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ON SUPER EDGE BIMAGIC TOTAL LABELING OF P_m X C_n AND ITS RELATED GRAPHS

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ABSTRACT

A Graph G(p, q) is said to have edge bimagic total labeling if there exists a bijection f: $V(G)\cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $e = (u,v) \in E(G)$, $f(u) + f(v) + f(e) = k_1$ or k_2, where k_1 and k_2 are two constants. Moreover, G is said to have super edge bimagic total labeling if $f(V(G)) = \{1, 2, \ldots, p\}$. In this paper we prove that the super edge bimagic total labeling for generalized prism P_m x C_n, the Mongolian Ger M(n,m), the generalized web W(m,n), the generalized web without centre $W_0(m,n)$ and its related graph.

Key Words: *edge bimagic total labeling, generalized prism, generalized web, super edge bimagic labeling and total labeling.*

AMS Subject Classification: *05C78.*

1. INTRODUCTION

As a standard notation, assume that G = G(V, E) is a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers). Edge bimagic labelings of graphs were introduced and studied by Baskar Babujee J. [1, 2]. Let x be any real number. Then $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. Terms and terminology as in Harary [4].

Definition 1.1 [1]: A graph G(p,q) is said to have edge bimagic total labeling with two common edge counts k_1 and k_2 if there exists a bijection $f : V \cup E \rightarrow \{1, 2, \ldots, p + q\}$ such that for each $e = (u, v)\in E$, $f(u)+f(v)+f(e) = k_1$ or k_2. A total edge bimagic graph is called super edge bimagic if f maps V onto $\{1, 2, \ldots, p\}$.

That is, a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ is said to be an edge bimagic labeling with two magic constants k_1 and k_2, if there exists an induced edge map $f^*: E(G) \rightarrow \{ k_1, k_2\}$ such that for every $e = (u, v) \in E$, $f^*(e) = f(u)+f(v)+f(e) = k_1$ or k_2.

Definition1.2: The **generalized prism P_m x C_n** is the graph with the vertex set $V(P_m \text{ x } C_n) = \{v_{i, j} : 1\le i \le m, 1\le j \le n\}$ and the edge set $E(P_m \text{ x } C_n) = \{v_{i, j}\, v_{i, j+1}: 1\le i \le m, 1\le j \le n\}\cup\{ v_{i, j}\, v_{i+1, j} : 1 \le i \le m-1, 1\le j \le n\}$,where j is taken modulo n (replacing 0 by n).

Definition1.3: The **generalized web W(m, n)** is the graph with vertex set $V(W(m, n)) =\{v_{i, j}:1\le i \le m+1, 1\le j \le n\}\cup\{v_0\}$ and the edge set $E(W(m, n)) = \{v_{i, j}\, v_{i, j+1}: 1\le i \le m, 1\le j \le n\} \cup\{ v_{i, j}\, v_{i+1, j} : 1\le i \le m, 1\le j \le n\} \cup\{v_0 v_{1,j}: 1\le j \le n\}$, where j is taken modulo n (replacing 0 by n).The generalized web without centre is denoted by $W_0(m, n)$.

Definition1.4: For any integer n > 2 and h > 1, the **Mongolian Ger is the graph M(n, h)** with the vertex set $V(M(n,h)) = \{v_0, v_{i, j} :1\le i \le h, 1\le j \le n\}$ and the edge set $E(M(n, h)) = \{v_0\, v_{1,j} : 1 \le j \le n\}\cup\{v_{i, j}\, v_{i, j+1}: 1\le i \le h, 1\le j \le n\}\cup\{v_{i, j}\, v_{i+1, j} : 1 \le i \le h-1, 1\le j \le n\}$,where j is taken modulo n (replacing 0 by n).

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In this paper we study the super edge bimagic total labeling for P_m x C_n, and some of its related graphs viz., the Mongolian Ger M(n, m), the generalized web W(m, n), the generalized web without centre W_0(m, n), etc.

2. MAIN RESULT

Theorem 2.1: For all m ≥ 2 and odd n ≥ 3, the generalized prism P_m x C_n has super edge bimagic total labeling.

Proof: Let V be the vertex set and E be the edge set of the graph P_m x C_n. Then |V| = mn and |E| =(2m−1)n. Denote the vertices of the innermost cycle of P_m x C_n as $v_{1,1}, v_{1,2}, \ldots, v_{1,n}$ and the vertices adjacent to $v_{1,1}, v_{1,2}, \ldots, v_{1,n}$ on the second cycle as $v_{2,1}, v_{2,2}, \ldots, v_{2,n}$ respectively. Next denote the vertices adjacent to $v_{2,1}, v_{2,2}, \ldots, v_{2,n}$ on the third cycle as $v_{3,1}, v_{3,2}, \ldots, v_{3,n}$ respectively and so on. Thus the vertices adjacent to $v_{m-1,1}, v_{m-1,2}, \ldots, v_{m-1,n}$ on the m th cycle as $v_{m,1}, v_{m,2}, \ldots, v_{m,n}$ respectively.

For 1 ≤ i ≤ m, denote

$$k(i) = \left\lfloor \frac{i-1}{n} \right\rfloor n^2 \tag{1}$$

$$\delta(i) = \begin{cases} a & ; \quad \text{if } i \equiv a \bmod n \ \& \ a > 0 \\ n & ; \quad \text{if } i \equiv 0 \bmod n \end{cases} \tag{2}$$

We define the bijection f: V ∪ E → {1, 2, …, (3m-1)n} as follows:

Initially we assign the label to the vertices of P_m x C_n.

For 1 ≤ i ≤ m , 1≤ j ≤ n

$$f(v_{i,j}) = \begin{cases} k(i) + (\delta(i)-1)(n+1) + j & ; \quad 1 \le j \le n - \delta(i) + 1 \\ k(i) + (\delta(i)-1)(n+1) - n + j & ; \quad n - \delta(i) + 2 \le j \le n \end{cases} \tag{3}$$

Let e = $(v_{i,j}, v_{s,t})$ be any edge in P_m x C_n and let $f'(e) = f(v_{i,j}) + f(v_{s,t})$ (4)

We denote the edges of P_m x C_n as follows:

For $1 \le j \le \frac{n-1}{2}$, denote the edge by $e_j^{(1)}$, if the sum of the labels of its end vertices is equal to 2j+1.

For 0 ≤ j ≤ (m-1)n, denote the edge by $e_j^{(2)}$, if the sum of the labels of its end vertices is equal to n+1+j.

For 0 ≤ j ≤ (m-1)n, denote the edge by $e_j^{(3)}$, if the sum of the labels of its end vertices is equal to mn + 2 + j.

For $1 \le j \le \frac{n-3}{2}$, denote the edge by $e_j^{(4)}$, if the sum of the labels of its end vertices is equal to 2mn – n + 2j+2.

Hence from equation (4),we have

$$\begin{aligned} f'(e_j^{(1)}) &= 2j + 1, & 1 \le j \le \frac{n-1}{2} \\ f'(e_j^{(2)}) &= n + 1 + j, & 0 \le j \le (m-1)n \\ f'(e_j^{(3)}) &= mn + 2 + j, & 0 \le j \le (m-1)n \\ f'(e_j^{(4)}) &= 2mn - n + 2(j+1), & 1 \le j \le \frac{n-3}{2}. \end{aligned} \tag{5}$$

Now, let us label the edges of P_m x C_n as

$$\begin{aligned} f(e_j^{(1)}) &= 2mn + 1 - 2j, & 1 \le j \le \frac{n-1}{2} \\ f(e_j^{(2)}) &= 2mn - n + 1 - j, & 0 \le j \le (m-1)n \\ f(e_j^{(3)}) &= 3mn - n - j, & 0 \le j \le (m-1)n \\ f(e_j^{(4)}) &= 2(mn - j), & 1 \le j \le \frac{n-3}{2}. \end{aligned}$$

The induced map f^* on E defined by $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)})$ for any edge $e_j^{(k)} \in E$ satisfies the conditions:

$$\begin{aligned} f^*(e_j^{(1)}) &= f'(e_j^{(1)}) + f(e_j^{(1)}) \\ &= (2j+1) + (2mn+1-2j) \\ &= 2mn+2, \qquad 1 \le j \le \frac{n-1}{2} \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(2)}) &= f'(e_j^{(2)}) + f(e_j^{(2)}) \\ &= (n+1+j) + (2mn-n+1-j) \\ &= 2mn+2, \qquad 0 \le j \le (m-1)n \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(3)}) &= f'(e_j^{(3)}) + f(e_j^{(3)}) \\ &= (mn+2+j) + (3mn-n-j) \\ &= 4mn-n+2, \qquad 0 \le j \le (m-1)n \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(4)}) &= f'(e_j^{(4)}) + f(e_j^{(4)}) \\ &= 2mn-n+2(j+1) + 2(mn-j) \\ &= 4mn-n+2, \qquad 1 \le j \le \frac{n-3}{2}. \end{aligned}$$

Clearly, it is observed that for each edge $e_j^{(k)} \in E$, $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)}) = 2mn+2$ or $4mn-n+2$. Since there exists two common edge counts $k_1 = 2mn+2$ and $k_2 = 4mn-n+2$, the graph P_m x C_n has edge-bimagic total labeling. Moreover $f(V) = \{1, 2, \ldots, mn\}$, f is a super labeling. Hence P_m x C_n has super edge bimagic total labeling with magic counts $2mn+2$ and $4mn-n+2$, where n is odd and $m \ge 2$.

Theorem 2.2: For all $m \ge 2$ and odd $n \ge 3$, the Mongolian Ger M(n,m) has super edge bimagic total labeling.

Proof: Let $G_1(V_1,E_1) \cong$ Mongolian Ger M(n, m).Then the vertex set $V_1 = V \cup \{v_0\}$ and the edge set $E_1 = E \cup E_0$, where V and E are as defined in the proof of the Theorem 2.1 and $E_0 = \{v_0 v_{1,j} / 1 \le j \le n\}$. Then $|V_1| = mn+1$ and $|E_1| = 2mn$. We define the bijection $f : V_1 \cup E_1 \to \{1, 2, \ldots, 3mn+1\}$ as follows:

Initially we assign labels to the vertices of G_1.

$f(v_0) = mn+1$ and f on the vertices in V is as in Theorem 2.1. Then $f(V_1) = \{1,2, \ldots, mn+1\}$.

Let $e = (v_{i,j}, v_{s,t})$ be any edge in G_1 and let $f'(e) = f(v_{i,j}) + f(v_{s,t})$ (6)

We denote the edges of G_1 as follows:

For $1 \le j \le n$, denote each edge $(v_0, v_{1,j}) \in E_0$ by $e_j^{(0)}$ and the edges in E as denoted in the proof of the Theorem 2.1.

Then from equation (6), we have

$$f'(e_j^{(0)}) = mn+1+j, \ 1 \le j \le n \text{ and}$$

for each edge $e_j^{(k)} \in E$, $f'(e_j^{(k)})$ is as defined in equation (5) of Theorem 2.1.

Now we assign labels to the edges of G_1.

$$\begin{aligned} f(e_j^{(0)}) &= mn+n+2-j, && 1 \le j \le n \\ f(e_j^{(1)}) &= 2mn+n+2-2j, && 1 \le j \le \frac{n-1}{2} \\ f(e_j^{(2)}) &= 2mn+2-j, && 0 \le j \le (m-1)n \\ f(e_j^{(3)}) &= 3mn+1-j, && 0 \le j \le (m-1)n \\ f(e_j^{(4)}) &= 2mn+n+1-2j, && 1 \le j \le \frac{n-3}{2}. \end{aligned}$$

Thus $f(E_1) = \{mn+2, mn+3, \ldots, 3mn+1\}$.

The induced edge map f^* on E_1 defined by $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)})$ for every edge $e_j^{(k)} \in E_1$ satisfies the conditions:

$$f^*(e_j^{(0)}) = f'(e_j^{(0)}) + f(e_j^{(0)}) = 2mn + n + 3, \qquad 1 \le j \le n$$

$$f^*(e_j^{(1)}) = f'(e_j^{(1)}) + f(e_j^{(1)}) = 2mn + n + 3, \qquad 1 \le j \le \frac{n-1}{2}$$

$$f^*(e_j^{(2)}) = f'(e_j^{(2)}) + f(e_j^{(2)}) = 2mn + n + 3, \qquad 0 \le j \le (m-1)n$$

$$f^*(e_j^{(3)}) = f'(e_j^{(3)}) + f(e_j^{(3)}) = 4mn + 3, \qquad 0 \le j \le (m-1)n$$

$$f^*(e_j^{(4)}) = f'(e_j^{(4)}) + f(e_j^{(4)}) = 4mn + 3, \qquad 1 \le j \le \frac{n-3}{2}.$$

Clearly it is observed that for each edge $e_j^{(k)} \in E_1$, $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)}) = 2mn + n + 3$ or $4mn+3$. Since there exists two common edge counts $k_1 = 2mn + n + 3$ and $k_2 = 4mn+3$, the graph G_1 has super edge bimagic total labeling. Hence for all $m \ge 2$ and odd $n \ge 3$, the Mongolian Ger M(n,m) has super edge bimagic total labeling.

Example 2.1: In Figure 2.1, we give a super edge bimagic total labeling for the Mongolian Ger M (5, 3).

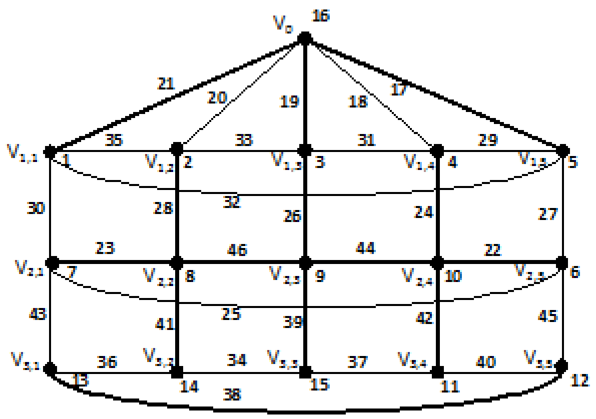

Fig. 2.1 : super edge bimagic total labeling of the Mongolian Ger M(5,3).

In the next section, we prove that for all m ≥ 2 and odd n ≥ 3, the generalized web without centre, $W_0(m, n)$ is super edge bimagic. The graph $W_0(m, n)$ is the same as the graph obtained from P_m x C_n by attaching a pendent vertex at each vertex of the outermost cycle of P_m x C_n. Denote the pendent vertices adjacent to $v_{m,1}, v_{m,2}, \ldots, v_{m,n}$ of the m th cycle of P_m x C_n as $v_{m+1,1}, v_{m+1,2}, \ldots, v_{m+1,n}$ respectively.

Theorem 2.3: For all $m \ge 2$ and odd $n \ge 3$, the graph $W_0(m, n)$ has super edge-bimagic total labeling.

Proof: Let V′ be the edge set and E′ be the edge set of $W_0(m, n)$. Then $V' = V \cup \{v_{m+1,j} / 1 \le j \le n\}$ and $E' = E \cup \{v_{m,j}\, v_{m+1,j} / 1 \le j \le n\}$, where V and E are as defined in Theorem 2.1. Then $|V'| = (m+1)n$ and $|E'| = 2mn$.

For $1 \le i \le m+1$, denote k(i) and δ(i) as defined in equations (1) and (2) of Theorem 2.1.

We define the bijection $f : V' \cup E' \to \{1, 2, \ldots, (3m+1)n\}$ as follows:

Initially we assign labels to the vertices $W_0(m, n)$.

For $1 \le i \le m+1$ and $1 \le j \le n$ each $v_{i,j} \in V'$, define $f(v_{i,j})$ as in equation (3) of Theorem 2.1.

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Then $f(V') = \{1, 2, \dots, (m+1)n\}$.

Let $e = (v_{i,j}, v_{s,t})$ be any edge in $W_0(m,n)$ and let $f'(e) = f(v_{i,j}) + f(v_{s,t})$ (7)

We denote the edges in E′ as follows:

For $1 \le j \le \frac{n-1}{2}$, denote the edge by $e_j^{(1)}$, if the sum of the labels of its end vertices is equal to 2j+1.

For $0 \le j \le mn$, denote the edge by $e_j^{(2)}$, if the sum of the labels of its end vertices is equal to n+1+j.

For $0 \le j \le (m-1)n$, denote the edge by $e_j^{(3)}$, if the sum of the labels of its end vertices is equal to (m+1)n + 2 + j.

For $1 \le j \le \frac{n-3}{2}$, denote the edge by $e_j^{(4)}$, if the sum of the labels of its end vertices is equal to 2(mn +j+1).

Then from equation (7), we have

$$\begin{aligned}
f'(e_j^{(1)}) &= 2j + 1, & 1 \le j \le \tfrac{n-1}{2} \\
f'(e_j^{(2)}) &= n + 1 + j, & 0 \le j \le mn \\
f'(e_j^{(3)}) &= (m+1)n + 2 + j, & 0 \le j \le (m-1)n \\
f'(e_j^{(4)}) &= 2(mn + j + 1), & 1 \le j \le \tfrac{n-3}{2}.
\end{aligned} \tag{8}$$

Now, let us label the edges of $W_0(m, n)$ by

$$\begin{aligned}
f(e_j^{(1)}) &= 2mn + 2n - 2j + 1, & 1 \le j \le \tfrac{n-1}{2} \\
f(e_j^{(2)}) &= 2mn + n + 1 - j, & 0 \le j \le mn \\
f(e_j^{(3)}) &= 3mn + n - j, & 0 \le j \le (m-1)n \\
f(e_j^{(4)}) &= 2(mn + n - j), & 1 \le j \le \tfrac{n-3}{2}.
\end{aligned}$$

Then $f(E') = \{mn+n+1, mn+n+2, \dots, 3mn+n\}$.

The induced map f^* on E′ defined by $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)})$ for every edge $e_j^{(k)} \in E'$ satisfies the conditions:

$$\begin{aligned}
f^*(e_j^{(1)}) &= f'(e_j^{(1)}) + f(e_j^{(1)}) \\
&= 2(mn + n + 1), \quad 1 \le j \le \tfrac{n-1}{2}
\end{aligned}$$

$$\begin{aligned}
f^*(e_j^{(2)}) &= f'(e_j^{(2)}) + f(e_j^{(2)}) \\
&= 2(mn + n + 1), \quad 0 \le j \le mn
\end{aligned}$$

$$\begin{aligned}
f^*(e_j^{(3)}) &= f'(e_j^{(3)}) + f(e_j^{(3)}) \\
&= 2(2mn + n + 1) \quad 0 \le j \le (m-1)n
\end{aligned}$$

$$\begin{aligned}
f^*(e_j^{(4)}) &= f'(e_j^{(4)}) + f(e_j^{(4)}) \\
&= 2(2mn + n + 1), \quad 1 \le j \le \tfrac{n-3}{2}.
\end{aligned}$$

Clearly, it is observed that for each edge $e_j^{(k)} \in E'$, $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)}) = 2(mn+n+1)$ or $2(2mn+n+1)$. Since there exists two common edge counts $k_1 = 2(mn+n+1)$ and $k_2 = 2(2mn+n+1)$, the graph $W_0(m, n)$ has super edge-bimagic total labeling , for all $m \ge 2$ and odd $n \ge 3$.

Theorem2.4: Let $G_2(V_2, E_2)$ be the graph obtained from P_m x C_n by attaching s pendent vertices at each vertex of the outermost cycle. Then the graph G_2 has a super edge bimagic total labeling for all $m \ge 2$ and odd $n \ge 3$.

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Proof: The graph $G_2(V_2, E_2)$ is the same as the graph obtained from $W_0(m, n)$ by appending s–1 pendent edges at each vertex $v_{m,j}$ (j = 1, 2, …, n) of the outermost cycle of $W_0(m,n)$. Denote the newly attached pendent vetices at $v_{m,j}$ as $v_j^{(l)}$, $v_j^{(2)}$, . . ., $v_j^{(s-1)}$ (j=1,2,…,n).Then $V_2= V' \cup V_3$ where $V_3=\{v_j^{(l)}; j =1,2,…,n ; l = 1,2,…,s–1\}$and the edge set $E_2 = E'\cup E_3$, where $E_3=\{v_{m,j}\, v_j^{(l)} \,/\, 1\leq j \leq n;\ 1\leq l \leq s–1\}$ and V′ and E′ are as defined in the proof of the Theorem 2.3. Then $|V_2| = (m+s)n$ and $|E_2| = (2m+s–1)n$. We define the bijection $f: V_2 \cup E_2 \to \{1, 2, …, (3m+2s–1)n\}$ as follows:

Initially we assign labels to the vertices of G_2.

For $1\leq i \leq m+1$and $1\leq j \leq n$ each $v_{i,j}\in V'$, define $f(v_{i,j})$ as in the equation (3) of Theorem 2.1and for $1\leq j\leq n$; $1\leq l \leq s–1$ each $v_j^{(l)} \in V_3$, define $f(v_j^{(l)} = f(v_{m+1,j}) + ln$. Then $f(V_2) =\{1,2, …,(m+s)n\}$.

Let $e = (v_{i,j} , v_{s,t})$ be any edge in G_2 and let $f '(e) = f(v_{i,j}) + f(v_{s,t})$ (9)

We denote the edges of G_2 as follows.

For $1 \leq j \leq \frac{n-1}{2}$, denote the edge by $e_j^{(1)}$, if the sum of the labels of its end vertices is equal to 2j+1.

For $0 \leq j \leq (m+s–1)n$, denote the edge by $e_j^{(2)}$, if the sum of the labels of its end vertices is equal to n+1+j.

For $0 \leq j \leq (m–1)n$, denote the edge by $e_j^{(3)}$, if the sum of the labels of its end vertices is equal to mn+sn + 2 + j.

For $1 \leq j \leq \frac{n-3}{2}$, denote the edge by $e_j^{(4)}$, if the sum of the labels of its end vertices is equal to (2m +s–1)+2j+2.

Then from equation (9), we have

$$\begin{aligned}
f '(e_j^{(1)}) &= 2j + 1 , && 1 \leq j \leq \tfrac{n-1}{2}\\
f '(e_j^{(2)}) &= n+ 1 + j , && 0 \leq j \leq (m+s–1)n\\
f '(e_j^{(3)}) &= mn+sn + 2 + j , && 0 \leq j \leq (m–1)n\\
f '(e_j^{(4)}) &= (2m +s–1)n+2j+2 , && 1 \leq j \leq \tfrac{n-3}{2} .
\end{aligned} \tag{10}$$

Now let us label the edges of G_2 by

$$\begin{aligned}
f(e_j^{(1)}) &= 2mn +2sn –2j+1 , && 1 \leq j \leq \tfrac{n-1}{2}\\
f(e_j^{(2)}) &= 2mn +2sn –n+1–j , && 0 \leq j \leq (m+s–1)n\\
f(e_j^{(3)}) &= 3mn +2sn–n – j , && 0 \leq j \leq (m-1)n\\
f(e_j^{(4)}) &= 2(mn +sn– j) , && 1 \leq j \leq \tfrac{n-3}{2} .
\end{aligned}$$

Then $f(E_2) = \{mn+sn+1, mn+sn+2, … , (3m+2s–1)n\}$.

The induced map f^* on E_2 defined by $f^*(e_j^{(k)}) = f '(e_j^{(k)}) + f (e_j^{(k)})$ for every edge $e_j^{(k)}\in E_2$ satisfies the conditions:

$$\begin{aligned}
f^*(e_j^{(1)}) &= f '(e_j^{(1)}) + f (e_j^{(1)})\\
&=2(mn+sn+1) = k_1 , \qquad 1 \leq j \leq \tfrac{n-1}{2}
\end{aligned}$$

$$\begin{aligned}
f^*(e_j^{(2)}) &= f '(e_j^{(2)}) + f (e_j^{(2)})\\
&=2(mn+sn+1) = k_1, \qquad 0 \leq j \leq (m+s–1)n
\end{aligned}$$

$$\begin{aligned}
f^*(e_j^{(3)}) &= f '(e_j^{(3)}) + f (e_j^{(3)})\\
&= (4m+3s–1)n+2 = k_2 , \qquad 0 \leq j \leq (m-1)n
\end{aligned}$$

$$\begin{aligned}
f^*(e_j^{(4)}) &= f '(e_j^{(4)}) + f (e_j^{(4)})\\
&= (4m+3s–1)n+2 = k_2 , \qquad 1 \leq j \leq \tfrac{n-3}{2} .
\end{aligned}$$

Clearly, it is observed that for every edge $e_j^{(k)} \in E_2$, $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)}) = 2(mn+sn+1)$ or $(4m+3s-1)n+2$. Since there exists two common edge counts $k_1 = 2(mn+sn+1)$ and $k_2 = (4m+3s-1)n+2$, the graph G_2 has super edge bimagic total labeling, for n odd and $m \geq 2$.

In Figure 2.3, we give a super edge bimagic total labeling for the graph G_2 in Theorem 2.4 with m =4, n =5 and s = 3.

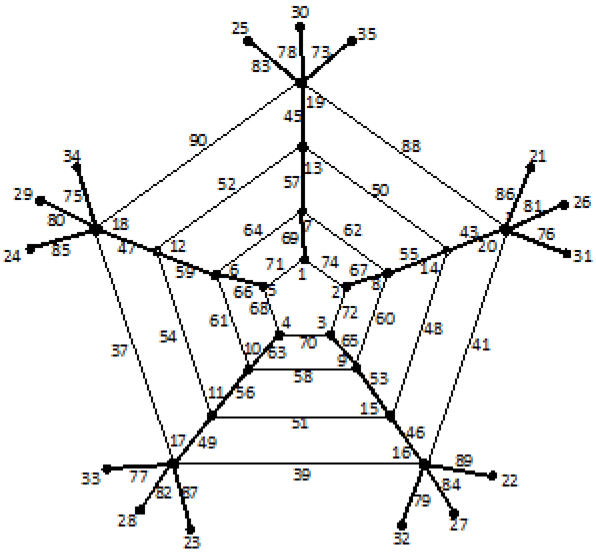

Figure 2.3

Theorem 2.5: For $m \geq 2$ and odd $n \geq 3$, the generalized web W(m, n) has super edge bimagic total labeling.

Proof: The generalized web W(m, n) is the same as the one obtained from $W_0(m, n)$ by joining each vertex of the innermost cycle to a new vertex v_0 by an edge. Let V'' be the vertex set and E'' be the edge set of W(m, n). Then $V'' = V' \cup \{v_0\}$ and $E'' = E' \cup E_0$, where V' and E' are as defined in Theorem 2.3 and $E_0 = \{v_0 v_{1,j} : 1 \leq j \leq n\}$. Then $|V''| = (m+1)n+1$ and $|E''| = (2m+1)n$.

For $1 \leq i \leq m+1$, denote k(i) and δ(i) as defined in equations (1) and (2) of Theorem 2.1.

We define the bijection $f: V'' \cup E'' \rightarrow \{1, 2, \ldots, (3mn+2)n+1\}$ as follows:

Initially we assign labels to the vertices of W(m, n).

For $1 \leq i \leq m+1$ and $1 \leq j \leq n$ each vertex $v_{i,j} \in V'$, define $f(v_{i,j})$ as in the equation (3) of Theorem 2.1 and $f(v_0) = mn+n+1$. Then $f(V'') = \{1, 2, \ldots, mn+n+1\}$.

Let $e = (v_{i,j}, v_{s,t})$ be any edge in W(m,n) and let $f'(e) = f(v_{i,j}) + f(v_{s,t})$ (11)

We denote the edges of W(m,n) as follows:

For $0 \leq j \leq n$, denote each edge $v_0 v_{1,j} \in E_0$ by $e_j^{(0)}$ and denote the edges in E' of W(m,n) as denoted in Theorem 2.3.

Then from the equation (11), we have

$$f'(e_j^{(0)}) = mn+n+1+j, \quad 0 \leq j \leq n$$

and for each $e_j^{(k)} \in E'$, $f'(e_j^{(k)})$ is as defined in equation (8) of Theorem 2.3.

Now, let us label the edges of W(m, n) by

$$f(e_j^{(0)}) = mn+2n+2-j, \; 0 \leq j \leq n$$

$$f(e_j^{(1)}) = 2mn+3n-2j+2, \; 1 \leq j \leq \frac{n-1}{2}$$

$$f(e_j^{(2)}) = 2mn+2n+2-j, \; 0 \leq j \leq mn$$

$f(e_j^{(3)}) = 3mn + 2n + 1 - j,\ 0 \le j \le (m-1)n$

$f(e_j^{(4)}) = 2mn + 3n + 1 - 2j,\ 1 \le j \le \frac{n-3}{2}$.

Then $f(E'') = \{mn+n+2, mn+n+3, \ldots, 3mn+2n+1\}$.

Define the induced map f^* on E'' by

$f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)})$ for every edge $e_j^{(k)} \in E''$.

It satisfies the conditions:

$$\begin{aligned} f^*(e_j^{(0)}) &= f'(e_j^{(0)}) + f(e_j^{(0)}) \\ &= (mn+n+1+j) + (mn+2n+2-j) \\ &= 2mn+3n+3 = k_1, \quad 0 \le j \le n. \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(1)}) &= f'(e_j^{(1)}) + f(e_j^{(1)}) \\ &= (2j+1) + (2mn+3n-2j+2) \\ &= 2mn+3n+3 = k_1,\ 1 \le j \le \frac{n-1}{2} \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(2)}) &= f'(e_j^{(2)}) + f(e_j^{(2)}) \\ &= (n+1+j) + 2mn + 2n + 2 - j \\ &= 2mn+3n+3 = k_1, \quad 0 \le j \le mn \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(3)}) &= f'(e_j^{(3)}) + f(e_j^{(3)}) \\ &= (mn+n+2+j) + (3mn+2n+1-j) \\ &= 4mn+3n+3 = k_2, \quad 0 \le j \le (m-1)n \end{aligned}$$

$$\begin{aligned} f^*(e_j^{(4)}) &= f'(e_j^{(4)}) + f(e_j^{(4)}) \\ &= 2(mn+j+1) + 2mn + 3n + 1 - 2j \\ &= 4mn+3n+3 = k_2, \quad 1 \le j \le \frac{n-3}{2}. \end{aligned}$$

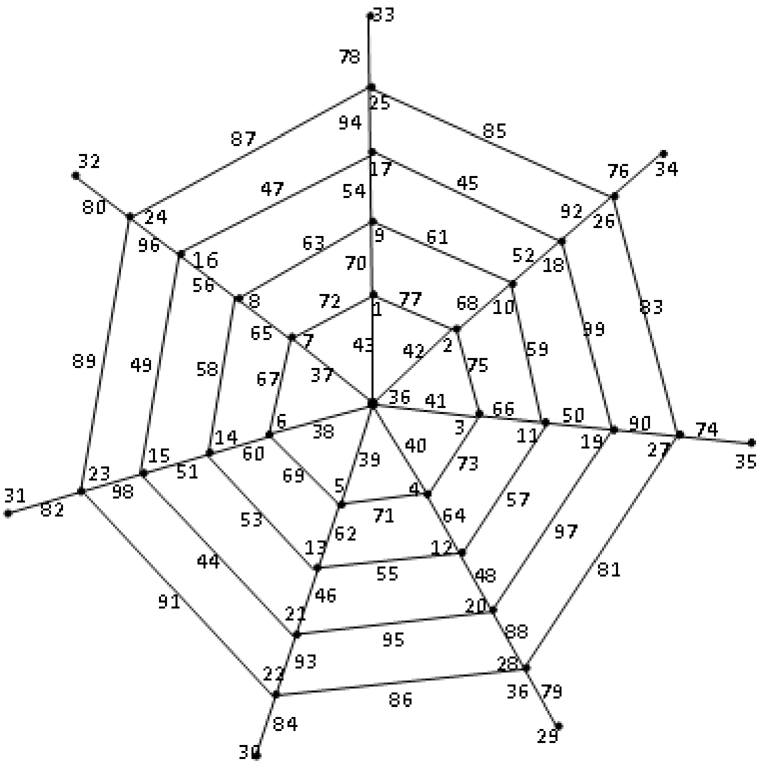

Fig.2.4: Generalized web W(4,7)

Clearly, it is observed that for each edge $e_j^{(k)} \in E'$, $f^*(e_j^{(k)}) = f'(e_j^{(k)}) + f(e_j^{(k)}) = 2mn + 3n + 3$ or $4mn+3n+3$. Since there exists two common edge counts $k_1 = 2(mn + n + 1)$ and $k_2 = 2(2mn+n+1)$, the generalized web W(m, n) has super edge bimagic total labeling for all $m \geq 2$ and odd $n \geq 3$.

In Figure 2.4, we give a super edge bimagic total labeling for the generalized web W(4,7).

CONCLUSION

In this paper, we have proved the super edge bimagic total labeling for generalized prism P_m x C_n, the Mongolian Ger M(n, m), the generalized web W(m, n), the generalized web without centre $W_0(m, n)$ and its related graph. Further we extend this work by examining the existence of certain other labelings for these graphs.

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