International Journal of Mathematical Archive-4(7), 2013, 274-281
IJMA Available online through www.ijma.info ISSN 2229 – 5046

A NEW RESULT ON FAITHFUL EXTENSION OF CONSTANT HEIGHT BETWEEN FINITE POSETS

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(Received on: 13-06-13; Revised & Accepted on: 09-07-13)

ABSTRACT

The problem of faithful extension with the condition of keeping constant height h, i.e. for h-inextensibility, seems more difficult and interesting than the brute extension of finite poset (partially ordered set). Two counter-examples of 2-inextensive posets can be used to formulate the faithful extension problem. A theorem in its general form of 2-inextensive posets $P=\left(\left|X\right|\ldots\right)$ with n- elements ($n\geq 2$) incomparable to the other ones is given to implement the presented counter examples.

Keywords: *Faithful extension, poset, extension, inextension, height, h-inextensive, h-inextensive.*

1. INTRODUCTION

This paper is concerned the problem of h-extension with height equal 2. The notions of *restriction, extension, isomorphism* and *mapping* between binary relations have been investigated.

Recall that a relation P is mapped into S or $P\leq S$ iff there exists an isomorphism from P onto a restriction of S. The problem of *faithful extension* (see [1]) is defined as follows:

Given two *posets* P and S such that P non $\leq$ S, then there exists a strict extension S^* of S (specially an extension with one additional element) which saves the non mapping P non$\leq$ S^*, (For more details, see [1]).

The faithful extension among infinite *chain* or *linearly ordered sets* is studied in [2, 3] together with a result on this faithful extension has been mentioned in [7]). Faithful extension between *bipartite graphs* and alias *bivalent tableaux* has been discussed in several works such as [4, 5, 6, 8]. In this present paper, the finite posets are considered only.

2. PRELIMINARY CONCEPTS

In what follows we give important and useful definitions in the representation of the problem of concern.

Definition 1 [1]: The poset P is an *extensive* iff for each poset S so that P non$\leq$ S (P not mapped into S) and there exists an extension S^* of S by adding one element to S such that P non$\leq$ S^*.

Definition 2 [1]: The poset P is *inextensive* by S iff P non$\leq$ S but P $\leq$ S^* where P is mapped into each poset S^*, (the extension of S by adding one element).

Definition 3 [9]: The *height* of finite poset P is the number of elements of maximal chains which are restriction of P.

Definition 4 [9]: The *faithful extension* between finite posets of constant height h is called *h-extension*.

Definition 5 [9]: The poset P is an *h-inextensive* for S iff P and S have a common height h and P non $\leq$ S but P is mapped into each poset S^* ($P\leq$ S^*) of the same height h where S^* is an extension of S by adding one element.

Definition 6 [9]: The poset P is an *h-extensive* iff for every S of height h so that P non $\leq$ S, and there exists an extension S^* of S by adding one element with height h and such that P non $\leq$ S^*.

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Definition 7 [1]: The elements a and b of a poset $(P, \leq)$ are called *comparable* if either $a \leq b$ or $b \leq a$. When a and b are elements of P such that neither $a \leq b$ nor $b \leq a$, a and b are called *incomparable* and denoted by $a \mid b$.

Proposition 1: If P is any finite poset and L is the set of elements in P which are incomparable with the other ones, then P\L be the restriction of P to the complement of L.

Proposition 2: Let P be a poset, then all the minimal elements of P are called the elements of rank 0.

Proposition 3: Let $|X|$ be the poset with two *minimal* elements and two elements of rank 0, then each minimal element being anterior to both elements of rank 1.

It should be noted that the notation ($|X|$ **.**) will be used to express the preceding poset with an additional incomparable element, while the notation ($|X|$ **. .**) is used for two incomparable elements.

3. REPRESENTATION OF THE PROBLEM

Many posets that are h-extensive and some posets that are 2-extensive have been studied in [9] together with one counter example of a poset of 2-inextensive has been given. In addition, it has been proved in [9] that P = ($|X|$ **. .**) is 2-inextensive by the poset S of cardinal 10.

The problem of extension is an open point of discussion for posets as well as for bipartite graphs, (see [1]). The problem addressed in the present paper as follows:

"*Does there exist a finite poset P of height h which is an h-inextensive for infinitely many finite poset S*" (considered up to isomorphism, equivalently having unlimited finite cardinals).

4. THE 2-INEXTENSIVE PROBLEM

The essential and important purpose of the work presented in this paper is to give a new counter-examples of a poset P ($|X|$ . . .) with L = 3 which is 2-inextensive by two posets S and T with the same cardinal 12, and poset P ($|X|$ . . . .) with L = 4 which is 2-inextensive by one posets S with cardinal 14, and poset P ($|X|$ . . . . .) with L = 5 which is 2-inextensive by four posets with the same cardinal 16.

Example 1: The *poset* P ($|X|$ **. . .**) is 2-inextensive.

Proof: First, we construct the poset S to be in form of four plates; each plate is isomorphic to $|X|$. Let a, b, c, d the four minimal elements (of rank 0) of these plates and a', b', c', d' the four maximal elements (of rank 1). The four minimals a, b, c, d and the four maximals a', b', c', d' can be expressed by a figure in which we put a' vertically to a, and similarly b' for b, c' for c, and d' for d. The first plate is formed by a, b, a', b'; the second plate by b, c, b', c'; the third by c, d, c', d'; and the last plate is formed by a, d, a', d'. Moreover we add two elements e and f of rank 0 to be anterior to b' only and incomparable to all other elements. In addition, e', f' two elements of rank 1 can be added to be posterior to d only and incomparable to all other elements.

It should be verified secondly that P non $\leq$ S, then we have the following:

(i) If the additional element s is from rank 1 and s > e, f then P $\leq$ S^* by s, e, f, b', c', e', f' which covers the sixteen cases: s > e, f ; then s>a, e, f; then s>b, e, f; then s>c, e, f; then s>d, e, f; then s>a, b, e, f; then s> a, c, e, f; then s>a, d, e, f; then s>b, c, e, f; then s> b, d, e, f; then s>c, d, e, f; then s>a, b, c, e, f; then s>a, c, d, e, f; then s>a, b, d, e, f; then s>b, c, d, e, f; then s>a, b, c, d, e, f.

(ii) If the additional element s is from rank 1 and s >a, e and $s \mid f$ then P $\leq$ S^* by s, e, a, b', c', e', f' which covers the eight cases: s> a, e; then s>a, b, e; then s>a, e, c; then s> a, e, d; then s> a, b, c, e; then s>a, b, d, e; then s>a, c, d, e; then s> a, b, c, d, e;

(iii) If s is from rank 1 and s>a, f and $s \mid e$ then $P \le S^*$ by s, f, a, b', c', e', f' which covers the eight cases: s>a, f; then s>a, b, f; then s> a, f, c; then s>a, f, d; then s>a, b, c, f; then s> a, b, d, f; then s>a, c, d, f; then s>a, b, c, d, f.

(iv) If s is from rank 1 and $s \mid b, c$ then $P \le S^*$ by b, c, b', c', s, e', f' which covers the seven cases: s>a; then s>e; then s>f; then s>d; then s>e, d; then s>f, d; then s>a, d.

(v) If s is from rank 1 and $s \mid a, b$ then $P \le S^*$ by the points a, b, a', b', s, e', f' which covers the six new cases: s>c; then s>c, d; then s>c, e; then s>c, f; then s>c, d, e; then s>c, d, f.

(vi) If s is from rank 1 and $s \mid a, d, e, f$ then $P \le S^*$ by a, d, a', d', s, e, f which covers the two cases: s>b; then s>b, c.

(vii) If s is from rank 1 and $s \mid c, d, e, f$ then $P \le S^*$ by c, d, c', d', s, e, f which covers the two cases: s incomparable to all elements; then s>a, b.

(viii) If s is from rank 1 and s>e, b and $s \mid f$ then $P \le S^*$ by s, e, b', b, e', f', d' which covers the four cases: s>e, b; then s>e, b, c; then s>e, b, d; then s>b, c, d, e.

(ix) If s is from rank 1 and s>f, b and $s \mid e$ then $P \le S^*$ by s, f, b', b, e', f', d' which covers the four cases: s>f, b; then s>f, b, c; then s>f, b, d; then s>b, c, d, f.

(x) If s is from rank 1 and s>a, c and $s \mid e, f$ then $P \le S^*$ by s, a, c, d', f, e', f' which covers the four cases: s>a, c; then s>a, b, c; then s>a, c, d; then s>a, b, c, d.

(xi) If s is from rank 1 and s>b, d and $s \mid a, e, f$ then $P \le S^*$ by s, b, d, c', a, e, f which covers the two cases: s>b, d; then s>b, c, d.

(xii) If s is from rank 1 and s>a, b, d and $s \mid e, f$ then $P \le S^*$ by s, a, b, a', f, e', f' which covers the sixty-four and last case: s>a, b, d.

Now if the additional element from rank 0, the proof is similar to the above one and we leave to the reader for completing the proof.

The same poset P ($|X| \ldots$) is also 2-inextensive by another poset T different from S but with same cardinal 12.

Example 2: The poset P ($|X| \ldots$) is *2*-inextnesive.

Proof: First, we construct the poset T to be in form of three plates; each plate is isomorphic to $|X|$. We call f, e, c, b are the four minimal elements (of rank 0) of these plates, and b', c', e', f' are the four maximal elements (of rank 1). These plates can be represented in figure in which we put b' vertically to f, and similarly c' for e; e' for c, and f' for b. The first plate is formed by f, e, b', c'; the second plate by c, e, c', e'; the third by c, b, e', f', then we connect between b' and b. Moreover, we add two elements a, d of rank 0 (a anterior to f' only and incomparable to all other elements). Let a', d' from rank 1, put a' posterior to f only (and incomparable to all other elements), and put also d' posterior to e only (and incomparable to all other elements).

It should be verified that P non $\le$ T, and we leave that to the reader.

(i) If the additional element s is from rank 0 and s < a', b' then $P \le T^*$ by the points a', b', s, f, d, c, a which covers the sixteen cases: s< a', b'; then s< a', b', c'; then s< a', b', d'; then s< a', b', e'; then s< a', b', f'; then s< a', b', c', d'; then s< a', b', c', e'; then s< a', b', c', f'; then s< a', b', d', e', f'; then s< a', b', d', f'; then s< a', b', e', f'; then s< a', b', c', d', e'; then s< a', b', d', e', f'; then s< a', b', c', d', f'; then s< a', b', c', e', f'; then s< a', b', c', d', e', f'.

(ii) If the additional element s is from rank 0 and s < c', d' then $P \leq T^*$ by the points c', d', s, c, d, b, a which covers the twelve cases: s< c', d'; then s< a', c', d'; then s< b', c', d'; then s< c', d', e'; then s< c', d', f'; then s< a', c', d', e'; then s< a', c', d', f'; then s< b', c', d', e'; then s< b', c', d', f'; then s< c', d', e', f'; then s< a', c', d', e', f'; then s< b', c', d', e', f'.

(iii) If the additional element s is from rank 0 and s < b', c' and $s \mid a', d'$ then $P \leq T^*$ by the points b', c', s, e, a', d, a which covers the four cases: s< b', c'; then s< b', c', e'; then s< b', c', f'; then s< b', c', e', f'.

(iv) If the additional element s is from rank 0 and s < c', e' and $s \mid a', d'$ then $P \leq T^*$ by the points c', e', s, e, a', d', a which covers the two cases: s< c', e'; then s< c', e', f'.

(v) If the additional element s is from rank 0 and s < a', c' then $P \leq T^*$ by the points a', c', s, f, d, b, a which covers the four cases: s< a', c'; then s< a', c', e'; then s< a', c', f'; then s< a', c', e', f'.

(vi) If the additional element s is from rank 0 and s < a' and $s \mid d', e', f'$ then $P \leq T^*$ by the points e', f', c, b, s, f, d' which covers the four cases: s incomparable to all elements; then s< a'; then s< b'; then s< c'.

(vii) If the additional element s is from rank 0 and s < d' and $s \mid b', c'$ then $P \leq T^*$ by the points b', c', f, e, s, d, a which covers the fourteen cases: s< d'; then s< e'; then s< f'; then s< a', d'; then s< a', e'; then s< a', f'; then s< d', e'; then s< d', f'; then s< e', f'; then s< a', d', e'; then s< a', d', f'; then s< a', e', f'; then s< d', e', f'; then s< a', d', e', f'

(viii) If the additional element s is from rank 0 and s < b', d' then $P \leq T^*$ by the points b', d', s, e, d, c, a which covers the four cases: s< b', d'; then s< b', d', e'; then s< b', d', f'; then s< b', d', e', f'.

(ix) If the additional element s is from rank 0 and s < b', f' and $s \mid a', d'$ then $P \leq T^*$ by the points b', f', s, b, d, d', a' which covers the two cases: s< b', f'; then s< b', e', f'.

(x) If the additional element s is from rank 0 and s < c', f' and $s \mid b', d'$ then $P \leq T^*$ by the points c', f', s, b, a, d, d' which covers the case: s< c', f'.

(xi) If the additional element s is from rank 0 and s < b', e' and $s \mid a', d'$ then $P \leq T^*$ by the points b', e', s, b, a, a', d' which covers the sixty-four and last case: s< b', e'.

Finally, if the additional element is from rank 1, then the proof can be caried out as before.

In what follows, we can put this conjecture about the 2-inextensive of the posets in example 1 and 2:

Conjecture: The poset P ($|X|$ **. . .**) is 2-inextensive by the only posets S and T of cardinal 12.

Example 3: The *poset* P ($|X|$ . . . .) is *2*-inextensive.

Proof: First, we construct the poset S to be in form of four plates; each plate is isomorphic to $|X|$. Let a, b, c, d the four minimal elements (of rank 0) of these plates and a', b', c', d' the four maximal elements (of rank 1). The four minimals a, b, c, d and the four maximals a', b', c', d' can be expressed by a figure in which we put a' vertically to a, and similarly b' for b, c' for c, and d' for d. The first plate is formed by a, b, a', b'; the second plate by b, c, b', c'; the third by c, d, c', d'; and the last plate is formed by a, d, a', d'. Moreover we add three elements e, f and g of rank 0 to be anterior to b' only and incomparable to all other elements. In addition e', f' and g' three elements of rank 1 can be added to be posterior to d only and incomparable to all other elements.

It should be verified secondly that P non $\leq$ S, then we have the following:

The proof is similar to that in example 1.

In what follows, we can put this conjecture about the 2-inextensive of this posets in example 3:

Conjecture: The poset P ($|X|$ . . . .) is 2-inextensive by the only posets S of cardinal 14.

Example 4: The *poset* P ($|X|$ . . . . .) is *2*-inextensive.

For this poset P ($|X|$ . . . . .) with L = 5 is 2-inextensive by four different posets with same cardinal 16. (The proof is very long for more details contact the author).

In what follows, we can put this conjecture about the 2-inextensive of this posets in example 4:

Conjecture: The poset P ($|X|$ . . . . .) is 2-inextensive by the only four posets of cardinal 16.

5. GENERAL FORM OF 2-INEXTENSIVE POSETS

At the end of this paper we give the general case of poset P $(|X|\ .\ .\ .\ .\ .)$ with L = n ($n \geq 2$) which is 2-inextensive and this result can be support the problem of Fraisse [1] of h-inextensivity of posets.

Theorem: The posets P ($|X|$ **…**) with L = n ($n \geq 2$) are 2-inextensive

Proof: Put P = **(**$|X|$ **. . .   … .)** and construct the poset U to be in the form of four plates, each plate is isomorphic to $|X|$; we call a, b, c, d are the four minimal elements of rank 0 of these plates, and a', b', c', d' are the four maximal of rank 1. In the figure of the plates, we put a' vertically to a, and similarly for b' and b, c' and c, d' and d. The first plate is formed by a, b, a', b'; the second plate by b, c, b', c'; the third by c, d, c', d'; and the last one by a, d, a', d'. Moreover we add (n-1) elements from rank 0, and (n-1) elements from rank 1. Let a_1, a_2, …..,a_{n-1} from rank 0, anterior to b' only and incomparable to all other elements. In addition, let $a'_1, a'_2, a'_3, ..., a'_{n-1}$ from rank 1, posterior to d only and incomparable to all other elements.

It should be verified that P non $\leq$ U.

(i) If $P \leq U$ by the points a', b', a, b, and the (L-1) elements of $P \leq a'_1, a'_2, a'_3, ..., a'_{n-1}$, but the nth element of L non $\leq$ in any element of U because each element of U is related with at least one of a', b', a, b or with the a_1, a_2, ..,a_{n-1} or with $a'_1, a'_2, a'_3, ..., a'_{n-1}$ of U.

(ii) If P $\leq$ U by the points b', c', b, c and the (L-1) elements of $P \leq a'_1, a'_2, a'_3, ..., a'_{n-1}$, but the nth element of L non $\leq$ in any element of U because each element of U is related with at least one of b', c', b, c or with the a_1, a_2, ...,a_{n-1} or with $a'_1, a'_2, a'_3, ..., a'_{n-1}$ of U.

(iii) Similarly if P $\leq$ U by the points c', d', c, d, and the (L-1) elements of P$\leq$ a_1, a_2,.,a_{n-1}, but the nth element of L but the nth element of L non $\leq$ in any element of U because each element of U is related with at least one of c', d', c, d or with the a_1, a_2, ..,$\mathbf{a_{n-1}}$ or with $a'_1, a'_2, a'_3, ..., a'_{n-1}$ of U. Or by the points a', d', a, d, and the (L-1) elements of P$\leq$ a_1, a_2, ..,a_{n-1}, but the nth element of L non $\leq$ in any element of U because each element of U is related with at least one of a', d', a, d or with the a_1, a_2, …..,a_{n-1} or with $a'_1, a'_2, a'_3, ..., a'_{n-1}$ of U. Thus P non $\leq$ U.

The proof of $P \leq U^*$ is completed by adding all the possible rows.

1.1. If the additional element s from rank 0, and s< at least two elements of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (for example let s< a'_1, a'_2) then $P \leq U^*$ by the points a'_1, a'_2, s, d, c, a_1, a_2,…….,a_{n-1} which covers the cases: s$\mid$ a', or b', or c', or d' and s < a', or b', or c', or d'.

1.2. Similarly, let s < any two elements of $a'_1, a'_2, a'_3, ..., a'_{n-1}$, the proof will be analogous to 1.1.

2.1. If the additional element s from rank 0, and s< one element only of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (let s< a'_1) and s< c', d', then $P \leq U^*$ by the points c', d', c, s, a'_2, a_1, a_2, a_3,...., a_{n-1} which covers the cases: s< c', d'; s $\mid$ a', or b'; s< a', or b'; s< a', b', c'; s< a', b', d'; s< a', b', c', d'.

2.3. If the additional element s from rank 0, and s< one element only of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (let s< a'_1) and s< a', b', then $P \leq U^*$ by the points a', a'_1, d, s, c, a_1, a_2, a_3,...., a_{n-1} which covers the cases: s< a', b'; s $\mid$ c', or d'; s< d'; s< a', b', d'.

2.4. If the additional element s from rank 0, and s< one element only of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (let s< a'_1) and s< a', c', then $P \leq U^*$ by the points a', c', b, s, a'_2, a_1, a_2, a_3,...., a_{n-1} which covers the cases: s< a', c'; s $\mid$ b', or d'; s< b'; s< d'; s< b', d'.

2.5. If the additional element s from rank 0, and s< one element only of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (let s< a'_1) and s< a', d', then $P \leq U^*$ by the points a', d', d, s, a'_2, a_1, a_2, a_3,...., a_{n-1} which covers the cases: s< a', d'; s $\mid$ b', or c'; s< b'; s< c'; s< b', c'.

2.6. If the additional element s from rank 0, and s< one element only of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (let s< a'_1) and s< b', c', and s $\mid$ a', then $P \leq U^*$ by the points c', a'_1, d, s, a', a_1, a_2, a_3,...., a_{n-1} which covers the cases: s< b', c'; s $\mid$ d'; s< d'; s< b', c', d'.

2.7. If the additional element s from rank 0, and s< one element only of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ (let s< a'_1) and s< b', d', and s $\mid$ a', then $P \leq U^*$ by the points d', d', a'_1, d, s, a', a_1, a_2, a_3,...., a_{n-1} which covers the cases: s< b', d'; s $\mid$ c'; s< c'

2.8. The proof is similar to precedent if s< a'_2; s< a'_3; ….; s< a'_{n-1}.

3.1. If the additional element s from rank 0, and s incomprable to any element of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers four cases:

3.1.1. If s< a', then $P \leq U^*$ by the points b', c', b, c, s, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< a'; s $\mid$ a'; s $\mid$ d'; s< d'; s $\mid$ a', d'; s< a', d'.

3.1.2. If s< b', then $P \leq U^*$ by the points c', d', c, d, s, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< b'; s$\mid$ b'; s $\mid$ a', b'.

3.1.3. If s< c', then P$\le$ U^* by the points a', b', a, b, s, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< c'; s $|$ c'; s $|$ c', d'; s< c', d'.

3.1.4. If s< d', then P$\le$ U^* by the points a', b', a, b, s, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s $|$ a', b', c', d'.

3.2. If the additional element s from rank 0, and s incomparable to any element of $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers six cases:

3.2.1. If s< a', b', and s $|$ c' then P$\le$ U^* by the points a', b', a, s, c', $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< a', b'; s $|$ c'; s $|$ d'; s< d'; s $|$ c', d'; s< a', b', d'.

3.2.2. If s< a', c' then P$\le$ U^* by the points a', c', b, s, a_1, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< a', c'; s $|$ b'; s<b'; s< d'; s $|$ b', d'; s< a', b', c'; s< a', b', c', d'.

3.2.3. If s< a', d', then P$\le$ U^* by the points a', d', d, s, a_1, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< a', d'; s< a', b', d'; s< a', c', d'.

3.2.4. If s< b', c', s $|$ a' then P$\le$ U^* by the points b', c', c, s, a, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers all the cases: s< b', c'; s $|$ a'; s< d'; s $|$ d'; s $|$ a', d'; s<b', c', d'.

3.2.5. If s< b', d', s $|$ a' then P$\le$ U^* by the points b', c', c, s, a, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers the case: s< b', d'.

3.2.6. If s< c', d', then P$\le$ U^* by the points c', d', d, s, a_1, $a'_1, a'_2, a'_3, ..., a'_{n-1}$ which covers the case: s< c', d'.

Finally**,** if the additional element s from rank 1; prove analogous to precedent, so the theorem is proved.

Finally, we can conclude our working by writing two important conjectures that also support the problem of Fraisse [1] of h-inextensivity of posets.

Conjecture 1: The poset $|X|$ with a number even of elements L (is the number of incomparable elements with other ones) is 2-inextensive by the only posets with cardinal $2n+6$, for $n \ge 2$.

Conjecture 2: The poset $|X|$ with a number odd of elements L (is the number of incomparable elements with other ones) is 2-inextensive by many different posets with same cardinal $2n+6$, for $n \ge 2$.

REFERENCES

[1] R.FRAISSE, *"Theory of Relation"*, Studies in Logic and the Foundation of Mathematics Number, N° 145, North Holland, Elsevier Science Publisher (1986).

[2] J. HAGENDORF, "Extension immediate et respectueuses de chaines et de relation", *C. R. Acad. Sci. Paris*, 275, serie A, p. 949-950 (1972).

[3] J. HAGENDORF, "Extension de chaines", *Z. Math. Logik Grundlag. Math*., 25, p. 423-444 (1979).

[4] G. LOPEZ, "Problem d'extension respectueuse", *C. R. Acad. Sci. Paris*, 277, serie A, p. 567-569 (1973).

[5] G. LOPEZ, "La p-extensivite d'un tableau bivalent pour $p \geq 5$", *C. R. Acad. Sci. Paris*, 284, serie A, p.1245-1248 (1977).

[6] C. RAUZY, "Sur l'extensivite des tableau bivalents a deux colonnes", *C. R. Acad. Sci. Paris*, 303, serie I, p. 721-724 (1986).

[7] J. ROSENSTEIN, "*Linear ordering*", Academic Press, (1982).

[8] WALIED H. SHARIF, "On the *p*-inextensivity of bivalent tables with two columns". Far East J. Math. Sci (FJMS), Vol. 3, No. 6, (2001).

[9] R. FRAISSE, and WALIED H. SHARIF, "L' extension respectueuse a hauteur constante entre posets finis". *C. R. Acad. Sci. Paris*, t. 316, serie I, p. 637-642,(1993).

Source of support: Nil, Conflict of interest: None Declared