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A FUZZY GOAL PROGRAMMING ALGORITHM FOR SOLVING BI-LEVEL MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEMS

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ABSTRACT

This paper studies a fuzzy goal programming (FGP) approach for solving bi-level multi-objective linear fractional programming problem. This paper makes an extension work of Pramanik and Dey [International Journal of Computer Applications 25 (11) (2011), 34-40] which deals with bi-level linear fractional programming problem based on FGP approach. In proposed approach, firstly we construct the linear fractional membership functions for objective functions of both levels decision makers (DMs). Then we transform the linear fractional membership functions into equivalent linear membership functions at the individual best solution point by using first order Taylor series. Thereafter, multi-objective decision making models are formulated for both level DMs and FGP technique is used to identify the satisfactory solution for each level DM. We again transform the linear fractional membership functions for objective functions of both level DMs into equivalent linear membership functions at the satisfactory solution point. Since the objectives of the DMs are conflicting in nature, the preference bounds on the decision variables under the control of both level DMs are considered. Finally, FGP approach is utilized to solve the problem. An illustrative numerical example is solved in order to clarify the proposed approach.

Keywords: *Bi-level multi-objective linear fractional programming; Fuzzy goal programming; Multi-objective decision making; Preference bounds; Taylor series*

AMS Subject Classification: *90C29; 90C32; 90C70.*

1. INTRODUCTION

Bi-level programming (BLP) is identified as a mathematical tool for modelling decentralized planning problems. BLP consists of the objective of the top level decision maker (TLDM) at its top level and that of the lower level decision maker (LLDM) at its lower level. In the decision making context, each level decision maker (DM) independently controls a set of decision variables and tries to optimize his/her own objective functions over a common feasible region. Also, each level DM should have an intention to cooperate with each other for the sake of the benefit of the hierarchical organization. Candler and Townsley [3] and Fortuny – Amat and McCarl [5] studied the traditional version of bi-level programming problem (BLPP) in the early eighties. Pramanik *et al.* [11] studied BLPP in intuitionistic fuzzy environment. Pramanik *et al.* [12] discussed decentralized bi-level multi-objective programming problem with fuzzy parameters based on fuzzy goal programming (FGP). In 1996, Lai [7] first incorporated the concept of tolerance membership function of fuzzy set theory to multi-level programming problem (MLPP) for obtaining satisfactory solution. Shih *et al.* [23] extended Lai's satisfactory solution concept and they proposed a supervised search approach by using non-compensatory max-min aggregation operator for MLPP. Shih and Lee [22] presented a solution methodology for MLPP by introducing the compensatory fuzzy operator. Sakawa *et al.* [20] studied interactive fuzzy programming (IFP) for solving MLPPs. Sinha [24, 25] proposed an alternative multi-level programming technique based on fuzzy mathematical programming to MLPPs. Pramanik and Roy [17] developed a FGP technique for MLPPs.

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When the objective functions of both level DMs of a BLPP are linear fractional in nature, then the BLPP is called bi-level linear fractional programming problem (BLLFPP). Malhotra and Arora [8] proposed an algorithm for solving BLLFPP through goal programming (GP) approach. Sakawa and Nishizaki [18, 19] developed IFP technique for solving BLLFPP as well as decentralized BLLFPP. Ahlatcioglu and Tiryaki [2] presented two new IFP approaches for solving decentralized BLLFPP by using analytic hierarchy process and suitable transformations. Mishra [9] studied weighting method for BLLFPP to obtain non-dominated solution. Toksari [26] presented Taylor series solution approach to BLLFPP based on the concept of Guzel and Sivri [6]. Dey and Pramanik [4] studied GP procedure to BLLFPP based on Taylor series approximation. Pramanik and Dey [16] also studied BLLFPP in fuzzy environment based on FGP approach for obtaining maximum degree of each of the membership goals by minimizing the negative deviational variables. Pramanik *et al*. [10] proposed FGP approach due to Pramanik and Roy [17] for solving decentralized BLLFPP with the help of Taylor series approximation.

In this paper, we have considered bi-level multi-objective linear fractional programming problem (BLMOLFPP) with a single TLDM with multiple objectives at the top level and a single LLDM with multiple objectives at the lower level. The objective functions of the DMs are linear fractional functions and the system constraints are linear functions. Abo-Sinna and Baky [1] presented a FGP model based on the method of variable change on the over - and under deviation variables of the membership goals to solve BLMOLFPP by utilizing linear GP methodology. Saraj and Safaei [21] investigated fuzzy BLMOLFPP by using Taylor series and Kuhn-Tucker conditions.

In the proposed approach, we construct the linear fractional membership functions for both level DMs by determining individual optimal solution of the objective functions. Then we linearize the linear fractional membership functions into equivalent linear membership functions at the individual best solution point by using first order Taylor series. We then construct the multi-objective decision making (MODM) models for both level DMs and apply FGP technique [13] in order to obtain the satisfactory solutions for both level DMs. The linear fractional membership functions of both level DMs are again linearized at the satisfactory solution point. Since the objectives of TLDM and LLDM are generally conflicting in nature, both level DMs provide the preference upper and lower bounds on the decision variables under their control in the decision making process. FGP model due to Pramanik and Dey [14] is utilized to obtain the compromise optimal solution for BLMOLFPP. Finally, a BLMOLFPP is solved in order to demonstrate the efficiency of the proposed FGP approach.

2. BLMOLFPP FORMULATION

Suppose that there are two levels in a hierarchy structure with a TLDM at the top level and a LLDM at the lower level. The TLDM controls the decision vector $x_1 = (x_{11}, x_{12}, ..., x_{1N_1}) \in R^{N_1}$ and the LLDM controls the decision vector $x_2 = (x_{21}, x_{22}, ..., x_{2N_2}) \in R^{N_2}$, where $N = N_1 + N_2$.

The BLMOLFPP of maximization-type objective functions at each level can be presented as:

[Top Level]

$$\max_{x_1} Z_1(x) = \max_{x_1} Z_1(x_1, x_2) = \max_{x_1} (z_{11}(x_1, x_2), z_{12}(x_1, x_2), ..., z_{1M_1}(x_1, x_2)) \tag{2.1}$$

[Lower Level]

$$\max_{x_2} Z_2(x) = \max_{x_2} Z_2(x_1, x_2) = \max_{x_2} (z_{21}(x_1, x_2), z_{22}(x_1, x_2), ..., z_{2M_2}(x_1, x_2)) \tag{2.2}$$

subject to

$$x \in S = \{x = (x_1, x_2) \in R^N \mid A_1 x_1 + A_2 x_2 \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} B, x \geq 0, B \in R^M\} \tag{2.3}$$

Where $z_{ij}(x_1, x_2) = \dfrac{p_{ij}x + \alpha_{ij}}{q_{ij}x + \beta_{ij}}$, (i = 1, 2; j = 1, 2, ..., M_i)

Here, S is the non empty convex constraint set, M_1 is the number of objective functions of TLDM, M_2 is the number of objective functions of LLDM, M is the total number of constraints of the problem. Also, A_i is the $M \times N_i$ matrix, i = 1, 2; p_{ij}, $q_{ij} \in R^N$; α_{ij}, β_{ij}, (i = 1, 2; j = 1, 2, ..., M_i) are scalars. We also assume that $q_{ij}x + \beta_{ij} > 0$, (i = 1, 2; j = 1, 2, ..., M_i) for all $x \in S$.

3. FORMULATION OF MEMBERSHIP FUNCTIONS FOR BOTH LEVEL DMs

Let $z_{ij}^{b} = \max_{x \in S} z_{ij}(x)$ and $z_{ij}^{w} = \min_{x \in S} z_{ij}(x)$, (i = 1, 2; j = 1, 2, ..., M_i) be individual best and worst solutions of the objective functions subject to the system constraints respectively. Then the fuzzy goals of both levels are appear as follows:

$$z_{ij}(x) \underset{\sim}{\geq} z_{ij}^{b}, \text{ (i = 1, 2; j = 1, 2,... } M_i) \tag{3.1}$$

Here, $\underset{\sim}{\geq}$ denotes the fuzziness of the aspiration levels.

Therefore, the linear fractional membership functions for the fuzzy objective goals can be formulated as:

$$\mu_{z_{ij}}(z_{ij}(x)) = \begin{cases} 0, & \text{if } z_{ij}(x) \leq z_{ij}^{w} \\ \dfrac{z_{ij}(x) - z_{ij}^{w}}{z_{ij}^{b} - z_{ij}^{w}}, & \text{if } z_{ij}^{w} \leq z_{ij}(x) \leq z_{ij}^{b} \\ 1, & \text{if } z_{ij}(x) \geq z_{ij}^{b} \end{cases}, \text{ i = 1, 2; j = 1, 2, ..., } M_i \tag{3.2}$$

Here, z_{ij}^{b} and z_{ij}^{w} (i = 1, 2; j = 1, 2, ..., M_i) represent the upper and lower tolerance limits respectively of the fuzzy objective goals.

4. LINEARIZATION OF LINEAR FRACTIONAL MEMBERSHIP FUNCTIONS FOR BOTH LEVEL DMs

Let $x^{ij0} = (x_1^{ij0}, x_2^{ij0}) = (x_{11}^{ij0}, x_{12}^{ij0}, ..., x_{1N_1}^{ij0}, x_{21}^{ij0}, x_{22}^{ij0}, ..., x_{2N_2}^{ij0})$, (i = 1, 2; j = 1, 2, ..., M_i) be the individual best solution of the linear fractional membership function $\mu_{z_{ij}}(z_{ij}(x))$, (i = 1, 2; j = 1, 2, ..., M_i) subject to the system constraints. We now transform the linear fractional membership functions into equivalent linear membership functions $\hat{\mu}_{z_{ij}}(z_{ij}(x))$ (i = 1, 2; j = 1, 2, ..., M_i) by using first order Taylor series as follows:

$$\mu_{z_{ij}}(z_{ij}(x)) \stackrel{\sim}{=} \mu_{z_{ij}}(z_{ij}(x^{ij0})) + (x_{11} - x_{11}^{ij0}) \left(\frac{\partial}{\partial x_{11}} \mu_{z_{ij}}(z_{ij}(x)) \right)_{\text{at } x = x^{ij0}} + (x_{12} - x_{12}^{ij0}) \left(\frac{\partial}{\partial x_{12}} \mu_{z_{ij}}(z_{ij}(x)) \right)_{\text{at } x = x^{ij0}} + ... + (x_{1N_1} - x_{1N_1}^{ij0}) \left(\frac{\partial}{\partial x_{1N_1}} \mu_{z_{ij}}(z_{ij}(x)) \right)_{\text{at } x = x^{ij0}} + (x_{21} - x_{21}^{ij0}) \left(\frac{\partial}{\partial x_{21}} \mu_{z_{ij}}(z_{ij}(x)) \right)_{\text{at } x = x^{ij0}} + (x_{22} - x_{22}^{ij0}) \left(\frac{\partial}{\partial x_{22}} \mu_{z_{ij}}(z_{ij}(x)) \right)_{\text{at } x = x^{ij0}} + ... + (x_{2N_2} - x_{2N_2}^{ij0}) \left(\frac{\partial}{\partial x_{2N_2}} \mu_{z_{ij}}(z_{ij}(x)) \right)_{\text{at } x = x^{ij0}} = \hat{\mu}_{z_{ij}}(z_{ij}(x)), \text{ (i = 1, 2; j = 1, 2, ..., } M_i). \tag{4.1}$$

5. SATISFACTORY SOLUTION FOR BOTH LEVEL DMs

In the decision making context, the individual best solutions of the objective functions of each level DM are generally different. So, each level DM desires to obtain his/her own satisfactory solution. We solve the following MODM models for obtaining satisfactory solutions for both level DMs as follows:

$$\max \ \hat{\mu}_{z_{ij}}(z_{ij}(x)), \text{ (i = 1, 2; j = 1, 2, ..., } M_i) \tag{5.1.1}$$

subject to

$$x \in S = \{x = (x_1, x_2) \in R^N \mid A_1 x_1 + A_2 x_2 \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} B, x \geq 0, B \in R^M \}.$$

Since the highest value of a membership goal is unity, so for the defined membership goal in (5.1.1), the flexible membership goal with aspiration level unity can be formulated as:

$$\hat{\mu}_{z_{ij}}(z_{ij}(x)) + d_{ij}^{-} - d_{ij}^{+} = 1, \text{ (i = 1, 2; j = 1, 2, ..., } M_i) \tag{5.1.2}$$

Here, d_{ij}^{-} ($\geq$ 0) and d_{ij}^{+} ($\geq$ 0), (i = 1, 2; j = 1, 2, ..., M_i) represent the negative deviational variable and positive deviational variable respectively.

Now following Pramanik and Dey [14], (5.1.2) can be presented as

$$\hat{\mu}_{z_{ij}}(z_{ij}(x)) + d_{ij}^{-} = 1, \ (i = 1, 2; j = 1, 2, ..., M_i) \tag{5.1.3}$$

Then we solve the MODM problem by utilizing the FGP approach [14] as follows:

$$\min \quad \lambda \tag{5.1.4}$$

subject to

$\hat{\mu}_{z_{ij}}(z_{ij}(x)) + d_{ij}^{-} = 1$, (i = 1, 2; j = 1, 2, ..., M_i)

$x \in S = \{x = (x_1, x_2) \in R^N \mid A_1 x_1 + A_2 x_2 \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} B, x \geq 0, B \in R^M \}$,

$\lambda \geq d_{ij}^{-}$, (i = 1, 2; j = 1, 2, ..., M_i), $d_{ij}^{-} \in [0, 1]$, i = 1, 2; j = 1, 2, ..., M_i.

Let, x^{i*} = ($x_{11}^{i*}, x_{12}^{i*}, ..., x_{1N_1}^{i*}, x_{21}^{i*}, x_{22}^{i*}, ..., x_{2N_2}^{i*}$) (i = 1, 2) be the satisfactory solution for both level DMs. Now we transforms the linear fractional membership function $\mu_{z_{ij}}(z_{ij}(x))$, (i = 1, 2; j = 1, 2, ..., M_i) into equivalent linear membership functions $\tilde{\mu}_{z_{ij}}(z_{ij}(x))$, (i = 1, 2; j = 1, 2, ..., M_i) by using first order Taylor series at the point x^{i*} = ($x_{11}^{i*}, x_{12}^{i*}, ..., x_{1N_1}^{i*}, x_{21}^{i*}, x_{22}^{i*}, ..., x_{2N_2}^{i*}$), (i = 1, 2).

6. PREFERENCE BOUNDS ON THE DECISION VARIABLES

Now the satisfactory solution of the TLDM and LLDM are revealed. However these two solutions are generally distinct, the direct compromise solution does not occur in the decision making situation. Therefore, both level DMs provide preference bounds on the decision variables under their control to get compromise optimal solution [15-16].

Let t_{1j}^{L} and t_{1j}^{R} , j = 1, 2, ..., N_1 be the preference lower and upper bounds on the decision variable x_{1j}^{1} (j = 1, 2, ..., N_1) respectively for TLDM such that

$$x_{1j}^{1*} - t_{1j}^{L} \leq x_{1j}^{1} \leq x_{1j}^{1*} + t_{1j}^{R}, \ j = 1, 2, ..., N_1 \tag{6.1}$$

Here, t_{1j}^{L} and t_{1j}^{R} (j = 1, 2, ..., N_1) are not generally equal.

Similarly, let t_{2j}^{L} and t_{2j}^{R} , j = 1, 2, ..., N_2 be the preference lower and upper bounds on the decision variable x_{2j}^{2} , (j = 1, 2, ..., N_2) respectively for LLDM such that

$$x_{2j}^{2*} - t_{2j}^{L} \leq x_{2j}^{2} \leq x_{2j}^{2*} + t_{2j}^{R}, \ j = 1, 2, ..., N_2 \tag{6.2}$$

Here, t_{2j}^{L} and t_{2j}^{R} , (j = 1, 2, ..., N_2) are not generally same.

7. FGP MODEL FOR SOLVING BLMOLFPP

Now in order to generate the solution for BLMOLFPP, we solve the following FGP model

$$\min \rho \tag{7.1}$$

subject to

$\tilde{\mu}_{z_{1j}}(z_{1j}(x)) + d_{1j}^{-} = 1$, (j = 1, 2, ..., M_1)

$\tilde{\mu}_{z_{2j}}(z_{2j}(x)) + d_{2j}^{-} = 1$, (j = 1, 2, ..., M_2)

$x \in S = \{x = (x_1, x_2) \in R^N \mid A_1 x_1 + A_2 x_2 \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} B,\ x \geq 0,\ B \in R^M\}$,

$\rho \geq d_{1j}^-$, j = 1, 2, ..., M_1, $\rho \geq d_{2j}^-$, j = 1, 2, ..., M_2,

$d_{1j}^- \in [0, 1]$, (j = 1, 2, ..., M_1), $d_{2j}^- \in [0, 1]$, (j = 1, 2, ..., M_2)

$x_{1j}^{1*} - t_{1j}^L \leq x_{1j}^1 \leq x_{1j}^{1*} + t_{1j}^R$, (j = 1, 2, ..., N_1)

$x_{2j}^{2*} - t_{2j}^L \leq x_{2j}^2 \leq x_{2j}^{2*} + t_{2j}^R$, (j = 1, 2, ..., N_2)

$x_{ij} \geq 0$, (i = 1, 2; j = 1, 2, ..., N_i).

8. NUMERICAL EXAMPLE FOR BLMOLFPP

Consider the following BLMOLFPP in order to demonstrate the effectiveness of the proposed approach

[Top Level]

$$\max_{x_1} \left(z_{11}(x) = \frac{5x_1 + 2x_2 + 3}{2x_1 - x_2 + 3},\ z_{12}(x) = \frac{2x_1 + 5x_2 + 3}{x_1 + 4x_2 + 4}\right)$$

[Lower Level]

$$\max_{x_2} \left(z_{21}(x) = \frac{x_1 + 3x_2}{x_1 + x_2 + 1},\ z_{22}(x) = \frac{-x_1 + 4x_2 + 3}{x_1 + 2x_2}\right)$$

subject to
$2x_1 + x_2 \leq 5$, $-x_1 + 3x_2 \leq 3$, $x_1 + x_2 \geq 1$,
$x_1 \geq 0$, $x_2 \geq 0$.

The individual best solution of the objective functions of TLDM subject to the constraints are z_{11}^b = 3.029 at (1.714, 1.571) and z_{12}^b = 1.231 at (2.5, 0) and the individual best solution of the objective functions of LLDM subject to the constraints are z_{21}^b = 1.5 at (1.191, 1.397) and z_{22}^b = 3.5 at (0, 1).

We also find the individual worst solution of the objective functions of TLDM subject to the constraints are z_{11}^w = 1.6 at (1, 0) and z_{12}^w = 1 at (0.251, 0.749) and the individual worst solution of the objective functions of LLDM subject to the constraints are z_{21}^w = 0.5 at (1, 0) and z_{22}^w = 0.2 at (2.5, 0).

Now the fuzzy objective goals of both levels appear as:

$z_{11}(x) \underset{\sim}{\geq} 3.029$, $z_{12}(x) \underset{\sim}{\geq} 1.231$, $z_{21}(x) \underset{\sim}{\geq} 1.5$, $z_{22}(x) \underset{\sim}{\geq} 3.5$

Then the linear fractional membership functions of the fuzzy objective goals are formulated as follows:

$$\mu_{z_{11}}(z_{11}(x)) = \begin{cases} 0, & \text{if } z_{11}(x) \leq 1.6 \\ \dfrac{z_{11}(x) - 1.6}{3.029 - 1.6}, & \text{if } 1.6 \leq z_{11}(x) \leq 3.029, \\ 1, & \text{if } z_{11}(x) \geq 3.029 \end{cases} \quad \mu_{z_{12}}(z_{12}(x)) = \begin{cases} 0, & \text{if } z_{12}(x) \leq 1 \\ \dfrac{z_{12}(x) - 1}{1.231 - 1}, & \text{if } 1 \leq z_{12}(x) \leq 1.231, \\ 1, & \text{if } z_{12}(x) \geq 1.231 \end{cases}$$

$$\mu_{z_{21}}(z_{21}(x)) = \begin{cases} 0, & \text{if } z_{21}(x) \leq 0.5 \\ \dfrac{z_{21}(x) - 0.5}{1.5 - 0.5}, & \text{if } 0.5 \leq z_{21}(x) \leq 1.5, \\ 1, & \text{if } z_{21}(x) \geq 1.5 \end{cases} \quad \mu_{z_{22}}(z_{22}(x)) = \begin{cases} 0, & \text{if } z_{22}(x) \leq 0.2 \\ \dfrac{z_{22}(x) - 0.2}{3.5 - 0.2}, & \text{if } 0.2 \leq z_{22}(x) \leq 3.5, \\ 1, & \text{if } z_{22}(x) \geq 3.5 \end{cases}$$

The linear fractional membership functions $\mu_{z_{1j}}(z_{1j}(x))$, (j = 1, 2) for TLDM subject to the constraints are maximal at the points (1.714, 1.571) and (2.5, 0) respectively and also the linear fractional membership functions $\mu_{z_{2j}}(z_{2j}(x))$, (j = 1, 2) for LLDM subject to the constraints are maximal at the points (1.191, 1.397) and (0, 1) respectively.

We now transform the linear fractional membership function $\mu_{z_{ij}}(z_{ij}(x))$, (i = 1, 2; j = 1, 2) into equivalent linear membership function $\hat{\mu}_{z_{ij}}(z_{ij}(x))$, (i = 1, 2; j = 1, 2) by using first order Taylor series at the maximal points. The transformed linear membership functions can be formulated as:

$$\mu_{z_{11}}(z_{11}(x)) \cong 1 + (x_1 - 1.714) \times (-0.152) + (x_2 - 1.571) \times (0.724) = \hat{\mu}_{z_{11}}(z_{11}(x)),$$

$$\mu_{z_{12}}(z_{12}(x)) \cong 1 + (x_1 - 2.5) \times (0.512) + (x_2 - 0) \times (0.051) = \hat{\mu}_{z_{12}}(z_{12}(x)),$$

$$\mu_{z_{21}}(z_{21}(x)) \cong 1 + (x_1 - 1.191) \times (-0.139) + (x_2 - 1.397) \times (0.418) = \hat{\mu}_{z_{21}}(z_{21}(x)),$$

$$\mu_{z_{22}}(z_{22}(x)) \cong 1 + (x_1 - 0) \times (-0.682) + (x_2 - 1) \times (-0.454) = \hat{\mu}_{z_{22}}(z_{22}(x))$$

We solve the MODM model to find the satisfactory solution of TLDM as follows:

min λ
subject to
$1 + (x_1 - 1.714) \times (-0.152) + (x_2 - 1.571) \times (0.724) + d_{11}^- = 1,$
$1 + (x_1 - 2.5) \times (0.512) + (x_2 - 0) \times (0.051) + d_{12}^- = 1,$
$2x_1 + x_2 \le 5,\ -x_1 + 3x_2 \le 3,\ x_1 + x_2 \ge 1,$
$\lambda \ge d_{11}^-,\ \lambda \ge d_{12}^-,\ d_{11}^- \in [0, 1],\ d_{12}^- \in [0, 1],$
$x_1 \ge 0,\ x_2 \ge 0.$

By solving the above model, we obtain the solution as follows:

$x_1^* = 1.875,\ x_2^* = 1.251.$

Now we again transform the original linear fractional membership function $\mu_{z_{1j}}(z_{1j}(x))$, (j = 1, 2) of TLDM into equivalent linear membership functions $\tilde{\mu}_{z_{1j}}(z_{1j}(x))$, (j = 1, 2) by using first order Taylor series at the satisfactory solution point (1.875, 1.251) as follows:

$$\mu_{z_{11}}(z_{11}(x)) \cong 0.774 + (x_1 - 1.875) \times (-0.052) + (x_2 - 1.251) \times (0.599) = \tilde{\mu}_{z_{11}}(z_{11}(x)),$$

$$\mu_{z_{12}}(z_{12}(x)) \cong 0.846 + (x_1 - 1.875) \times (0.320) + (x_2 - 1.251) \times (0.087) = \tilde{\mu}_{z12}(z_{12}(x))$$

Again, we solve the following MODM model to get the satisfactory solution of LLDM as:
min λ
subject to
$1 + (x_1 - 1.191) \times (-0.139) + (x_2 - 1.397) \times (0.418) + d_{21}^- = 1$
$1 + (x_1 - 0) \times (-0.682) + (x_2 - 1) \times (-0.454) + d_{22}^- = 1,$
$2x_1 + x_2 \le 5,\ -x_1 + 3x_2 \le 3,\ x_1 + x_2 \ge 1,$
$\lambda \ge d_{21}^-,\ \lambda \ge d_{22}^-,\ d_{21}^- \in [0, 1],\ d_{22}^- \in [0, 1],$
$x_1 \ge 0,\ x_2 \ge 0.$

By solving the above MODM model, we get the solution as follows:

$x_1^* = 0,\ x_2^* = 1.$

Now we again transform the linear fractional membership function $\mu_{z_{2j}}(z_{2j}(x))$, (j = 1, 2) of LLDM into equivalent linear membership functions $\tilde{\mu}_{z_{2j}}(z_{2j}(x))$, (j = 1, 2) by using first order Taylor series at the satisfactory solution point (0, 1) as follows:

$$\mu_{z_{21}}(z_{21}(x)) \cong 1 + (x_1 - 0) \times (-0.25) + (x_2 - 1) \times (0.75) = \tilde{\mu}_{z_{21}}(z_{21}(x)),$$

$$\mu_{z_{22}}(z_{22}(x)) \cong 1 + (x_1 - 0) \times (-0.682) + (x_2 - 1) \times (-0.454) = \tilde{\mu}_{z_{22}}(z_{22}(x))$$

Let, the preference upper and lower bounds assigned by the TLDM on the decision variable x_1 be $1.4 \le x_1 \le 1.8$.

Also, the preference upper and lower bounds provided by the LLDM on the decision variable x_2 be $1.09 \le x_2 \le 1.2$

Finally, the proposed FGP model for BLMOLFPP is presented as follows:

$$
\begin{aligned}
&\min \rho \\
&\text{subject to} \\
&0.774 + (x_1 - 1.875) \times (-0.052) + (x_2 - 1.251) \times (0.599) + d_{11}^{-} = 1, \\
&0.846 + (x_1 - 1.875) \times (0.320) + (x_2 - 1.251) \times (0.087) + d_{12}^{-} = 1, \\
&1 + (x_1 - 0) \times (-0.25) + (x_2 - 1) \times (0.75) + d_{21}^{-} = 1, \\
&1 + (x_1 - 0) \times (-0.682) + (x_2 - 1) \times (-0.454) + d_{22}^{-} = 1, \\
&2x_1 + x_2 \le 5,\ -x_1 + 3x_2 \le 3,\ x_1 + x_2 \ge 1, \\
&\rho \ge d_{11}^{-},\ \rho \ge d_{12}^{-}, \\
&\rho \ge d_{21}^{-},\ \rho \ge d_{22}^{-}, \\
&d_{11}^{-} \in [0, 1],\ d_{12}^{-} \in [0, 1], \\
&d_{21}^{-} \in [0, 1],\ d_{22}^{-} \in [0, 1], \\
&1.4 \le x_1 \le 1.8, \\
&1.09 \le x_2 \le 1.2, \\
&x_1 \ge 0,\ x_2 \ge 0.
\end{aligned}
$$

By solving the above FGP model, the compromise optimal solution of the BLMOLFPP is shown in Table 1.

Table 1: Compromise optimal solution of BLMOLFPP

Optimal solution	Decision variables x_1, x_2	Objective functions of TLDM z_{11}, z_{12}	Objective functions of LLDM z_{21}, z_{22}	Membership values $\mu_{z_{11}}(z_{11})$, $\mu_{z_{12}}(z_{12})$, $\mu_{z_{21}}(z_{21})$, $\mu_{z_{22}}(z_{22})$
$\rho = 0.99566$	$x_1 = 1.4$, $x_2 = 1.09$	$z_{11} = 2.586$, $z_{12} = 1.153$	$z_{21} = 1.338$, $z_{22} = 1.665$	$\mu_{z_{11}}(z_{11}) = 0.69$, $\mu_{z_{12}}(z_{12}) = 0.661$, $\mu_{z_{21}}(z_{21}) = 0.838$, $\mu_{z_{22}}(z_{22}) = 0.444$.

Note: All the solutions of the problem are obtained by using the software Lingo, version 6.0.

9. CONCLUSIONS

This paper has proposed a FGP approach for solving BLMOLFPP to produce compromise optimal solution. In proposed approach, firstly the linear fractional membership functions associated with linear fractional objective functions are transformed into equivalent linear membership functions by using first order Taylor series at the individual best solution point. MODM models are formulated in order to obtain satisfactory solution for each level DM. The linear fractional membership functions are transformed once again into equivalent linear membership functions at the satisfactory solution point. Each level DM assigns preference upper and lower bounds on the decision variables under his/her control for smooth functioning of the hierarchical organization. Finally, FGP model for BLMOLFPP is developed. Then the model is solved by minimizing the negative deviational variables in search of compromise optimal solution of BLMOLFPP. In addition, an illustrated numerical example is solved in order to clarify the proposed FGP approach. However, we hope that the proposed approach can be effective in dealing with the area of multi-level optimization problems such as decentralized BLMOLFPP, three level multi-objective linear fractional programming problems, BLMOLFPP with fuzzy parameters, etc.

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