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# A METHOD TO SOLVE THE DIOPHANTINE EQUATION $x^{2}-12 y^{2}+3=0$ 

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#### Abstract

Diophantine equation are fascinating to analyze and realize the study of Diophantine equation and to find their solutions would continue to puzzle both mathematicians and amateurs alike . we consider the equation (1) $x^{2}-12 y^{2}+3=0$ with $a, b \in N^{*}$ and $c \in z^{*}$

It is a particular case of Pell's equation: $x^{2}-D y^{2}=1$. Here, we show that: if the equation has an integer solution and $1.12=12$ is not a perfect square, then (1) has infinitude of integer solution, in this case we find a closed expression for $\left(x_{n}, y_{n}\right)$, the general positive integer solution ,by an original method. more, we generalize it for any Diophantine equation of second degree and with two unknowns.


METHOD TO SOLVE: Let $\left(x_{0}, y_{0}\right)$ and $\left(x_{1}, y_{1}\right)$ be the smallest positive integer solution for
(1) with $0 \leq x_{0}<x_{1}$ we construct recurrent sequences
(2) $x_{n+1}=\alpha x_{n}+\beta y_{n}$

$$
y_{n+1}=\gamma x_{n}+\delta y_{n}
$$

Putting the condition (2) verify (1), it results
(3) $\alpha \beta=12 \gamma \delta$
(4) $\alpha^{2}-12 \gamma^{2}=1$
(5) $\beta^{2}-12 \delta^{2}=-12$

Having unknown $\alpha, \beta, \gamma, \delta$

We pull out $\alpha^{2}, \beta^{2}$ from (4) and (5) respectively and replace them in (3) at the sequence it obtains
(6) $\delta^{2}-12 \gamma^{2}=1$

Subtract (6) from (4) and find
(7)

$$
\alpha= \pm \delta
$$

Replacing (7) in (3) it obtains

$$
\beta= \pm \frac{12}{1} \gamma
$$

(8) $\beta= \pm 12 \gamma$

Afterwards replacing (7) in (4) and (8) in (5) if find the same sequence
(9) $\alpha^{2}-12 \gamma^{2}=1$
because we work with positive solutions only we take
$x_{n+1}=\alpha_{0} x_{n}+12 \gamma_{0} y_{n}$
$y_{n+1}=\gamma_{0} x_{n}+\alpha_{0} y_{n}$
Where $\left(\alpha_{0}, \gamma_{0}\right)$ is the smallest positive integer solution of (9) such that $\alpha_{0} \gamma_{0} \neq 0$
So the $\alpha_{0}=7, \gamma_{0}=2$ and $7.2=14 \neq 0$
Let $\quad \mathrm{A}=\left[\begin{array}{cc}\alpha_{0} & 12 \gamma_{0} \\ \gamma_{0} & \alpha_{0}\end{array}\right] \in M_{2}(Z)$

$$
\mathrm{A}=\left[\begin{array}{cc}
7 & 24 \\
2 & 7
\end{array}\right] \in M_{2}(Z)
$$

Of course if ( $\mathrm{x}^{\prime}, \mathrm{y}^{\prime}$ ) is an integer solution for (1) then $\mathrm{A}\binom{x^{\prime}}{y^{\prime}}, A^{-1}\binom{x^{\prime}}{y^{\prime}}$ are another ones-where $A^{-1}$ is the inverse matrix of A hence, if (1) has an integer solution it has an infinite ones $A^{-1} \in M_{2}(Z)$

The general positive integer solutions of the equation (1)
$\left(x_{n}{ }^{\prime}, y_{n}{ }^{\prime}\right)=\left(\mathrm{Ix}_{\mathrm{n}} \mathrm{I}, \mathrm{Iy}_{\mathrm{n}} \mathrm{I}\right)$
$\left(\mathrm{GS}_{1}\right)$ with $\binom{x_{n}}{y_{n}}=A^{n}\left[\begin{array}{l}x_{0} \\ y_{0}\end{array}\right]$ for all $\mathrm{n} \in Z$
Where by conversion $A^{0}=I$ and $\mathrm{A}^{-\mathrm{n}}=\mathrm{A}^{-1} \ldots \ldots . . \mathrm{A}^{-1}$ of n time In problem it is better to write general solution (GS) as $\binom{x_{n}{ }^{\prime}}{y_{n}{ }^{\prime}}=A^{n}\binom{x_{0}}{y_{0}} \mathrm{n} \in \mathrm{N}$
$\left(\mathrm{GS}_{2}\right)$ and
$\binom{x_{n}{ }^{\prime \prime}}{y_{n}{ }^{\prime \prime}}=A^{n}\binom{x_{1}}{y_{1}} \mathrm{n} \in \mathrm{N}^{*}$
We proof by reduction ad absurdum $\left(\mathrm{GS}_{2}\right)$ is a general positive integer solution for (1) let ( $u$, v ) be a positive integer particular solution for (1) if $\exists k_{0} \in N:(\mathrm{u}, \mathrm{v})=A^{k}\left[\begin{array}{l}x_{0} \\ y_{0}\end{array}\right]$ or $\exists k_{1} \in N^{*}:(\mathrm{u}, \mathrm{v})=A^{k_{1}}\left[\begin{array}{l}x_{1} \\ y_{1}\end{array}\right]$ then (u, v) $\in\left(\mathrm{GS}_{2}\right)$
Contrary to this, we calculate
$\left(\mathrm{u}_{\mathrm{i}+1}, \mathrm{v}_{\mathrm{i}+1}\right)=A^{-1}\binom{u_{i}}{v_{i}}$ for $\mathrm{i}=0,1,2 \ldots \ldots \ldots \ldots \ldots$.
Where $u_{0}=u, v_{0}=v$ clearly $u_{i+1}<u_{i}$ for all I after a certain rank $x_{0}<u_{i 0}<x_{1}$ for all I it finds either $0<u_{i 0}<x_{0}$ but that is absured
$G S_{3}\binom{x_{n}}{y_{n}}=\left[\begin{array}{cc}7 & 24 \\ 2 & 7\end{array}\right]^{n}\left[\begin{array}{l}3 \\ \varepsilon\end{array}\right]$

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