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A METHOD TO SOLVE THE DIOPHANTINE EQUATION $x^2 - 12y^2 + 3 = 0$

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ABSTRACT

Diophantine equation are fascinating to analyze and realize the study of Diophantine equation and to find their solutions would continue to puzzle both mathematicians and amateurs alike . we consider the equation
(1) $x^2 - 12y^2 + 3 = 0$ *with* $a, b \in N^*$ *and* $c \in z^*$

It is a particular case of Pell's equation: $x^2 - Dy^2 = 1$. *Here, we show that: if the equation has an integer solution and 1.12=12 is not a perfect square, then (1) has infinitude of integer solution, in this case we find a closed expression for* $(x_{n,} y_n)$ *,the general positive integer solution ,by an original method. more, we generalize it for any Diophantine equation of second degree and with two unknowns.*

METHOD TO SOLVE: Let $(x_{0,} y_0)$ and $(x_{1,} y_1)$ be the smallest positive integer solution for (1) with $0 \le x_0 < x_1$ we construct recurrent sequences

(2) $x_{n+1} = \alpha x_n + \beta y_n$
$y_{n+1} = \gamma x_n + \delta y_n$

Putting the condition (2) verify (1), it results
(3) $\alpha\beta = 12\gamma\delta$
(4) $\alpha^2 - 12\gamma^2 = 1$
(5) $\beta^2 - 12\delta^2 = -12$

Having unknown $\alpha, \beta, \gamma, \delta$

We pull out α^2, β^2 from (4) and (5) respectively and replace them in (3) at the sequence it obtains

(6) $\delta^2 - 12\gamma^2 = 1$

Subtract (6) from (4) and find

(7) $\alpha = \pm\delta$

Replacing (7) in (3) it obtains

$$\beta = \pm\frac{12}{1}\gamma$$

(8) $\beta = \pm 12\gamma$

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Afterwards replacing (7) in (4) and (8) in (5) if find the same sequence

$$(9) \quad \alpha^2 - 12\gamma^2 = 1$$

because we work with positive solutions only we take

$$x_{n+1} = \alpha_0 x_n + 12\gamma_0 y_n$$
$$y_{n+1} = \gamma_0 x_n + \alpha_0 y_n$$

Where (α_0, γ_0) is the smallest positive integer solution of (9) such that $\alpha_0\gamma_0 \neq 0$

So the $\alpha_0 = 7, \gamma_0 = 2$ and 7.2 $= 14 \neq 0$

Let $A = \begin{bmatrix} \alpha_0 & 12\gamma_0 \\ \gamma_0 & \alpha_0 \end{bmatrix} \in M_2(Z)$

$A = \begin{bmatrix} 7 & 24 \\ 2 & 7 \end{bmatrix} \in M_2(Z)$

Of course if (x', y') is an integer solution for (1) then $A\begin{pmatrix} x' \\ y' \end{pmatrix}$, $A^{-1}\begin{pmatrix} x' \\ y' \end{pmatrix}$ are another ones-where A^{-1} is the inverse matrix of A hence, if (1) has an integer solution it has an infinite ones $A^{-1} \in M_2(Z)$

The general positive integer solutions of the equation (1)

$(x_n{}', y_n{}') = (|x_n|, |y_n|)$

(GS_1) with $\begin{pmatrix} x_n \\ y_n \end{pmatrix} = A^n \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ for all $n \in Z$

Where by conversion $A^0 = I$ and $A^{-n} = A^{-1}$………A^{-1} of n time In problem it is better to write general solution (GS) as $\begin{pmatrix} x_n{}' \\ y_n{}' \end{pmatrix} = A^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ $n \in N$

(GS_2) and

$\begin{pmatrix} x_n{}'' \\ y_n{}'' \end{pmatrix} = A^n \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $n \in N^*$

We proof by reduction ad absurdum (GS_2) is a general positive integer solution for (1) let (u, v) be a positive integer particular solution for (1) if $\exists\ k_0 \in N$: (u, v) = $A^k \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ or $\exists\ k_1 \in N^*$: (u, v) = $A^{k_1} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ then (u, v) $\in (GS_2)$

Contrary to this, we calculate

$(u_{i+1}, v_{i+1}) = A^{-1}\begin{pmatrix} u_i \\ v_i \end{pmatrix}$ for i=0, 1, 2……………..

Where $u_0 = u$, $v_0 = v$ clearly $u_{i+1} < u_i$ for all I after a certain rank $x_0 < u_{i0} < x_1$ for all I it finds either $0 < u_{i0} < x_0$ but that is absured

$GS_3 \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{bmatrix} 7 & 24 \\ 2 & 7 \end{bmatrix}^n \begin{bmatrix} 3 \\ \varepsilon \end{bmatrix}$,

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