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UNSTEADY MHD FLOW OF CONDUCTING WALTER'S VISCO-ELASTIC FLUID THROUGH POROUS MEDIUM IN A LONG UNIFORM RECTANGULAR CHANNEL

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ABSTRACT

The unsteady MHD flow of conducting Walter's visco-elastic fluid through porous medium in a long uniform straight channel of rectangular cross-section under the influence of time varying pressure gradient and uniform magneticfield applied perpendiculary to the flow of fluid has been studied. The exact solution for the velocity of fluid has been obtained by using integral transform technique. Some particular cases of pressure gradient have been discussed in detail. Also we have discussed the case when porous medium is withdrawn. Besides, the corresponding viscous flow problem has been derived as a limiting case when the relaxation time parameter tends to become zero. We have also derived the cases (i) when porous medium is withdrawn i.e. if $K\to\infty$ (ii) when magnetic field is withdrawn i.e. if $M\to\infty$ (iii) when magnetic field and porous medium both are withdrawn i.e. $M\to\infty$, $K\to\infty$ both.

INTRODUCTION

The flow of visco-elastic fluid between two parallel plates under the influence of uniform, exponential or periodic pressure gradient has been discussed by Pal and Sengupta(1986) and Das (1991). Roy, Sen and Lahiri (1990),Ghosh and Sengupta (1996), Das (2001, 2002), Kundu and Sengupta (2001), Kumar Singh and Sharma (2009,2011) and others have been studied the flow problems of visco-elastic fluid through channels of various cross-sections.Sengupta and Banerjee (2005) studied the unsteady MHD flow of visco-elastic Rivlin-Ericksen and Walter's fluid througha straight tube. Kumar, Gupta and Jain (2010) and Rajput, Mishra and Varshney (2011) have considered the flow problems concerned with the Walter's fluid In the present paper, the unsteady flow of Walter's visco-elastic fluid through porous medium in a long uniform rectangular channel under the influence of time dependent pressure gradient has been studied. Various particular cases have also been discussed in detail. We have also derived the cases. (i) when porous medium is withdrawn i.e. if $K\to\infty$ (ii) when magnetic field is withdrawn i.e. if $M\to\infty$ (iii) when magnetic field and porous medium both are withdrawn i.e. $M\to\infty$, $K\to\infty$ both.

FORMULATION OF THE PROBLEM

Here we are considering the motion of conducting visco-elastic Walter's fluid through porous medium inside a long uniform rectangular tube and under transverse uniform magnetic field.

The boundary walls of rectangular tube considered to be the planes $x=\pm a$, $y=\pm b$. The motion is under the influence of time dependent pressure gradient. Let the motion of the fluid along z-axis i.e. along the axis of rectangular channel.

According to the Navier-Stockes equation of motion for visco-elastic Walter's fluid through porous medium under the influence of uniform magnetic field applied perpendicularly to the flow of fluid is given by

$$\frac{\partial W}{\partial t} = -\frac{1}{\rho}.\frac{\partial p}{\partial z} + \nu\left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right) - \frac{\nu W}{K} - \frac{\sigma B_0^2}{\rho}W \tag{1}$$

where W(x, y, t) is the velocity of the fluid in z-direction, μ_1 the kinematical coefficient of visco-elasticity, ρ the density of the fluid, $\nu\left(=\frac{\mu}{\rho}\right)$the coefficient of viscosity, K the permeability of porous medium,σ the electrical conductivity and B_0 is the magnetic inductivity.

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Introducing the following non-dimensional quantities:

$$x^* = \frac{x}{a}, y^* = \frac{y}{a}, z^* = \frac{z}{a}, t^* = \frac{\nu}{a^2}t, p^* = \frac{a^2}{\rho\nu^2}p$$

$$W^* = \frac{a}{\nu}W,\ \mu_1^* = \frac{\nu}{a^2}\mu_1, K^* = \frac{1}{a^2}K$$

In equation (1), we get(after dropping stars)

$$\frac{\partial W}{\partial t} = -\frac{\partial p}{\partial z} + \left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) - \left(\frac{1}{K} + M^2\right)W \qquad (2)$$

where $M^2 = B_0 a\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number)

Here, the initial and boundary conditions are

$$W(x, y, 0) = 0 \qquad (3)$$

$$\left.\begin{array}{ll} W(1,y,t)=0, \quad 0\le y\le l, & t>0 \\ \frac{\partial W}{\partial x}=0, & x=0 \end{array}\right] \qquad (4)$$

$$\left.\begin{array}{ll} W(x,l,t)=0, \quad 0\le x\le 1 & t>0 \\ \frac{\partial W}{\partial x}=0, & y=0 \end{array}\right] \qquad (5)$$

where $l = \frac{b}{a}$

Solution of the problem

For solving eqn. (2), we use the following finite Fourier cosine transforms defined as:

$$W_c(i, y, t) = \int_0^1 W(x, y, t)\cos(p_i x)\,dx \qquad (6)$$

$$W_{\bar{c}}(x, j, t) = \int_0^l W_c(x, y, t)\cos(p_j y)\,dy, \qquad (7)$$

where

$p_i = (2i + 1)\frac{\pi}{2}, \qquad p_j = (2j + 1)\frac{\pi}{2l}$

Consequently, we have the following inverse of finite Fourier cosine transforms:

$$W(x, y, t) = 2\sum_{i=0}^{\infty} W_c(i, y, t)\cos(p_i x) \qquad (8)$$

$$W_c(i, y, t) = \frac{2}{l}\sum_{j=0}^{\infty} W_{\bar{c}}(i, j, t)\cos(p_j y) \qquad (9)$$

We use transforms (6) and (7) to initial condition (3) we get

$$W_{\bar{c}}(i, j, 0) = 0 \qquad (10)$$

Also taking finite Fourier cosine transform to boundary conditions, we have

$$\left.\begin{array}{l} W_c(i,l,t)=0 \\ \frac{\partial W_c}{\partial y}(i,0,t)=0 \end{array}\right] \qquad (11)$$

Applying transforms (6) and (7) to the equation of motion (2) and using initial and boundary conditions (10) and (11) we get

$$\zeta_1 \frac{\partial W_{\bar{c}}}{dt} + \xi_1 W_{\bar{c}} = \frac{(-1)^{i+j}F(t)}{p_i p_j} \qquad (12)$$

where

$$W_{\bar{c}} = \int_0^1 \int_0^l W\,(x, y, t) \cos(p_i x) \cos(p_j y)\, dx\, dy$$

$$\frac{\partial p}{\partial z} = -\,F(t)$$

$$\zeta_1 = 1 - \mu_1\left(p_i^2 + p_j^2\right)$$

and $\quad \xi_1 = \dfrac{1}{K} + M^2 + p_i^2 + p_j^2$

Then using the Laplace transform defined as:

$$\left.\begin{array}{l} \overline{W}_{\bar{c}}(s) = \int_0^\infty W_{\bar{c}} e^{-st}\, dt \\ \overline{F}(s) = \int_0^\infty F(t) e^{-st}\, dt \end{array}\right] \tag{13}$$

and by condition (11) on equation (12) we get

$$\zeta_1 \overline{W}_{\bar{c}} + \xi_1 \overline{W}_{\bar{c}} = \frac{(-1)^{i+j}\overline{F}(s)}{p_i p_j} \tag{14}$$

Now, by Laplace inversion formula and using convolution theorem, we get

$$\overline{W}_{\bar{c}} = \frac{(-1)^{i+j}}{p_i p_j \zeta_1} \int_0^t F\,(t-\lambda) e^{-(\xi_1/\zeta_1)\lambda} d\lambda \tag{15}$$

Thus, by Fourier cosine inversion formula as in equation (8) and (9), the expression of velocity becomes

$$W(x, y, t) = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j}}{p_i p_j \eta} \left\{\int_0^t F(t-\lambda)\, e^{-c_1 \lambda} d\lambda\right\} \cos(p_i x) \cos\left(p_j y\right)\right] \tag{16}$$

where $\qquad c_1 \dfrac{\xi_1}{\zeta_1}, \quad p_i = (2i+1)\dfrac{\pi}{2}, \quad p_j = (2j+1)\dfrac{\pi}{2l}$

We discuss the nature of velocity for following different particular cases:

Case I: Flow under constant pressure gradient:

Let,

$F(t) = F_0 \qquad$ (a constant)

From equation (16) the velocity will be

$$W = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j} F_0}{p_i p_j \xi_1} \left(1 - e^{-c_1 t}\right) \cos(p_i x) \cos\left(p_j y\right)\right] \tag{17}$$

Case II: Flow under impulsive pressure gradient:

Let

$F(t) = f_0 \delta(t)$

Where $\delta(t)$ is the unit impulse function defined as

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

So, from equation (16), we get the velocity

$$W = \frac{4}{l}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\frac{(-1)^{i+j}f_0}{p_i p_j \zeta_1} e^{-c_1 t}\cos(p_i x)\cos(p_j y)\right] \tag{18}$$

Case III: Flow under transient pressure gradient:

Let,

$F(t) = f_1 e^{-Nt}, (N > 0),$

Where f_1 is a constant.

So, from equation (16), the velocity takes form

$$W = \frac{4}{l}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\frac{(-1)^{i+j}f_1 e^{-Nt}}{p_i p_j(\xi_1 - N\zeta_1)}\left\{1 - e^{-(c_1 - N)t}\right\}\cos(p_i x)\cos(p_j y)\right] \tag{19}$$

Case IV: Flow under periodic pressure gradient:

Let,

$F(t) = \mathrm{Re}(F_1 e^{i\omega t}),$

Where F_1 is a constant,

From equation (16), the velocity becomes

$$W = \frac{4}{l}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\frac{(-1)^{i+j}F_1}{p_i p_j({\xi_1}^2 - \omega^2{\zeta_1}^2)}\left\{\omega\zeta_1\sin\omega t + \xi_1(\cos\omega t - 1)\right\}\right] x\cos(p_i x)(p_j y) \tag{20}$$

Case V: When the fluid is purely viscous:

For purely viscous fluid the kinematical co-efficient of visco-elasticity $\mu_1 = 0$ and we get,

$$W = \frac{4}{l}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\frac{(-1)^{i+j}}{p_i p_j}\int_0^t F(t-\theta)e^{-\xi_1\theta}d\theta\right]\cos(p_i x)(p_j y) \tag{21}$$

where

$\xi_1 = \frac{1}{K} + M^2 + p_i^2 + p_j^2, \qquad c_1 = \xi_1$

Case VI: When porous medium is withdrawn i.e. $K \to \infty$.

We get all results for Walter's fluid motion in the absence of porous medium and in the presence of magnetic field.

The values of ξ_1 and ζ_1 are given by,

$$\xi_1 = M^2 + p_i^2 + P_j^2 \text{ and } \zeta_1 = 1 - \mu_1(p_i^2 + p_j^2) \tag{22}$$

Case VII: when magnetic field is withdrawn i.e. $M \to 0$

We get all results for Walter's fluid motion in the absence of magnetic field and in the presence of porous medium. The values of ξ_1 and ζ_1 are given by

$\xi_1 = \frac{1}{K} + p_i^2 + P_j^2$ and $\zeta_1 = 1 - \mu_1(p_i^2 + p_j^2)$

Case VIII: When magnetic field and porous medium both are withdrawn i.e.

$M \to 0$, $K \to \infty$ both

We get all results for Walter's fluid motion in absence of magnetic field and porous medium. The values of ξ_1 and ζ_1 are given by

$\xi_1 = p_i^2 + P_j^2$ and $\zeta_1 = 1 - \mu_1(p_i^2 + p_j^2) = 1 - \mu_1\xi_1$

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