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PERISHABLE INVENTORY MODEL WITH FINITE RATE OF REPLENISHMENT HAVING WEIBULL LIFETIME AND TIME DEPENDENT DEMAND

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ABSTRACT

In this paper we develop and analyse an inventory model for deteriorating items with Weibull rate of decay having finite rate of replenishment and time dependent demand with shortages. Using the differential equations, the instantaneous state of inventory at time't', the amount of deterioration etc. are derived. With suitable cost considerations the total cost function and profit rate function are also obtained by maximizing the profit rate function, the optimal ordering and pricing policies of the model are derived. The sensitivity of the model with respect to the parameters is discussed through numerical illustration. It is observed that the deteriorating parameters have a tremendous influence on the optimal selling price and ordering quantity.

Key Words: *Perishability, Instantaneous rate of deterioration, Total cost function, Profit rate function, the optimal ordering and pricing policies.*

AMS Subject Classification Number (2000): *90B05.*

1. INTRODUCTION

Much work has been reported regarding inventory models for deteriorating items in recent years. In many of the inventory systems the major consideration is regarding the pattern of demand and supply. [5] have reviewed inventory models for deteriorating items. They classified the literature by the self life characteristic of the inventory of goods. They also developed on the basis of demand variations and various other conditions or constraints. Various functional forms are considered for describing the demand pattern. [1] has developed an inventory model with the weibull rate of decay having selling price dependent demand. [3], [4], [6] have developed inventory models with shortages taking time proportional demand, i.e, the demand varies as a function of time. Many researchers have developed various inventory models with time dependent demand. [2], [7].

However, no serious attempt was made to develop inventory models for deteriorating items having weibull rate of decay and time dependent demand with finite rate of replenishment, which are very useful in many practical situations arising at oil and natural gas industry, photo chemical industries, chemical processes, etc. Hence, in this paper we develop and analysed an inventory model with the assumption that the lifetime of the commodity is random and follows a three-parameter weibull distribution having demand as a function of time with finite rate of replenishment and shortages. Using differential equations the instantaneous state of on hand inventory is obtained. With suitable cost considerations the total cost function is derived. The optimal ordering policies are also obtained. The sensitivity of the model is analysed though numerical illustration. This model includes some of the earlier models as particular cases for specific or limiting values of the parameters. The inventory model for deteriorating items without shortages having weibull rate of decay and time dependent demand is also analysed as a limiting case.

2. ASSUMPTIONS AND NOTATIONS

We adopt the following assumptions and notations for the models to be discussed.

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2.1. Assumptions

Assumption 1: Replenishment rate is finite.
Assumption 2: Lead time zero
Assumption 3: Shortages are allowed and fully back logged
Assumption 4: A deteriorating item is lost
Assumption 5: The production rate is finite
Assumption 6: T is the fixed duration of a production cycle
Assumption 7: The lifetime of the commodity is random and follows a three parameter weibull distribution of the form $f(t)= \alpha\beta(t-\gamma)^{\beta-1} e^{\alpha(t-\gamma)\beta}$ for $t>\gamma$ where α,β,γ are parameters.

Hence, the instantaneous rate of deterioration h(t) is $h(t)= \alpha\beta(t-\gamma)^{\beta-1}$ (2.1)

2.2 Notations

Q	:	The ordering quantity I one cycle of length T.
A	:	The cost of placing an order.
C	:	The cost price of one unit.
h	:	The inventory holding cost per unit per unit time.
Π	:	The shortage cost per unit for unit time.
r	:	The rate of demand.
n	:	The pattern index.

3. INVENTORY MODEL

Consider an inventory system in which the amount of stock is zero at time t = 0. Replenishment starts at t = 0 and stops at $t = t_1$ The deterioration of the item start after a certain fixed lifetime 'γ'. Since the perishability starts after 'γ' the decrease in the inventory is due to demand during the period (0,γ), demand and deterioration during the period (γ, t_1). During (t_1, t_2) the inventory level gradually decreases mainly to meet up demand and partly due to deterioration. By this process the stock reaches zero level at $t = t_2$ .Now shortages occur and accumulate to the level $t = t_3$. Replenishment starts again at $t = t_3$ and the backlogged demand is cleared at t = T. The cycle then repeats itself after time 'T'.

Let I (t) be the inventory level of the system at time t ($0 \le t \le T$). Then, the differential equations describing the instantaneous state of I(t) over the cycle of length T are,

$$\frac{d}{dt}I(t) = k - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \quad 0 \le t \le \gamma \tag{3.1}$$

$$\frac{d}{dt}I(t) + h(t)\,I(t) = k - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \quad \gamma \le t \le t_1 \tag{3.2}$$

$$\frac{d}{dt}I(t) + h(t)\,I(t) = - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \quad t_1 \le t \le t_2 \tag{3.3}$$

$$\frac{d}{dt}I(t) = - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \quad t_2 \le t \le t_3 \tag{3.4}$$

$$\frac{d}{dt}I(t) = k - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \quad t_3 \le t \le T \tag{3.5}$$

with the initial conditions I (0) = 0, I (t_2) = 0, I (T) = 0.

Solving the above differential equations (3.1) to (3.5), the on hand inventory at time t can be obtained as,

$$I(t) = kt - \frac{rt^{\frac{1}{n}}}{T^{\frac{1}{n}}}, \quad 0 \le t \le \gamma \tag{3.6}$$

$$I(t) = e^{-\alpha(t-\gamma)^{\beta}} \left\{ \int_{\gamma}^{t} \left[k - \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right] e^{-\alpha(u-\gamma)^{\beta}} du + k\gamma - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}} \right\}, \quad \gamma \le t \le t_1 \tag{3.7}$$

$$I(t) = e^{-\alpha(t-\gamma)^{\beta}} \frac{r}{nT^{\frac{1}{n}}} \left[\int_{t_1}^{t^2} t^{\frac{1}{n}-1} e^{\propto(t-\gamma)^{\beta}} du - \int_{t_1}^{t} u^{\frac{1}{n}-1} e^{\propto(u-\gamma)^{\beta}} du \right], \quad t_1 \le t \le t_2 \tag{3.8}$$

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$$I(t)=\frac{r}{t^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}}-t^{\frac{1}{n}}\right],\quad t_2\le t\le t_3 \tag{3.9}$$

$$I(t) = k(t-T) - \frac{r}{t^{\frac{1}{n}}}\left[t^{\frac{1}{n}} - T^{\frac{1}{n}}\right],\ t_3 \le t \le T \tag{3.10}$$

The stock loss due to deterioration in the interval (0, T) is

$$I(t) = kt_1 - \frac{rt^{\frac{1}{n}}}{nT^{\frac{1}{n}}}t_2 \tag{3.11}$$

The backlogged demand at time t is

$$B(t) = (t_3 - t_2)\ \frac{rt^{\frac{1}{n}}}{nt^{\frac{1}{n}}} \tag{3.12}$$

The ordering quantity in a cycle of length T is obtained as

$$Q = Kt_1 + k(T-t_3) \tag{3.13}$$

4. OPTIMAL POLICIES OF THE PERISHABLE INVENTORYMODEL HAVING DETERMINISTIC DEMAND AS POWER FUNCTION OF TIME

For obtaining the optimal polices of the perishable inventory model having deterministic demand as power function of time and with shortages, the expected total cost per unit time k (t_1, t2) is obtained. Since the expected total cost is sum of the setup cost, cost of the units, inventory holding cost and shortage cost k (t_1, t2) is obtained as

$$k(t_1,t_2) = \frac{A}{T} + \frac{C}{T}\left[kt_1 + k(T-t_3)\right] + \frac{h}{T}\int_o^r\left[kt - \frac{r.t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]dt$$
$$+\frac{h}{T}\left\{\int_r^{t_1} e^{-a(t-r)^\beta}\left[\int_r^t\left(k-\frac{r.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}}\right)e^{\propto(t-r)^\beta}dt + k\gamma - \frac{r.\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]dt\right\}$$
$$+.\frac{hr}{n.T^{\frac{1}{n}+1}}\left\{\int_{t_1}^{t_2} e^{-\propto(t-r)^\beta}\left[\int_{t_1}^{t_2} t^{\frac{1}{n}-1}.e^{\propto(t-r)^\beta}dt - \int_{t_1}^{t} u^{\frac{1}{n}-1}e^{\propto(u-r)^\beta}dt\right]dt\right\} + \frac{\pi r}{T^{\frac{1}{n}+1}}\int_{t_2}^{t_3}\left[t^{\frac{1}{n}}{}_2 - t^{\frac{1}{n}}\right]dt$$

Using Tailors series expansion, integrating and neglecting the higher order terms of α and substituting $t_3 = \dfrac{T+t_2}{2}$ and on simplification, we have

$$K(t_1,t_2) = \frac{A}{T} + \frac{ck}{2T}(2t_1 + T - t_2) + \frac{h}{T}k\left[\frac{t_1^2}{2} + \frac{2.\propto}{(\beta+1)(\beta+2)}(t_1-\gamma)^{\beta+2} - \frac{\propto.t_1(t_1-\gamma)^{\beta+1}}{\beta+1}\right]$$
$$-.\frac{hn.r}{T^{\frac{1}{n}+1}}\left\{\frac{\propto}{n\beta+1}\left[\frac{-\gamma^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta+1} - \gamma^{\frac{1}{n}+\beta}(t_1-\gamma)\right] - \frac{\propto\beta\gamma}{n\beta-n+1}\left[\frac{-\gamma^{\frac{1}{n}+\beta}}{\frac{1}{n}+\beta} - \gamma^{\frac{1}{n}+\beta-1}(t_1-\gamma)\right]\right.$$
$$\left.+\propto.\frac{\gamma^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta+1} - \left[\frac{\propto\beta\gamma^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta}\right]\right\} + .\frac{hr}{T^{\frac{1}{n}+1}}\left\{t_2^{\frac{1}{n}+1} - t_1t_{2^{\frac{1}{n}}} - \frac{t_2^{\frac{1}{n}+1}}{\frac{1}{n}+1} + \frac{\propto}{n\beta+1}\left[t_2^{\frac{1}{n}+\beta}(t_2-t_1) - \frac{t_2^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta+1}\right]\right.$$
$$\left.-\frac{\propto\beta\gamma}{n\beta-n+1}\left[t_2^{\frac{1}{n}+\beta-1}(t_2-t_1) - \frac{t_2^{\frac{1}{n}+\beta}}{\frac{1}{n}+\beta}\right] - \frac{\propto t_2^{\frac{1}{n}}}{\beta+1}\left[(t_{2-}\gamma)^{\beta+1} - (t_1-r)^{\beta+1}\right]\right.$$

$$+\propto.\frac{t_2^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta+1}-\propto\beta\gamma\frac{t_2^{\frac{1}{n}+\beta}}{\frac{1}{n}+\beta}\Bigg\}+.\frac{\pi r}{T^{\frac{1}{n}+1}}\left\{\frac{T}{2}.t_2^{\frac{1}{n}}-\frac{1}{2}.t_2^{\frac{1}{n}+1}-\frac{n}{n+1}\left[\left(\frac{T+t_2}{2}\right)^{\frac{1}{n}+1}-t_2^{\frac{1}{n}+1}\right]\right\} \quad (4.1)$$

To find the optimal values of t_1, t_2 equate the first order partial derivatives of K (t_1, t_2) with respect to t_1, t_2 to zero.

By differentiating K (t_1, t_2) with respect to t_1 and equating to zero, we get

$$\frac{ck}{T}+\frac{hk}{T}\left[t_1+\frac{\alpha}{\beta+1}-(t_{1-}\gamma)^{\beta+1}-\propto t_1(t_1-\gamma)^{\beta}\right]-\frac{hr}{T^{\frac{1}{n}+1}}\left[\frac{-\alpha.\gamma^{\frac{1}{n}+\beta}}{n\beta+1}+\frac{\alpha\beta\gamma^{\frac{1}{n}+\beta}}{n\beta-n+1}\right]$$
$$+\frac{hr}{T^{\frac{1}{n}+1}}\left[-t_2^{\frac{1}{n}}-\frac{\alpha}{n\beta+1}t^{\frac{1}{n}+\beta}+\frac{\alpha\beta\gamma}{n\beta-n+1}t_2^{\frac{1}{n}-\beta+1}+\alpha t_2^{\frac{1}{n}}(t_1-r)^{\beta}\right]=0 \quad (4.2)$$

By differentiating K (t_1, t_2) With respect to t_2 and equating to zero, we get

$$\frac{-ck}{2T}+\frac{h.r}{.T^{\frac{1}{n}+1}}\left[\frac{n+1}{n}.t_2^{\frac{1}{n}}-\frac{1}{n}.t_1.t_2^{\frac{1}{n}-1}-t_2^{\frac{1}{n}}+\frac{\alpha}{n\beta+1}\left[(\frac{1}{n}+\beta+1).t^{\frac{1}{n}+\beta}-t_1.t_2^{\frac{1}{n}+\beta-1}(\frac{1}{n}+\beta)-t_2^{\frac{1}{n}+\beta}\right]\right.$$
$$-\frac{\alpha\ \beta\ \gamma}{n\beta-n+1}\left[(\frac{1}{n}+\beta).t_2^{\frac{1}{n}+\beta-1}-(\frac{1}{n}+\beta-1)t_1t_2^{\frac{1}{n}+\beta}-t_2^{\frac{1}{n}+\beta+1}\right]$$
$$-\alpha t_2^{\frac{1}{n}}(t_2-\gamma)^{\beta}-\frac{\alpha}{\beta+1}.\frac{1}{n}t_1^{\frac{1}{n}-1}(t_2-\gamma)^{\beta+1}+\frac{\alpha}{\beta+1}\frac{1}{n}t_2^{\frac{1}{n}-1}(t_1-\gamma)^{\beta+1}+\alpha t_2^{\frac{1}{n}+\beta}$$
$$\left.-\alpha\beta\gamma t_2^{\frac{1}{n}+\beta+1}+\frac{\pi r}{T^{\frac{1}{n}+1}}\left[\frac{T}{2}.\frac{1}{n}.t_2^{\frac{1}{n}-1}-\frac{1}{2}\frac{n+1}{n}.t_2^{\frac{1}{n}}-\frac{1}{2}\left(\frac{T=t_2}{2}\right)^{\frac{1}{n}}-t_2^{\frac{1}{n}}\right]=0 \quad (4.3)$$

Solving equations (4.2) and (4.3) numerically for given values of T, $\alpha, \beta, \gamma, r, n, A, c, h$ and π one can obtain the optimal values t_1^* of t_1, t_2^* of t_2 and also obtain the optimal expected total cost per unit time. Even though T is fixed the expected total cost is a function of T. So the expected cost function is effected by the various values of T. Hence the optimal cycle length can also be obtained by using search methods after substituting the optimal value of t_1^* t_2^* and Q* as initial values.

5. NUMBERICAL ILLUSTRATION

For various values of α, β, γ and T the optimal values of t_1^* t_2^* and Q* are computed by solving the equations (4.2) and (4.3) using the MATHECAD on IBM PC computer. From Table (1.1), it is observed that the decision variables of the system namely the unit selling price, the ordering quantity and the time of stopping the production are influenced by the production rate' K'. As 'K' increases the unit selling price is decreasing, and the optimal time period of production is increasing for fixed values of the parameters. That is, if the capacity utilization of the production process is maximum then the selling price and the ordering quantity are optimal. From Table (1.1) it is observed that the optimal ordering quantity, the time at which shortage occur and the expected total cost per unit time are much influenced by the parameters, various costs and cycle length. It is observed that as cycle.

Table -1.1

Optimum values of production time, time at which shortages occur, total cost and ordering quantity

C	J	R	N	π	k	T	α	β	γ	A	.t_1	.t_2	K	Q
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936
5	0.4	3	2	3	5	10	3	2	0	50	1.319	6.617	16.154	90210
6	0.4	3	2	3	5	10	3	2	0	50	1.466	6.617	18.154	9.314
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936
4	0.5	3	2	3	5	10	3	2	0	50	1.104	5.362	14.470	8.945
4	0.6	3	2	3	5	10	3	2	0	50	1.082	4.973	14.786	8.994
4	0.4	2	2	3	5	10	3	2	0	50	1.310	6.493	13.369	8.576
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936

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4	0.4	4	2	3	5	10	3	2	0	50	0.869	5.403	14.939	9.102
4	0.4	3	1	3	5	10	3	2	0	50	1.399	4.973	14.860	8.524
4	0.4	3	2	3	5	10	3	2	0	50	1.290	5.552	14.154	8.936
4	0.4	3	3	3	5	10	3	2	0	50	0.927	6.718	13.729	9.012
4	0.4	3	2	2	5	10	3	2	0	50	1.349	5.598	14.191	8.714
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.976
4	0.4	3	2	4	5	10	3	2	0	50	0.787	6.041	14.117	9.102
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936
4	0.4	3	2	3	4	10	3	2	0	50	0.945	5.521	12.794	8.726
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936
4	0.4	3	2	3	6	10	3	2	0	50	1.226	6.155	15.514	9.102
4	0.4	3	2	3	5	8	3	2	0	50	1.023	5.722	15.655	8.412
4	0.4	3	2	3	5	9	3	2	0	50	1.082	5.809	14.813	8.216
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936
4	0.4	3	2	3	5	10	2	2	0	50	1.217	6.988	13.866	8.512
4	0.4	3	2	3	5	10	3	2	0	50	1.129	5.882	14.154	8.936
4	0.4	3	2	3	5	10	4	2	0	50	1.063	5.205	14.442	9.124
4	0.4	3	2	3	5	10	3	2.0	0	50	1.129	5.882	14.154	8.936
4	0.4	3	2	3	5	10	3	2.5	0	50	1.278	4.391	17.012	9.126
4	0.4	3	2	3	5	10	3	2.0	0	50	1.293	3.567	24.432	9.234
4	0.4	3	2	3	5	10	3	2	0	50	1.129	2.736	14.154	8.936
4	0.4	3	2	3	5	10	3	2	0.1	50	1.860	3.905	14.213	9.128
4	0.4	3	2	3	5	10	3	2	0.2	50	1.984	3.154	14.245	9.224
4	0.4	3	2	3	5	10	3	2	0	50			14.154	8.936
4	0.4	3	2	3	5	10	3	2	0	75			16.745	12.428
4	0.4	3	2	3	5	10	3	2	0	100			19.245	16.156

Length increases, the optimal ordering quantity and the time at which shortages occur are increasing when the other parameters and costs are fixed. The optimal ordering quantity and the time at which shortages occur are also influenced by the mean lifetime of the commodity. The optimal ordering quantity and the time at which shortages occur are very sensitive to the penalty cost, when other parameters are fixed. When the penalty cost is increasing, the optimal ordering quantity is increasing. The same phenomenon can be visualized in case of the time at which shortages occur. This is very close to the practical situations where one wishes to minimize the shortages when the penalty is more. So by ordering the optimal ordering quantity one can reduce the cost due to shortages and hence can increase the profits. However if the increase in the penalty cost is in proportion to the increase in the cost per unit, then the optimal ordering quantity is an increasing function of the penalty cost. If the cost per unit is much higher than the penalty cost, then the optimal ordering quantity is a decreasing function of the cost per unit. Similarly when other costs and parameters are fixed. The optimal ordering quantity is a decreasing function of the holding cost. However even if the holding cost is much less compared to the penalty cost then the optimal ordering quantity is decreasing function of the holding cost. So by properly choosing the penalty cost one can have the optimal inventory management.

It is also observed that the parameters of the model influencing the mean lifetime of the commodity and hence the rate of deterioration, which results in the change of the optimal ordering quantity and time at which shortage occur. The optimal ordering quantity is a decreasing function of the location parameter γ, when the other parameters and costs are fixed. When γ increases, the time at which shortages occur is increasing. As a result of the decrease in the optimal ordering quantity and increase in the time at which shortages occur, the total expected cost per unit time is decreasing. Since the location and shape parameters of the lifetime of the commodity have a significant influence on the rate of deterioration, there is a need to analyze the influence of these two parameters on the optimal values of their inventory models with respect to their inter-relationship. Eventhough, T the cycle length has vital influence on the optimal ordering quantity and the expected total cost per unit time for deriving the optimal ordering quantity one has to properly estimate the parameters involved in the lifetime of the commodity.

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