1.5	2.5	3.5

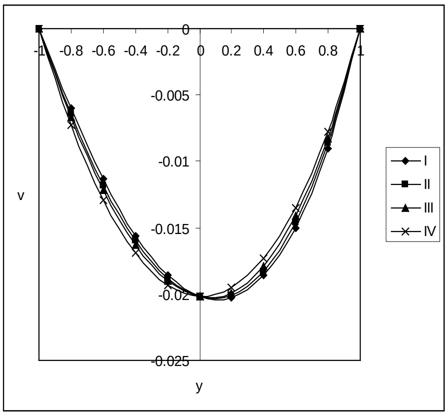


Fig. 6 : Variation of v with Ec

	I	II	III	IV
Ec	0.02	0.04	0.06	0.09

The secondary velocity (v) which arises due to the non-uniform boundary temperature is shown in figs (4 – 6) for different parametric values. It is observed that the secondary velocity v is directed towards the mid region for all variations. From fig(4) we find that |v| enhances with increase in Sc thus lesser the molecular diffusivity larger |v| in the entire flow region. With respect to γ and q_1 we find an enhancement in |v| with increase in γ or Q_1. Thus the effect of radiation absorption is to enhance the magnitude of v in the flow region (fig 5). The variation of v with Eckert number Ec is shown in fig(6). It is found that higher the dissipative heat larger |v| in the left half and smaller in the right half of the channel.

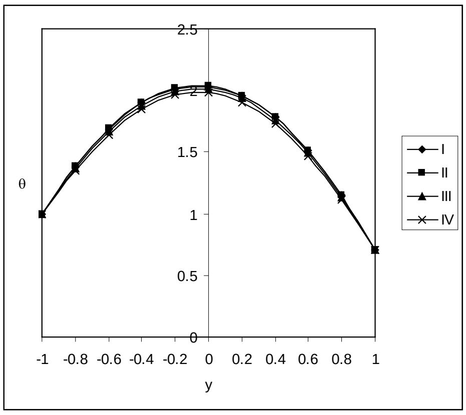


Fig. 7 : Variation of θ with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

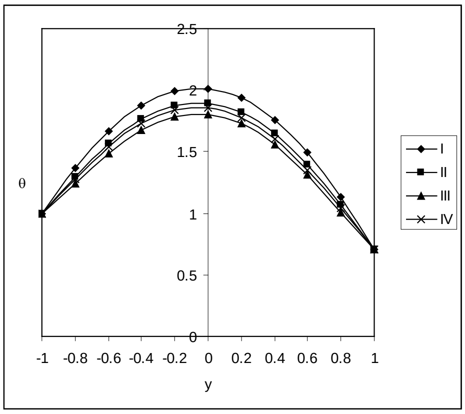


Fig. 8 : Variation of θ with γ

	I	II	III	IV
γ	0.5	1.5	2.5	3.5

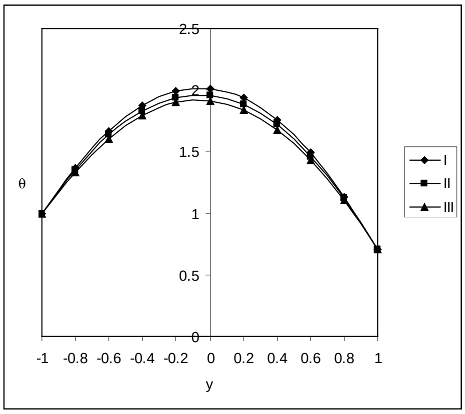


Fig. 9: Variation of θ with Q_1

	I	II	III
Q_1	1.5	2.5	3.5

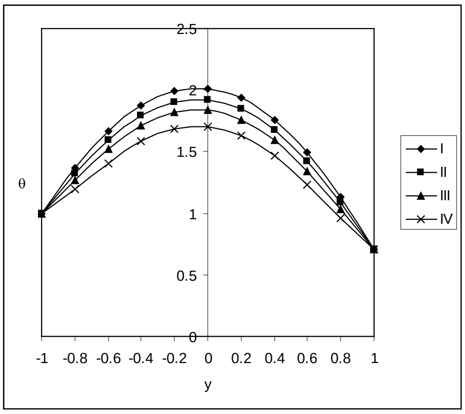


Fig. 10 : Variation of θ with Ec

	I	II	III	IV
Ec	0.02	0.04	0.06	0.09

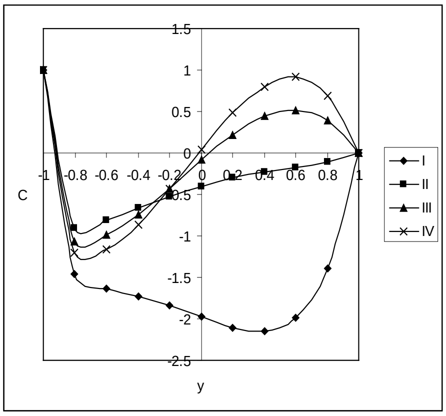


Fig. 11 : Variation of C with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

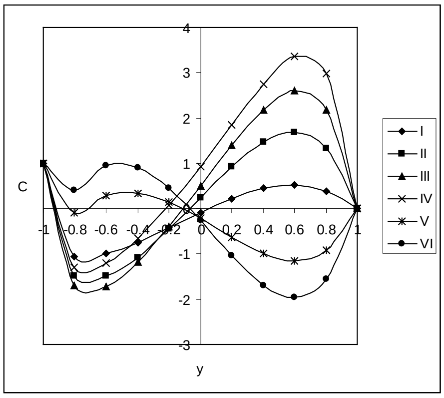


Fig. 12 : Variation of C with γ & Q_1

	I	II	III	IV	V
γ	0.5	1.5	2.5	0.5	0.5
Q_1	1.5	1.5	1.5	2.5	3.5

The non-dimensional temperature distribution (θ) is shown in figures (7 – 10) for different parametric values. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature is greater/lesser than T_2. From Fig. (7) we observe that lesser the molecular diffusivity ($Sc \le 0.6$) larger the actual temperature and for further lowering of the diffusivity smaller the temperature in the flow region (fig 7). The variation of θ with chemical reaction parameter γ indicates that the actual temperature reduces with increase in $\gamma \le 2.5$ and enhances with $\gamma \ge 3.5$ (fig 8). Fig (9) shows that the effect of radiation absorption is to decrease the actual temperature in the flow region. The graphs of θ with Eckert number Ec indicates that higher the dissipative heat smaller the actual temperature in the flow region (fig 10).

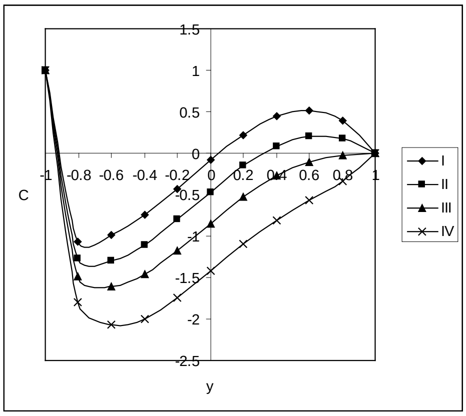


Fig. 13 : Variation of C with Ec

	I	II	III	IV
Ec	0.02	0.04	0.06	0.09

The non-dimensional concentration distribution (C) is shown in figures (11 – 13) for different parametric values. We follow the convention that the non-dimensional concentration is positive or negative according as the actual concentration is greater / lesser than C_2. With respect to Sc it is observed that lesser the molecular diffusivity larger the actual concentration in the entire flow region and for further lowering of the diffusivity the concentration reduces in the left half and enhances in the right half (fig 11). The variation of c with chemical reaction parameter γ shows that the actual concentration depreciates in the left half and enhances in the right half with increase in $\gamma \leq 2.5$ and enhances with higher $\gamma = 3.5$. The effect of radiation absorption (q_1) is to enhance the actual concentration in the left half and depreciates it in the right half (fig 12). The variation of c with Ec shows that higher the dissipative heat smaller the actual concentration in the flow region (fig 13).

Table 1: Nusselt number (Nu) at y = +1

G	I	II	III	IV	VI	VII	VIII
100	-3.0292	-6.2198	-1.6362	-1.1316	-0.8592	-0.7833	-0.7914
300	-0.1616	-5.6171	-1.6550	-0.5485	0.4080	0.3518	0.7742
-100	0.5880	-10.0260	-1.6174	-1.3094	-1.1581	-1.1043	-1.1411
-300	0.9695	18.3427	-1.5986	-1.3799	-1.1324	-1.0750	-1.1013
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
γ	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table 2: Nusselt number (Nu) at y = +1

G	I	II	III	IV	V	VI	VII
100	-1.6362	-2.0284	-3.3203	-4.6579	-1.4341	-1.3268	-1.4514
300	-1.6550	-1.3087	-3.2420	-4.9022	-0.0837	-4.1023	-3.1343
-100	-1.6174	-3.2798	-5.6409	-4.0005	-1.6868	-1.7636	-1.6023
-300	-1.5986	-6.3028	2.7378	-2.3190	-5.4461	-1.1880	-1.0196
Q1	0.5	1.5	2.5	3.5	0.5	0.5	0.5
Ec	0.02	0.02	0.02	0.02	0.04	0.06	0.09

Table 3: Nusselt number (Nu) at y = -1

G	I	II	III	IV	VI	VII	VIII
100	-1.9449	-2.9956	-1.7085	0.3240	0.3402	0.3650	0.4263
300	-0.1553	-3.3925	-1.7390	-1.0798	-2.0899	-3.9704	-2.2728
-100	0.1450	-6.0286	-1.6776	0.5752	0.7539	0.9195	0.9035
-300	0.0180	2.1908	-1.6464	-0.0526	-12.7350	1.0952	1.0375
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
γ	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table 4: Nusselt number (Nu) at y = -1

G	I	II	III	IV	V	VI	VII
100	-1.7085	-0.4428	-1.5439	-1.8747	0.0990	-0.0873	-0.3988
300	-1.7390	-0.5734	-1.5949	-2.5118	-1.3767	1.9240	3.8540
-100	-1.6776	-0.3476	-1.7091	14.5546	0.4192	0.4087	0.6886
-300	-1.6464	-0.5050	0.5268	0.6143	0.0626	0.9508	1.4588
Q1	0.5	1.5	2.5	3.5	0.5	0.5	0.5
Ec	0.02	0.02	0.02	0.02	0.04	0.06	0.09

The rate of heat transfer (Nusselt number) at y = ± 1 is shown in tables (1 – 4) for different parametric values. It is found that the rate of heat transfer enhances with G > 0 and depreciates with G < 0 at y = ± 1. From tables (1 & 3) (Sc & γ) we find that lesser the molecular diffusivity larger |Nu| and for further lowering of the diffusivity smaller |Nu| at both the walls. When the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer depreciates at y = ± 1 when the buoyancy forces act in the same direction and for the forces acting in opposite direction |Nu| depreciates in the heating case and enhances in the cooling case. An increase in chemical reaction parameter $\leq \gamma$ 2.5 results in a depreciation in |Nu| at y = ± 1 and for higher $\gamma = 3.5$ we notice an enhancement in |Nu| at y = +1 and a depreciation at y = -1 (Tables 1 & 3). With respect to Q_1 we find the rate of heat transfer at y = +1 depreciates for G > 0 and enhances for G < 0 with $q_1 \leq 1.5$ and for higher $Q_1 \geq 2.5$ it enhances for G > 0 and it reduces for G < 0. At y = -1, the rate of heat transfer depreciates with increase in $Q_1 \leq 1.5$ and enhances with higher $Q_1 \geq 2.5$ (Tables 2 &4). The variation of Nu with Ec shows that at y = +1 |Nu| depreciates for G > 0 and enhances for G<0 with Ec$\leq$ 0.04 and for higher Ec = 0.06 it enhances for G > 0 and depreciates for G < 0. While at y = -1 |Nu| depreciates with Ec$\leq$ 0.06 and enhances with higher Ec = 0.09.

It is found that the rate of mass transfer with respect to chemical reaction parameter γ we find that the rate of mass transfer at y = +1 enhances in the heating case and reduces in the cooling case while at y = -1 |Sh| reduces with $\gamma \leq 1.5$ and for further higher $\gamma \geq 2.5$ it reduces in the heating case and enhances in the cooling case (Tables 5 & 6).

Table 5: Sherwood number (Sh) at y = +1

Sc	I	II	III	IV
0.24	-2.5044	-2.6016	-3.1457	-3.8601
0.6	-2.5190	-4.1394	-6.8194	-10.5469
1.3	-3.3985	-10.7174	-8.8910	-2.4593
2.01	-4.9870	-9.4170	-1.3648	-0.5496
γ	0.5	1.5	2.5	3.5

Table 6: Sherwood number (Sh) at y = -1

Sc	I	II	III	IV
0.24	4.0036	1.2239	0.6681	0.4301
0.6	1.4905	0.3538	0.1120	-0.0021
1.3	0.5246	-0.1141	-0.3611	-1.1850
2.01	0.1380	-0.5387	1.2657	0.0862
Γ	0.5	1.5	2.5	3.5

6. CONCLUSION

(1) The flow exhibits a reversal flow in the entire flow region with Sc = 0.24 and for higher Sc = 0.6 it is confined to the regions adjacent to $y = \pm 1$ and for still higher $Sc \geq 1.3$ it disappears in the entire flow region. |u| depreciates with increase in $Sc \leq 0.6$ and enhances with higher $Sc \geq 1.3$.

(2) An increase in γ or Q_1 leads to an enhancement in u in the entire flow region.

(3) Higher the dissipative heat smaller the axial velocity in the flow region

(4) Lesser the molecular diffusivity larger |v| in the entire flow region.

(5) With respect to γ and Q_1 we find an enhancement in |v| with increase in γ or Q_1.

(6) Higher the dissipative heat larger |v| in the left half and smaller in the right half of the channel

(7) Lesser the molecular diffusivity ($Sc \leq 0.6$) larger the actual temperature and for further lowering of the diffusivity smaller the temperature in the flow region.

(8) The actual temperature reduces with increase in $\gamma \leq 2.5$ and enhances with $\gamma \geq 3.5$.

(9) The effect of radiation absorption is to decrease the actual temperature in the flow region. (10) Higher the dissipative heat smaller the actual temperature in the flow region.

(11) Lesser the molecular diffusivity higher the actual concentration in the entire flow region and for further lowering of the diffusivity the concentration reduces in the left half and enhances in the right half.

(12) The actual concentration depreciates in the left half and enhances in the right half with increase in $\gamma \leq 2.5$ and enhances with higher $\gamma = 3.5$. The effect of radiation absorption (Q_1) is to enhance the actual concentration in the left half and depreciates it in the right half.

(13) Higher the dissipative heat smaller the actual concentration in the flow region.

(14) An increase in chemical reaction parameter $\gamma \leq 2.5$ results in a depreciation in |Nu| at $y=\pm 1$ and for higher $\gamma = 3.5$ we notice an enhancement in |Nu| at y = +1 and a depreciation at y = -1. With respect to Q_1 we find the rate of heat transfer at y = +1 depreciates for G > 0 and enhances for G < 0 with $Q_1 \leq 1.5$ and for higher $Q_1 \geq 2.5$ it enhances for G > 0 and it reduces for G < 0. At y = -1, the rate of heat transfer depreciates with increase in $Q_1 \leq 1.5$ and enhances with higher $Q_1 \geq 2.5$. The variation of Nu with Ec shows that at y = +1 |Nu| depreciates for G > 0 and enhances for G<0 with $Ec \leq 0.04$ and for higher Ec = 0.06 it enhances for G > 0 and depreciates for G < 0. While at y = -1 |Nu| depreciates with $Ec \leq 0.06$ and enhances with higher Ec = 0.09.

(15) The rate of mass transfer at y = +1 enhances in the heating case and reduces in the cooling case while at y = -1 |Sh| reduces with $\gamma \leq 1.5$ and for further higher $\gamma \geq 2.5$ it reduces in the heating case and enhances in the cooling case

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