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MHD FREE CONVECTIVE OSCILLATORY FLOW AND MASS TRANSFER PAST A POROUS PLATE EMBEDDED IN A POROUS MEDIUM IN THE PRESENCE OF RADIATION FOR AN OPTICALLY THIN FLUID

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ABSTRACT

An attempt has been made to study the two-dimensional MHD free convective oscillatory flow and mass transfer of an optically thin gray fluid with electrically conducting incompressible viscous fluid past an infinite vertical porous plate embedded in a porous medium, through which suction occurs with constant velocity. A uniform magnetic field is assumed to be applied transversely to the direction of the free stream taking into account of induced magnetic field. The governing equations involved in the present analysis are solved by using the perturbation method. The velocity, temperature and concentration fields are studied for different parameters such as Radiation parameter (S), Grashof number (Gr), modified Grashof number (Gc), Magnetic field parameter (M), Permeability parameter (k), Schmidt number (Sc), Prandtl number (Pr) and Eckert number (Ec) etc.

Key words: *Free convection, mass transfer, radiation, oscillatory flow, MHD, Porous medium etc.*

INTRODUCTION

Many processes in engineering areas occur at high temperature making the knowledge of thermal radiation heat transfer becomes very important. Plasma physics, gas turbines, and the various propulsion devices for aircraft, missiles, satellites and space vehicles, flow through a porous medium in the presence of radiation and glass production are some examples of such engineering areas.

The influence of magnetic field on viscous incompressible flow of an electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications are in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth’s core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Jonah Philliph *et al.* (10) studied the effects of thermal radiation and MHD on the unsteady free convection and mass transform flow past an exponentially accelerated vertical plate with variable temperature. Raptis (12) discussed the free convective oscillatory flow and mass transfer past a porous plate in the presence of radiation for an optically thin fluid. Kishore *et al*. (11) have analyzed the effects of thermal radiation and viscous dissipation on MHD heat and mass

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diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions. Israel – Cookey *et al.* (9) have studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Cookey *et al.* (6) have analyzed influence of viscous dissipation and radiation on unsteady MHD free convective flow past an infinite heated vertical plate in a porous medium with time dependent suction.

Chamkha (4) discussed unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat generation. Ahmed (2) looked the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Sharma and Singh (15) have discussed the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Sharma *et al.* (14) have analyzed the heat and mass transfer effects on unsteady MHD free convective flow along a vertical porous plate with internal heat generation and variable suction. Soundalgekar (16) investigated the unsteady free convection flow along vertical porous plate with different boundary conditions and viscous dissipation effect. Hemanth Poonia and Chaudhary (8) have analyzed the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation.

Convection in porous medium has important applications in many areas including thermal energy storage, flow through filtering devices, utilization of geothermal energy, oil extraction, high performance insulation for buildings, paper industry etc. Hence combined study may give some vital information which will surely be helpful in developing other relevant areas. Chaudhary and Arpita Jain (5) have discussed the MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium. Seethamahalakshmi *et al.* (13) have examined the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow fast a semi-infinite vertical moving in a porous medium with heat source and suction. Abdel-Nasser Osman *et al.* (1) have investigated the analytical solution of thermal radiation and chemical reaction effects on unsteady MHD convection through porous medium with heat source/sink. Chamkha and Khaled (3) have looked the effects of hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in fluid saturated porous medium. Sudheer Babu *et al.* (17) have examined the radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation. Girish Kumar *et al.* (7) have examined the mass transfer effects on MHD flows exponentially accelerated vertical plate in the presence of chemical reaction through porous media.

The main objective of the present analysis is to study the two-dimensional MHD free convective oscillatory flow and mass transfer of an optically thin gray fluid with electrically conducting incompressible viscous fluid past an infinite vertical porous plate embedded in a porous medium, through which suction occurs with constant velocity. The equations of continuity, momentum, energy and diffusion which govern the flow field are solved to the best possible solution.

Nomenclature

C	-	dimensionless concentration, [-]
C'	-	concentration, [molm^{-3}]
C'_w	-	species concentration at the plate, [molm^{-3}]
C'_∞	-	species concentration far away from the plate, [molm^{-3}]
c_p	-	specific heat at constant pressure, [$\mathrm{Jkg^{-1}K^{-1}}$]
D	-	chemical diffusivity, [$\mathrm{m^2s^{-1}}$]
Ec	-	Eckert number, [-]
$g_{x'}$	-	acceleration due to gravity, [$\mathrm{ms^{-2}}$]
Gc	-	modified Grashof number, [-]
Gr	-	Grashof number, [-]
B_0	-	applied magnetic field, [-]
k'	-	permeability parameter, [-]
κ	-	thermal conductivity, [$\mathrm{Wm^{-1}K^{-1}}$]
Pr	-	Prandtl number, [-]
p'	-	pressure, [$\mathrm{kgm^{-1}s^{-2}}$]
q'	-	heat flux at the plate, [$\mathrm{Wm^{-2}}$]
Sc	-	Schmidt number, [-]
T	-	dimensionless fluid temperature, [-]
T'	-	fluid temperature, [K]
T'_∞	-	temperature of the fluid far away from the plate, [K]
t	-	dimensionless time, [-]
t'	-	the time, [s]
U_0	-	mean free stream velocity, [$\mathrm{ms^{-1}}$]
u	-	dimensionless velocity of the fluid at the x'- direction, [-]

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$u^{'}$	-	velocity of the fluid at the $x^{'}$- direction, [ms^{-1}]
$v^{'}$	-	velocity of the fluid at the $y^{'}$- direction, [-]
v_0	-	suction velocity, [ms^{-1}]
$x^{'}$	-	co-ordinate axis along the plate, [-]
$y^{'}$	-	co-ordinate axis normal to the plate, [-]

Greek symbols

α	-	absorption coefficient, [m^{-1}]
β	-	coefficient of thermal expansion, [K^{-1}]
β^{*}	-	coefficient of concentration expansion, [(molm^{-3})$^{-1}$]
v	-	kinematic viscosity, [m^{2}s^{-1}]
ρ	-	fluid density, [kgm^{-3}]
σ^{*}	-	Stefan – Boltzman constant, [Wm^{-2}K^{-4}]
ω	-	dimensionless frequency of vibration of the fluid, [-]
$\omega^{'}$	-	frequency of vibration of the fluid, [rads^{-1}]

FORMULATION OF THE PROBLEM

We consider the unsteady two-dimensional MHD free convective oscillatory flow and mass transfer of an optically thin gray fluid with electrically conducting incompressible viscous fluid past an infinite vertical porous plate embedded in a porous medium, through which suction occurs with constant velocity. The $x^{'}$ - axis is along the plate in the upward direction and the $y^{'}$ - axis is normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). There is a chemical reaction between the diffusing species and the fluid. The foreign mass present in the flow is assumed to be a low level and hence Soret and Dufour effects are negligible. Under these assumptions, the governing equations of the flow field are:

Continuity equation

$$\frac{\partial v^{'}}{\partial y^{'}} = 0 \tag{1}$$

Momentum equation

$$\rho\left(\frac{\partial u^{'}}{\partial t^{'}} + v^{'}\frac{\partial u^{'}}{\partial y^{'}}\right) = -\frac{\partial p^{'}}{\partial x^{'}} - \rho g_{x^{'}} + v\rho\frac{\partial^2 u^{'}}{\partial y^{'2}} - \left(\sigma B_0^2 + \rho\frac{v^{'}}{k^{'}}\right)(u^{'}) \tag{2}$$

Energy equation

$$\frac{\partial T^{'}}{\partial t^{'}} + v^{'}\frac{\partial T^{'}}{\partial y^{'}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \frac{v}{C_p}\left(\frac{\partial u^{'}}{\partial y^{'}}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^{'}} \tag{3}$$

Diffusion equation

$$\frac{\partial C^{'}}{\partial t^{'}} + v^{'}\frac{\partial C^{'}}{\partial y^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}} \tag{4}$$

Where $u^{'}$ and $v^{'}$ are the components of the velocity parallel and perpendicular to the plate, $t^{'}$ - the time, $p^{'}$ - the pressure, ρ - the fluid density, $g_{x^{'}}$ - the acceleration due to gravity, B_0 - the applied magnetic field, $k^{'}$ - the permeability parameter, $T^{'}$ - the fluid temperature, v- the kinematic viscosity, C_p- the specific heat at constant pressure, κ - the thermal conductivity, $C^{'}$ - the concentration and D- the chemical diffusivity, q_r- the radiative heat flux in the $y^{'}$ direction

The boundary conditions are:

$$\left.\begin{array}{c} u^{'} = 0, v^{'} = -v_0, \frac{\partial T^{'}}{\partial y^{'}} = -\frac{q^{'}}{\kappa}, C^{'} = C_w^{'} \; at \; y^{'} = 0 \\ u^{'} \to U^{'} = U_0\left(1 + \varepsilon e^{iw^{'}t^{'}}\right), T^{'} \to T_\infty^{'}, C^{'} \to C_\infty^{'} \; as \; y^{'} \to \infty \end{array}\right\} \tag{5}$$

Where v_0 is the constant suction velocity and the negative sign indicates that it is towards the plate, $q^{'}$ - the constant heat flux, $T_\infty^{'}$ - the fluid temperature far away from the plate, $C_w^{'}$ - the species concentration at the plate, $C_\infty^{'}$ - the species concentration far away from the plate, U_0 - the mean free stream velocity, $\omega^{'}$ - the frequency of vibration of the fluid, and ε $(\varepsilon < 1)$ - a constant quantity.

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For the free stream, equation (2) becomes:

$$\rho\frac{dU'}{dt'} = -\frac{\partial p'}{\partial x'} - \rho_\infty g_{x'} - \sigma B_0^2 U' - \rho\frac{v'}{k'}U' \tag{6}$$

On eliminating $\frac{\partial p'}{\partial x'}$ between (2) and (6) we get:

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = \rho\frac{dU'}{dt'} + g_{x'}(\rho_\infty - \rho) + v\rho\frac{\partial^2 u'}{\partial y'^2} - \left(\sigma B_0^2 + \rho\frac{v'}{k'}\right)\left(u' - U'(t')\right) \tag{7}$$

The state equation is

$$g_{x'}(\rho_\infty - \rho) = g_{x'}\rho\beta(T' - T'_\infty) + g_{x'}\rho\beta^*(C' - C'_\infty) \tag{8}$$

Where β is the coefficient of thermal expansion and β^* is the coefficient of concentration expansion

From (7) and (8) we have

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g_{x'}\beta(T' - T'_\infty) + g_{x'}\beta^*(C' - C'_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{v'}{k'}\right)\left(u' - U'(t')\right) \tag{9}$$

In the case of an optically thin gray fluid the local radiant absorption is expressed as:

$$-\frac{\partial q_r}{\partial y'} = 4d\sigma^*(T'^4_\infty - T'^4) \tag{10}$$

where d is the absorption coefficient and σ^* the Stefan-Boltzman constant.

We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{10}$$

Equation (9) through (10) takes the form:

$$-\frac{\partial q_r}{\partial y'} = 16d\sigma^* T'^3_\infty(T'_\infty - T') \tag{11}$$

From the equations (3) and (11) we have

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\rho C_p}16d\sigma^* T'^3_\infty(T'_\infty - T') \tag{12}$$

Equation (1) gives:

$$v' = -v_0(v_0 > 0) \tag{13}$$

On substituting equation (13), in equations (9), (12) and (4) we take:

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g_{x'}\beta(T' - T'_\infty) + g_{x'}\beta^*(C' - C'_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{v'}{k'}\right)\left(u' - U'(t')\right) \tag{14}$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{16d\sigma^* T'^3_\infty}{\rho C_p}(T'_\infty - T') \tag{15}$$

$$\frac{\partial C'}{\partial t'} - v_0\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{16}$$

Using the transformations:

$$\left.\begin{array}{l} y = \frac{y' v_0}{v}, t = \frac{t' v_0^2}{4v}, u = \frac{u'}{U_0}, U = \frac{U'}{U_0}, \omega = \frac{4v\omega'}{v_0^2} T = \frac{T' - T'_\infty}{\frac{vq'}{kv_0}} \\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gr = \frac{g_{x'}\beta v^2 q'}{kU_0 v_0^3}, Gc = \frac{v g_{x'}\beta^*(C'_w - C'_\infty)}{U_0 v_0^2}, \Pr = \frac{\rho v C_p}{k}, \\ Ec = \frac{kU_0^2 v_0}{C_p v q'}, M = \frac{\sigma B_0^2 v}{\rho v_0^2}, Sc = \frac{v}{D}, \ k = \frac{k' v^2}{v_0^2}, S = 16d\sigma^* T'^3_\infty v^3 \end{array}\right\} \tag{17}$$

With the help of the non-dimensional quantities (17), equations (14)-(16) reduce to:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4}\frac{dU}{dt} + GrT + GcC + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{k}\right)(u - U) \tag{18}$$

$$Pr\left(\frac{1}{4}\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} + PrEc\left(\frac{\partial u}{\partial y}\right)^2 - ST \tag{19}$$

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$$Sc\left(\frac{1}{4}\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}\right)=\frac{\partial^2 C}{\partial y^2} \tag{20}$$

With the boundary conditions:

$$\left.\begin{array}{l} u=0,\quad \frac{\partial T}{\partial y}=-1,\quad C=1 \quad at \quad y=0 \\ u\to U(t)=1+\varepsilon e^{i\omega t}, T\to 0, C\to 0\ as\ y\to\infty \end{array}\right\} \tag{21}$$

SOLUTION OF THE PROBLEM

Equations (18) – (20) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as:

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+\cdots \tag{22}$$

$$T(y,t)=T_0(y)+\varepsilon e^{i\omega t}T_1(y)+\cdots \tag{23}$$

$$C(y,t)=C_0(y)+\varepsilon e^{i\omega t}C_1(y)+\cdots \tag{24}$$

On substituting equations (22)-(24) in equations (18)-(20) we get the following system of differential equations:

$$\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-\left(M+\frac{1}{k}\right)u_0=-\left[GrT_0+GcC_0+\left(M+\frac{1}{k}\right)\right] \tag{25}$$

$$\frac{d^2u_1}{dy^2}+\frac{du_1}{dy}-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)u_1=-\left[GrT_1+GcC_1+\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)\right] \tag{26}$$

$$\frac{d^2T_0}{dy^2}+Pr\frac{dT_0}{dy}-ST_0=-PrEc\left(\frac{du_0}{dy}\right)^2 \tag{27}$$

$$\frac{d^2T_1}{dy^2}+Pr\frac{dT_1}{dy}-\frac{i\omega}{4}PrT_1-ST_1=-2PrEc\left(\frac{du_0}{dy}\right)\left(\frac{du_1}{dy}\right) \tag{28}$$

$$\frac{d^2C_0}{dy^2}+Sc\frac{dC_0}{dy}=0 \tag{29}$$

$$\frac{d^2C_1}{dy^2}+Sc\frac{dC_1}{dy}=0 \tag{30}$$

The corresponding boundary conditions (21) become:

$$\left.\begin{array}{l} u_0=0, u_1=0, \frac{dT_0}{dy}=-1, \frac{dT_1}{dy}=0, C_0=1, C_1=0\ at\ y=0 \\ u_0\to 1, u_1\to 1, T_0\to 0, T_1\to 0, C_0\to 0, C_1\to 0 \quad as \quad y\to\infty \end{array}\right\} \tag{31}$$

In order to solve the system of the differential equations (25)-(28) we put:

$$\left.\begin{array}{l} u_0(y)=u_{01}(y)+Ecu_{02}(y) \\ T_0(y)=T_{01}(y)+EcT_{02}(y) \end{array}\right\} \tag{32}$$

and

$$\left.\begin{array}{l} u_1(y)=u_{11}(y)+Ecu_{12}(y) \\ T_1(y)=T_{11}(y)+EcT_{12}(y) \end{array}\right\} \tag{33}$$

In this system, equating the coefficients of Ec^0 and Ec^1 we get:

$$\frac{d^2u_{01}}{dy^2}+\frac{du_{01}}{dy}-\left(M+\frac{1}{k}\right)u_{01}=-\left(GrT_{01}+GcC_{01}+\left(M+\frac{1}{k}\right)\right) \tag{34}$$

$$\frac{d^2u_{02}}{dy^2}+\frac{du_{02}}{dy}-\left(M+\frac{1}{k}\right)u_{02}=-(GrT_{02}+GcC_{02}) \tag{35}$$

$$\frac{d^2T_{01}}{dy^2}+Pr\frac{dT_{01}}{dy}-ST_{01}=0 \tag{36}$$

$$\frac{d^2T_{02}}{dy^2}+Pr\frac{dT_{02}}{dy}-ST_{02}=-2PrEc\left(\frac{du_{01}}{dy}\right)^2 \tag{37}$$

$$\frac{d^2u_{11}}{dy^2}+\frac{du_{11}}{dy}-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)u_{11}=-\left(\frac{i\omega}{4}+GrT_{11}+GcC_{11}+M+\frac{1}{k}\right) \tag{38}$$

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$$\frac{d^2u_{12}}{dy^2}+\frac{du_{12}}{dy}-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)u_{11}=-(GrT_{12}+GcC_{12}) \quad (39)$$

$$\frac{d^2T_{11}}{dy^2}+Pr\frac{dT_{11}}{dy}-\left(\frac{i\omega}{4}Pr+S\right)T_{11}=0 \quad (40)$$

$$\frac{d^2T_{12}}{dy^2}+Pr\frac{dT_{12}}{dy}-\left(\frac{i\omega}{4}Pr+S\right)T_{12}=-2PrEc\left(\frac{du_{01}}{dy}\right)\left(\frac{du_{11}}{dy}\right) \quad (41)$$

The corresponding boundary conditions (31) become:

$$\left.\begin{array}{l}\left.\begin{array}{l}u_{00}=0,\ \ u_{01}=0,\ \ \ u_{11}=0,\ \ u_{12}=0\\ \frac{dT_{00}}{dy}=-1,\frac{dT_{01}}{dy}=0,\frac{dT_{11}}{dy}=0,\frac{dT_{12}}{dy}=0\\ \qquad C_0=1, C_1=0\end{array}\right\}at\ y=0\\ \left.\begin{array}{l}u_{00}\to 1,\ u_{01}\to 0,\ u_{11}\to 1,\ u_{12}\to 0\\ T_{00}\to 0,\ T_{01}\to 0,\ T_{11}\to 0,\ T_{12}\to 0\\ \qquad C_0\to 0, C_1\to 0\end{array}\right\}as\ y\to\infty\end{array}\right\} \quad (42)$$

Solving the differential equations (29), (30) and (34) – (41), using boundary conditions (42) we get

$$u_{10}=\alpha_5e^{\alpha_2y}+\alpha_3e^{\beta_2y}+\alpha_4e^{\gamma_2y}+1 \quad (43)$$

$$u_{02}=\alpha_{13}e^{\alpha_2y}+\alpha_6e^{\beta_2y}+\alpha_7e^{2\alpha_2y}+\alpha_8e^{2\beta_2y}+\alpha_9e^{2\gamma_2y}+\alpha_{10}e^{(\alpha_2+\beta_2)y}+\alpha_{11}e^{(\beta_2+\gamma_2)y}+\alpha_{12}e^{(\gamma_2+\alpha_2)y} \quad (44)$$

$$u_{11}=-e^{\alpha_{15}y}+1 \quad (45)$$

$$u_{12}=\alpha_{20}e^{\alpha_{15}y}+\alpha_{16}e^{\beta_{11}y}+\alpha_{17}e^{(\alpha_2+\alpha_{15})y}+\alpha_{18}e^{(\beta_2+\alpha_{15})y}+\alpha_{19}e^{(\gamma_2+\alpha_{15})y} \quad (46)$$

$$T_{01}=\frac{-1}{\beta_2}e^{\beta_2y} \quad (47)$$

$$T_{02}=\beta_9e^{\beta_2y}+\beta_3e^{2\alpha_2y}+\beta_4e^{2\beta_2y}+\beta_5e^{2\gamma_2y}+\beta_6e^{(\alpha_2+\beta_2)y}+\beta_7e^{(\beta_2+\gamma_2)y}+\beta_8e^{(\alpha_2+\gamma_2)y} \quad (48)$$

$$T_{11}=0 \quad (49)$$

$$T_{12}=\beta_{15}e^{\beta_{11}y}+\beta_{12}e^{(\alpha_2+\alpha_{15})y}+\beta_{13}e^{(\beta_2+\alpha_{15})y}+\beta_{14}e^{(\gamma_2+\alpha_{15})y} \quad (50)$$

$$C_0=e^{Scy} \quad (51)$$

$$C_1=0 \quad (52)$$

With the help of (43) – (50) the equations (32) and (33) becomes

$$\begin{aligned}u_0=&\left(\alpha_5e^{\alpha_2y}+\alpha_3e^{\beta_2y}+\alpha_4e^{\gamma_2y}+1\right)\\&+Ec\left(\alpha_{13}e^{\alpha_2y}+\alpha_6e^{\beta_2y}+\alpha_7e^{2\alpha_2y}+\alpha_8e^{2\beta_2y}+\alpha_9e^{2\gamma_2y}+\alpha_{10}e^{(\alpha_2+\beta_2)y}+\alpha_{11}e^{(\beta_2+\gamma_2)y}+\alpha_{12}e^{(\gamma_2+\alpha_2)y}\right)\end{aligned} \quad (53)$$

$$T_0=\left(\frac{-1}{\beta_2}e^{\beta_2y}\right)+Ec\left(\beta_9e^{\beta_2y}+\beta_3e^{2\alpha_2y}+\beta_4e^{2\beta_2y}+\beta_5e^{2\gamma_2y}+\beta_6e^{(\alpha_2+\beta_2)y}+\beta_7e^{(\beta_2+\gamma_2)y}+\beta_8e^{(\alpha_2+\gamma_2)y}\right) \quad (54)$$

$$u_1=(-e^{\alpha_{15}y}+1)+Ec\left(\alpha_{20}e^{\alpha_{15}y}+\alpha_{16}e^{\beta_{11}y}+\alpha_{17}e^{(\alpha_2+\alpha_{15})y}+\alpha_{18}e^{(\beta_2+\alpha_{15})y}+\alpha_{19}e^{(\gamma_2+\alpha_{15})y}\right) \quad (55)$$

$$T_1=Ec\left(\beta_{15}e^{\beta_{11}y}+\beta_{12}e^{(\alpha_2+\alpha_{15})y}+\beta_{13}e^{(\beta_2+\alpha_{15})y}+\beta_{14}e^{(\gamma_2+\alpha_{15})y}\right) \quad (56)$$

Finally from the above equations (51) – (56) and with the help of equations (22), (23) and (24) we obtain the velocity, temperature and concentration fields are as follows:

$$\begin{aligned}u(y,t)=&\ u_0+\varepsilon(\cos(wt)+isin(wt))u_1\\=&\Big(\left(\alpha_5e^{\alpha_2y}+\alpha_3e^{\beta_2y}+\alpha_4e^{\gamma_2y}+1\right)\\&+Ec\big(\alpha_{13}e^{\alpha_2y}+\alpha_6e^{\beta_2y}+\alpha_7e^{2\alpha_2y}+\alpha_8e^{2\beta_2y}+\alpha_9e^{2\gamma_2y}+\alpha_{10}e^{(\alpha_2+\beta_2)y}+\alpha_{11}e^{(\beta_2+\gamma_2)y}\\&+\alpha_{12}e^{(\gamma_2+\alpha_2)y}\big)\Big)+\varepsilon(\cos(wt)+isin(wt))(-e^{\alpha_{15}y}+1)\\&+Ec\left(\alpha_{20}e^{\alpha_{15}y}+\alpha_{16}e^{\beta_{11}y}+\alpha_{17}e^{(\alpha_2+\alpha_{15})y}+\alpha_{18}e^{(\beta_2+\alpha_{15})y}+\alpha_{19}e^{(\gamma_2+\alpha_{15})y}\right)\end{aligned} \quad (57)$$

$$T(y,t) = T_0 + \varepsilon\big(\cos(wt) + i\sin(wt)\big)T_1$$
$$= \left(\left(\frac{-1}{\beta_2} e^{\beta_2 y}\right) + Ec\big(\beta_9 e^{\beta_2 y} + \beta_3 e^{2\alpha_2 y} + \beta_4 e^{2\beta_2 y} + \beta_5 e^{2\gamma_2 y} + \beta_6 e^{(\alpha_2+\beta_2)y} + \beta_7 e^{(\beta_2+\gamma_2)y} + \beta_8 e^{(\alpha_2+\gamma_2)y}\big)\right)$$
$$+\varepsilon(\cos(wt) + i\sin(wt))\left(Ec\big(\beta_{15} e^{\beta_{11} y} + \beta_{12} e^{(\alpha_2+\alpha_{15})y} + \beta_{13} e^{(\beta_2+\alpha_{15})y} + \beta_{14} e^{(\gamma_2+\alpha_{15})y}\big)\right) \quad (58)$$

$$C(y,t) = C_0 + \varepsilon(\cos(wt) + i\sin(wt))C_1 = e^{\gamma_2 y} \quad (59)$$

RESULTS AND DISCUSSIONS

The chemical reaction effects on MHD free convective oscillatory flow past a porous plate embedded in a porous medium and in the presence of heat source have been studied. The governing equations are solved by using perturbation method and approximate solutions are obtained for velocity, temperature and concentration fields. The effects of the flow parameters such as Radiation parameter (S), Prandtl number (Pr), Eckert number (Ec), Schmidt number (Sc), Grashof number for heat and mass transfer (Gr, Gc), magnetic parameter or Hartmann number (M) and permeability parameter (k) on the velocity, temperature and concentration profiles of the flow field are presented with help of velocity, temperature and concentration profile.

For different values of the radiation parameter S the velocity and temperature profiles are plotted in Figs. 1(a) and 1(b). It is obvious that an increase in the radiation parameter S results a decrease in the velocity and temperature profiles within the boundary layer. This decrease of temperature may be attributed to the loss of heat energy due to radiation as well as low diffusivity.

Figs. 2(a) and 2(b) are shown that the behavior of the velocity (u) and temperature (T) for different values of the Prandtl number Pr. The numerical results show that the effect of increasing values of Pr results in a decreasing velocity. From Fig. 2(b), it is observed that an increase in Pr results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers as the thermal boundary layer is thicker and therefore the rate of heat transfer is reduced.

The influence of the viscous dissipation parameter i.e., the Eckert number (Ec) on velocity and temperature are shown in Fig. 3(a) and 3(b). The velocity and temperature both are increases with increasing Eckert number. It expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy in to internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity.

The effects of Schmidt number on the velocity (u) and concentrations (C) are displays in Figs. 4(a) and 4(b). As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

The velocity profiles for different values of the thermal Grashof number Gr are described in Fig. 5(a). It is observed that an increase in Gr leads to arise in the values of velocity. Hence the positive values of Gr correspond to cooling of the plate. In addition, it is observed that the velocity increases rapidly near wall of the plate as Grashof number increases and then decays to the free stream velocity. For the case of different values of the solutal Grashof number Gc, the velocity profiles in the boundary layer are shown in Fig. 5(b). It is observed that an increase in Gc, leads to a rise in the values of velocity.

The influence of magnetic parameter or Hartmann number M, on the velocity (u) is shown in Fig.6. An increase in M reduces the velocity. The application of a transverse magnetic field to an electrically conducting field gives rise to a resistive type of force called Lorentz force. This force has the tendency to slow down the fluid. This trend is apparent from Fig.7.

Fig. 7 depicts the velocity profiles for different values of permeability parameter k. Clearly as k increases the peak value of velocity across the boundary layer tends to increase rapidly near wall of the porous plate

CONCLUSIONS

We summarize below the following results of physical interest on the velocity, temperature and concentration distribution of the flow field.

1. The velocity decreases with the increase of the radiation parameter.

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2. A growing magnetic parameter or Prandtl number or Schmidt number or chemical reaction parameter retards the velocity of the flow field at all points.
3. The effect of increasing Grashof number or modified Grashof number or permeability parameter or Eckert number is to accelerate velocity of the flow field at all points.
4. A growing Prandtl number decreases temperature of the flow field at all points
5. The growing Schmidt number decreases the concentration of the flow field at all points.

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APPENDIX

$$\alpha_1 = \frac{-1+\sqrt{1+4\left(M+\frac{1}{k}\right)}}{2},\ \alpha_2 = \frac{-1-\sqrt{1+4\left(M+\frac{1}{k}\right)}}{2}, \alpha_3 = \frac{Gr}{\beta_2\left(\beta_2^2+\beta_2-\left(M+\frac{1}{k}\right)\right)}, \alpha_4 = \frac{Gc}{\gamma_2^2+\gamma_2-\left(M+\frac{1}{k}\right)},$$

$$\alpha_5 = -[1+\alpha_3+\alpha_4], \alpha_6 = \frac{-Gr\,\beta_9}{\beta_2^2+\beta_2-\left(M+\frac{1}{k}\right)}, \alpha_7 = \frac{-Gr\,\beta_3}{(2\alpha_2)^2+(2\alpha_2)-\left(M+\frac{1}{k}\right)},$$

$$\alpha_8 = \frac{-Gr\,\beta_4}{(2\beta_2)^2+(2\beta_2)-\left(M+\frac{1}{k}\right)}, \alpha_9 = \frac{-Gr\,\beta_5}{(2\gamma_2)^2+(2\gamma_2)-\left(M+\frac{1}{k}\right)}, \alpha_{10} = \frac{-Gr\,\beta_6}{(\alpha_2+\beta_2)^2+(\alpha_2+\beta_2)-\left(M+\frac{1}{k}\right)},$$

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$$\alpha_{11} = \frac{-Gr\,\beta_7}{(\gamma_2+\beta_2)^2+(\gamma_2+\beta_2)-\left(M+\frac{1}{k}\right)},\ \alpha_{12} = \frac{-Gr\,\beta_8}{(\gamma_2+\alpha_2)^2+(\gamma_2+\alpha_2)-\left(M+\frac{1}{k}\right)}$$

$$\alpha_{13} = -[\alpha_6+\alpha_7+\alpha_8+\alpha_9+\alpha_{10}+\alpha_{11}+\alpha_{12}],$$

$$\alpha_{14} = \frac{-1+\sqrt{1+4\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)}}{2},\ \alpha_{15} = \frac{-1-\sqrt{1+4\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)}}{2},\ \alpha_{16} = \frac{-Gr\,\beta_{15}}{\beta_{11}^2+\beta_{11}-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)},$$

$$\alpha_{17} = \frac{-Gr\,\beta_{12}}{(\alpha_2+\alpha_{15})^2+(\alpha_2+\alpha_{15})-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)},\ \alpha_{18} = \frac{-Gr\,\beta_{13}}{(\beta_2+\alpha_{15})^2+(\beta_2+\alpha_{15})-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)}$$

$$\alpha_{19} = \frac{-Gr\,\beta_{14}}{(\gamma_2+\alpha_{15})^2+(\gamma_2+\alpha_{15})-\left(\frac{i\omega}{4}+M+\frac{1}{k}\right)},\ \alpha_{20} = -[\alpha_{16}+\alpha_{17}+\alpha_{18}+\alpha_{19}]$$

$$\beta_1 = \frac{-Pr+\sqrt{Pr^2+4S}}{2},\ \beta_2 = \frac{-Pr-\sqrt{Pr^2+4S}}{2},\ \beta_3 = \frac{-2PrEc\,(\alpha_2\alpha_5)^2}{(2\alpha_2)^2+Pr\,(2\alpha_2)-S},$$

$$\beta_4 = \frac{-2PrEc\,(\beta_2\alpha_3)^2}{(2\beta_2)^2+Pr\,(2\beta_2)-S},\ \beta_5 = \frac{-2PrEc\,(\gamma_2\alpha_4)^2}{(2\gamma_2)^2+Pr\,(2\gamma_2)-S},\ \beta_6 = \frac{-4PrEc\,\alpha_2\alpha_3\alpha_5\beta_2}{(\alpha_2+\beta_2)^2+Pr\,(\alpha_2+\beta_2)-S},$$

$$\beta_7 = \frac{-4PrEc\,\beta_2\gamma_2\alpha_3\alpha_4}{(\beta_2+\gamma_2)^2+Pr\,(\beta_2+\gamma_2)-S},\ \beta_8 = \frac{-4PrEc\,\alpha_2\gamma_2\alpha_4\alpha_5}{(\alpha_2+\gamma_2)^2+Pr\,(\alpha_2+\gamma_2)-S},$$

$$\beta_9 = \frac{-1}{\beta_2}[2\alpha_2\beta_3+2\beta_2\beta_5+2\gamma_2\beta_5+(\alpha_2+\beta_2)\beta_6+(\beta_2+\gamma_2)\beta_7+(\alpha_2+\gamma_2)\beta_8]$$

$$\beta_{10} = \frac{-Pr+\sqrt{Pr^2+4\left(\frac{i\omega}{4}Pr+S\right)}}{2},\ \beta_{11} = \frac{-Pr-\sqrt{Pr^2+4\left(\frac{i\omega}{4}Pr+S\right)}}{2},$$

$$\beta_{12} = \frac{2PrEc\,\alpha_2\alpha_5\alpha_{15}}{(\alpha_2+\alpha_{15})^2+Pr\,(\alpha_2+\alpha_{15})-\left(\frac{i\omega}{4}Pr+S\right)},\ \beta_{13} = \frac{2PrEc\,\beta_2\alpha_3\alpha_{15}}{(\beta_2+\alpha_{15})^2+Pr\,(\beta_2+\alpha_{15})-\left(\frac{i\omega}{4}Pr+S\right)}$$

$$\beta_{14} = \frac{2PrEc\,\gamma_2\alpha_4\alpha_{15}}{(\gamma_2+\alpha_{15})^2+Pr\,(\gamma_2+\alpha_{15})-\left(\frac{i\omega}{4}Pr+S\right)},$$

$$\beta_{15} = \frac{-1}{\beta_{11}}[\beta_{12}(\alpha_2+\alpha_{15})+\beta_{13}(\beta_2+\alpha_{15})+\beta_{14}(\gamma_2+\alpha_{15})]$$

$$\gamma_1 = \frac{-Sc+\sqrt{Sc^2+4KrSc}}{2},\ \gamma_2 = \frac{-Sc-\sqrt{Sc^2+4KrSc}}{2},\ \gamma_3 = \frac{-Sc+\sqrt{Sc^2+4\left(\frac{i\,\omega}{4}+Kr\right)Sc}}{2},$$

$$\gamma_4 = \frac{-Sc-\sqrt{Sc^2+4\left(\frac{i\,\omega}{4}+Kr\right)Sc}}{2}$$

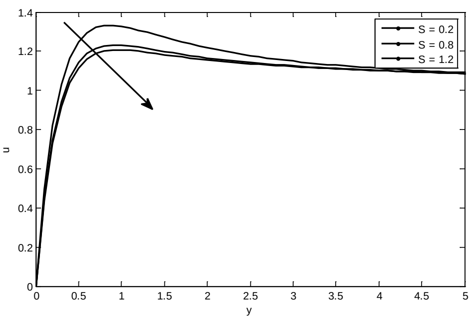

Figure - 1(a): Velocity profiles for different values of Radiation parameter 'S' when Gr = 5, Gc = 2, Ec = 0.001, Sc = 0.22, M = 1.0, k = 0.1.

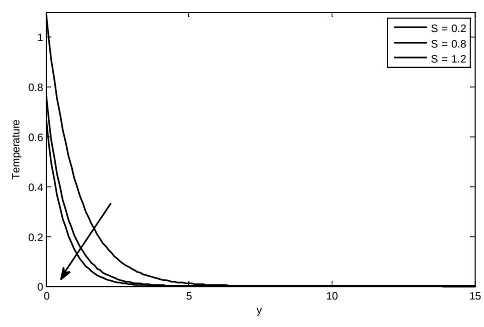

Figure - 1(b): Temperature profiles for different values of Prandtl number 'Pr' when Gr = 5, Gc = 2, Ec = 0.001, Sc = 0.22, M = 1.0, k = 0.1.

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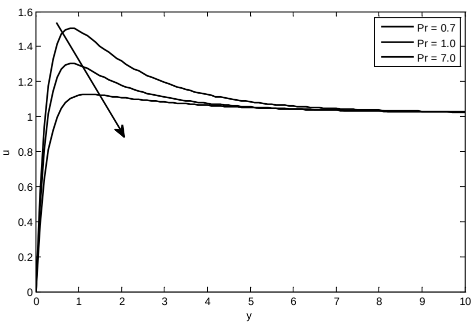

Figure - 2(a): Velocity profiles for different values of Prandtl number 'Pr' when Gr = 5, Gc = 2, Ec = 0.001, Sc = 0.22, M = 1.0, k = 0.1

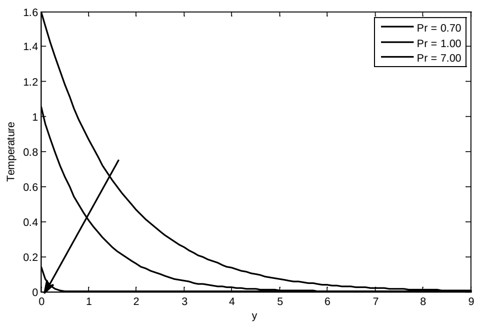

Figure - 2(b): Temperature profiles for different values of Prandtl number 'Pr' when Gr = 5, Gc = 2, Ec = 0.001, Sc = 0.22, M = 1.0, k = 0.1.

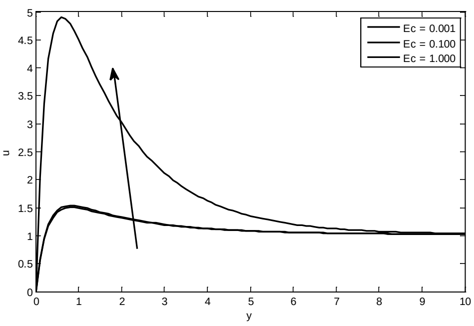

Figure - 3(a): Velocity profiles for different values of Eckert number 'Ec' when Gr = 5, Gc = 2, Pr = 0.7, Sc = 0.22, M = 1.0, k = 0.1.

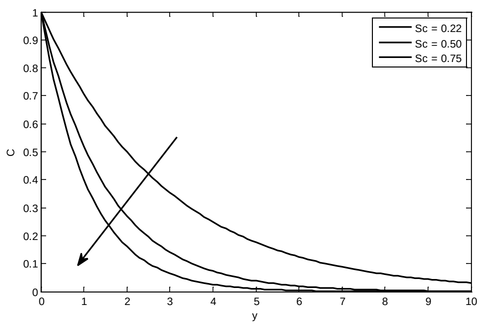

Figure - 4(b): Concentration profile for different values of Schmidt number 'Sc'

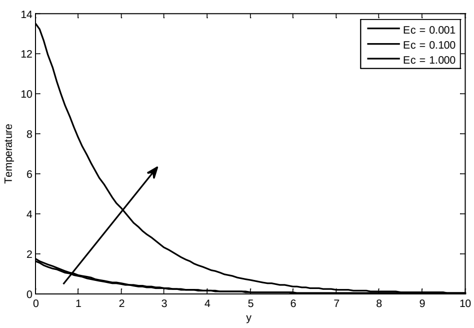

Figure - 3(b): Temperature profiles for different values of Eckert number 'Ec' when Gr = 5, Gc = 2, Pr = 0.7, Sc = 0.22, M = 1.0, k = 0.1.

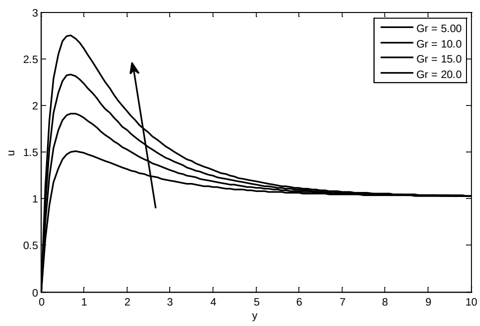

Figure - 5(a): Velocity profiles for different values of Grashof number 'Gr' when Gc = 2, Ec = 0.001, Pr = 0.7, Sc = 0.22, M = 1.0, k = 0.1.

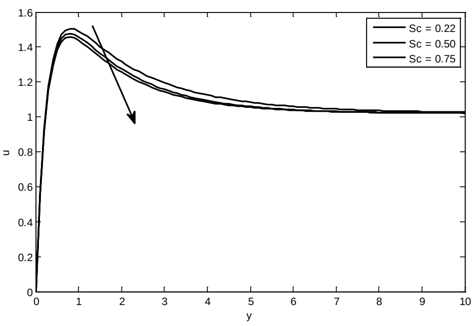

Figure - 4(a): Velocity profiles for different values of Schmidt number 'Sc' when Gr = 5, Gc = 2, Ec = 0.001, Pr = 0.7, M = 1.0, k = 0.1.

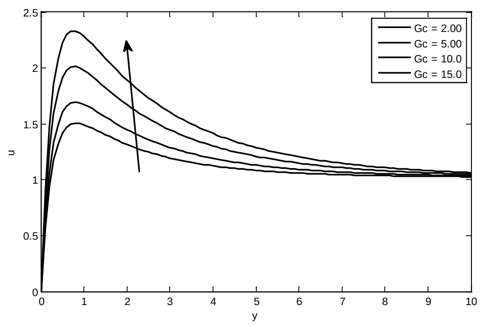

Figure - 5(b): Velocity profiles for different values of modified Grashof number 'Gc' when Gr = 5, Ec = 0.001, Pr = 0.7, Sc = 0.22, M = 1.0, k = 0.1.

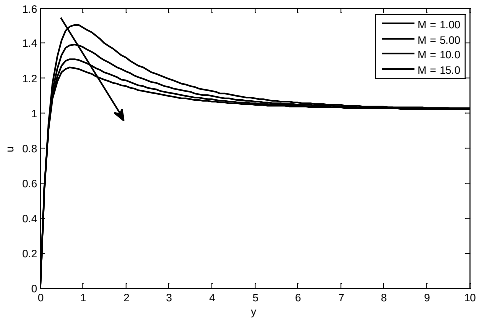

Figure - 6: Velocity profiles for different values of magnetic parameter 'M' when Gr = 5, Gc = 2, Ec = 0.001, Pr = 0.7, Sc = 0.22, k = 0.1.

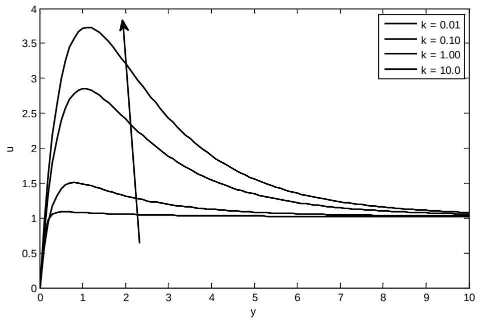

Figure - 7: Velocity profiles for different values of permeability parameter 'k' when Gr = 5, Gc = 2, Ec = 0.001, Pr = 0.7, Sc = 0.22, M = 1.0.

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