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A NOTE ON THE LEVITZKI RADICAL OF A NEAR FIELDS (LR - NF)

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ABSTRACT

In this paper we study and obtain some results on Levitzki radical of a near-field over a defined near-ring earlier. It is known that a near-field N the Levitzki radical L(N) i.e., the sum of all locally nilpotent ideals is the intersection of all the prime ideals P in near-field N such that N / P has zero Levitzki radical. The purpose of these note is to prove that L(N) is the intersection of a certain class of prime ideals called 1-prime ideals. Every 1-prime ideal P is such that N / Phas zero Levitzki radical. We also introduce an 1-prime ideal if and only if N / P has zero Levitzki radical of the nearfield as the intersection all the 1-semi-prime ideals.

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SECTION 1 - INTRODUCTION - PRELIMINARIES AND DEFINITIONS

In this section we study about near-fields called as locally nilpotent ideals and Levitzki radical of near-field N. Also extending about the study of features like l-prime ideals, l-semi-prime ideals, l-system, w-system and l-radical, w-radical of a near-field N.

For that preliminaries and basic definitions required and are as follows:

Definition 1.1: A Near Field N with 1 is a semi simple, or Left semi simple to be precise. If the free left N-module underlying N is a sum of simple N-module.



Fig. 1

Definition 1.2: A Near field N with 1 is simple or left simple to be precise, if N is semi simple and any two simple left ideals (i.e., any two simple left sub near-fields of N) are isomorphic.

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Note 1.3: A near-field N is semi simple if and only if there exists a near-field S and semi simple S-module M of finite length such that $N \cong \text{End}_{S}(M)$.

Note. 1.4: Let N be a semi simple near-field. Then N is isomorphic to a finite direct product Π N_i for all i=1, 2...,n where each N_i is a simple near-field.

Note 1.5: Let N be a simple near-field. Then there exists a division field D and a positive integer n such that $N \cong M_n$ (D).

Definition 1.6: Let N be a near-field with 1. Define radical of N to be the intersection of all maximal left ideals of N. The above defined definition uses left N-modules emphasized by me that η is the left radical of near-field N.



Note 1.7: The radial of a semi simple near-field is zero.

Note 1.8: Let N be a simple near-field. Then N has no non-trivial two sided ideals and its radical is zero.

Definition 1.9: A near-field is an algebraic system, $(N, +, \cdot)$ satisfying (i) (N, +) is a group, (ii) (N, \cdot) is a semi group and (iii) $(x + y) z = xz + yz \forall x, y, z \in N$. we abbreviate $(N, +, \cdot)$ as N a near-field.

Note 1.10: If P and Q are subsets of near-field N, we denote the set $\{pq / p \in P, q \in Q\}$ by PQ. For $n \in \eta$, the definition of Pⁿ is then clear.

Definition 1.11: A normal subgroup I of (N, +) is called an ideal of a near-field N ($I \triangleleft N$) if IN $\subseteq I$ and n(n' + i) - nn' $\in I$ for n,n' $\in N$ and all $i \in I$.



Definition 1.12: P of N is called a prime ideal if for any ideals L and M of N; $LM \subseteq P$ implies either $L \subseteq P$ or $M \subseteq P$.



Definition 1.13: An ideal P of a near-field N is called a semi-prime ideal if for any ideal L of N, $L^2 \subseteq P$ implies $L \subseteq P$.

Definition 1.14: by Bhandari and saxena [1], we call a near-field N locally nilpotent if every finite subset of N is nilpotent. Let us denote the sum of all locally nilpotent ideals in N by L (N).

Definition 1.15: The class of locally nilpotent near-fields is a hereditary radical class.

Definition 1.16: If $L(N) = \bigcap \{P \mid P \text{ is a prime ideal with } L(N/P) = (0) \}$ then it is called the Levitzki radical of near-field N and denoted by L(N).

Definition 1.17: By [01], now we define that, for associate near-fields, L(N) coincides with a certain class of prime ideals called l-prime ideals.

Definition 1.18: By [05], we define the intersection of all the l-semi-prime ideals of a near-field N coincides with L(N). We now extend some of these results to near-fields.

Definition 1.19: A set of elements L of a near-field is called an l-system if to every element $a \in L$ is assigned a finite number of elements $a_1, a_2, a_3, \dots, a_{n(a)}$ in the principal ideal generated by the element a, such that the following condition is satisfied. If $a, b \in L$ then for every n > 1 ($n \in \eta$) there exists a product of $N \ge n$ factors, consisting of a_i 's and b_i 's, which is in L. ϕ is defined to be an l-system.



Definition 1.20: An ideal P in a near-field N is an l-prime ideal if and only if the complement C(P) of P in N is an l-system.



Definition 1.21: A set of elements W of a near-field N is called w-system if to every element $a \in W$ is assigned a finite number of elements $a_{1,a_{2},a_{3},...,a_{n(a)}}$ such that the following is satisfied. If $a \in W$, then for every n > 1 ($n \in \eta$) there exists a product of $N \ge n$ factors, consisting of the a_{i} 's, which is in W. ϕ is defined to be an w-system.

Definition 1.22: An ideal Q of N is an l-semi-prime ideal if and only if the complement C(Q) of Q in near-field N is a w-system.



Fig. 8

Note 1.23: In a Near-field N, every l-prime ideal is an l-semi – prime ideal and every l-semi-prime ideal is a semi-prime ideal.

Definition 1.25: The l-radical [w-radical] l(H) [w(H)] of the ideal H of the near-field N is the set of all elements $r \in N$ with the property that every l-system [w-system] which contains r contains an element of A.

Definition 1.26: the l-radical [w-radical] of the near-field N is l((0)) [w((0))].

SECTION 2 MAIN RESULTS ON THE LEVITZKI RADICAL OF A NEAR-FIELD N

In this section, myself and guide Dr. T V Pradeep Kumar we study and obtained main results by considering, analyzing and extending features cum similarities of the rings with the help of Lemma 1 of vander walt [02]) to the near-fields. By making the necessary adjustments and using similar techniques proof follows for the near-fields mentioned here in this section by us which are very useful for future assumptions over near-rings of near-fields.

Theorem 2.1: Let H be any ideal in the near-field N. Then l(H) [w(H)] is the intersection of all the l-prime [l-semi-prime] ideals which contain H.

Proof: We shall prove the theorem for the l-radical of a near-field N. The proof for w-radical is quite analogous. Let $h \in l(H)$ and suppose $h \in C(K)$ where K is an l-prime ideal containing H. Now $C(K) \cap H = \Phi$ (empty), contradicting the definition of l(H). Thus l(H) is contained in the intersection of all the l-prime ideals containing H in near-field N.

Now $h \notin l(H)$. Hence by the definition of l(H), there exists an l-system L containing h such that $L \cap H = \Phi$. From lemma 1.1 there exists an l-prime ideal K such that $H \subseteq K$ and $H \cap K = \Phi$, i.e., $h \notin K$. thus h cannot be in the intersection of all the l-prime ideals in a near-field N containing H, and this completes the proof.

Lemma 2.2: Let L[W] be an l-system [w-system] in a near-field N, and H be an ideal which does not meet L[W]. Then H is contained in an ideal K which is maximal in the class of ideals not meeting L[W]. K necessarily an l-prime [l-semi-prime] ideal.

Proof: by using lemma 1.1 of N J Groenewald and P C Potgieter [06] the proof follows similarity to that for near-rings in A P J Wander walt,[01 & 02]. This completes the proof.



Fig. 9

Theorem 2.3: Let N be any near-field. L((0)) [w((0))] coincides with the levitzki radical L(N) of the near-field N.

Proof: refer theorem 2 [01].

Theorem 2.4: Let N be any near-field. If Q is an ideal in a near-field N, then Q is 1-semi-prime if and only if N/Q contains no non-zero locally nilpotent ideals.

Proof: suppose Q is 1-semi-prime. Hence C(Q) is a w-system. Let H/Q be any non-zero ideal of N/Q. since H/Q is non-zero there exists an $h \in H$, $h \notin Q$. Because Q is an 1-semi-prime ideal and $h \notin Q$ there exists elements $a_1, a_2, a_3, \dots, a_{n(a)} \in (a)$ such that for every n > 1 there is a product of $N \ge n$ factors consisting of the a_i 's which is not in Q. There thus exists a finite set $\{a_1+Q, a_2+Q, a_3+Q \dots a_{n(a)}+Q\} \subseteq H/Q$ such that $\{a_1+Q, a_2+Q, a_3+Q, \dots a_{n(a)}+Q\}^m \neq Q$ for every m. Hence H/Q is not locally nilpotent.

Now suppose N/Q contains no non-zero locally nilpotent ideals. Let $p \in C(Q)$ be an arbitrary. Since, $(0) \neq (p) / (p) \cap Q \leq N/Q$, it follows from our assumption that $(p) / (p) \cap Q$ is not locally nilpotent. Hence there exists $p_1, p_2, p_{3,..}, p_n \in (p)$, $p_i \notin Q$, such that $\{p_1+Q, p_2+Q, p_3+Q, \ldots, p_{n(a)}+Q\}$ is not nilpotent. Therefore, for every $p \in C(Q)$, we can find elements $p_1, p_2, p_{3,\ldots, m}, p_n \in (p)$ such that for every $n > 1(n \in \eta)$ there exists a product of $N \ge n$ factors consisting of the p_i 's which is on C(Q). Hence C(Q) is a w-system. This completes the proof.

Note 2.5: If $(S_k)_{k \in K}$ is a family of l-semi-prime ideals in a near-field N the $S = \bigcap_{k \in K} S_k$ is also a l-semi-prime ideal in a near-field N.

Note 2.6: any intersection of l-prime ideals is l-semi-prime.

Theorem 2.7: Let N be a near-field. Q is an l-semi-prime ideal in a near-field N if and only if L(Q) = Q.

Proof: \Rightarrow If L(Q) = Q it follows from Theorem 1.2 and the cor. Lemma 1.5 [06] that Q is an l-semi-prime ideal. \Leftarrow suppose now Q is an l-semi-prime ideal of a near-field N. from the definition of l(Q) we have Q \subseteq l(Q).

Furthermore, it follows from Th. 2.3 and Th. 2.4 that $l(Q) \subseteq Q$. This completes the proof.

We now make the following general conclusions. We have the following characterization of the Levitzki radical L(N) of the near-field N over near-rings.

Note 2.8: If N is any near-field, then L(N) coincides with the intersection of all l-semi-prime ideals in N, i.e., L(N) is an l-semi-prime ideal which is contained in every l-semi-prime ideal in a near-field N.

Note 2.9: by Bhandari and saxena [05], L(N) is the smallest ideal H of near-field N such that N / H has no non-zero locally nilpotent ideals.

Corollary 2.10: L(N) = (0) if and only if N has no non-zero locally nilpotent ideals.

Proof: obvious from the definition of L(N).

Theorem 2.11: If N is a near-field and H is any ideal of near-field N, the Levitzki radical of the near-field H is $H \cap L(N)$.

Proof: \Rightarrow Let K be any l-semi-prime ideal in a near-field N. C(K) is a w-system and it is easy to show that C(K) \cap H is a w-system on H. Hence K \cap H is an l-semi-prime ideal in H. From theorem 2.3 it now follows that, if we denote the l-radical of the near-field H by P, then P $\subseteq l(N) \cap H$.

 \Leftarrow (converse) if $h \in H \cap l(N)$, then every 1-system in a near-field N which contains h, also contains 0. Hence $h \in H$ and $H \cap l(N) \subseteq P$. We have therefore, shown that $P = H \cap l(N) = H \cap L(N)$. This completes the proof.

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