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SOME NEW RESULTS ON HARMONIC MEAN GRAPHS

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ABSTRACT

A Graph G = (V, E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels from 1,2….q+1 in such a way that when each edge e = uv is labeled with f(e = uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic Mean labeling of G.

In this paper we prove the Harmonic mean labeling behaviour for some new graphs.

Key words: *Graph, Harmonic mean graph, Path, Cycle, Star graph, Union of graphs, Comb, Crown.*

INTRODUCTION

We begin with simple, finite and undirected graph G = (V, E) with p vertices and q edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G). For standard terminology and Notations we follow Harary [1].

S. Somasundaram and S. S. Sandhya introduced Harmonic mean labeling of graphs in [2] and studied their behaviour in [3], [4] and [5]. In this paper, we investigate Harmonic mean labeling for some new graphs.

We will provide brief summary of definitions and other informations which are necessary for the present investigation.

Definition 1.1: A Graph G with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1, 2…..q+1 in such a way that when each edge e=uv is labeled with f(e = uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)+f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are district. In this case f is called Harmonic mean labeling of G.

Definition 1.2: The union of two graphs G_1 and G_2 is graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup (G_2)$

Definition 1.3: The Corona $G_1 A G_2$ of two graphs G_1 and G_2 is defined as the Graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Definition: 1.4: The graph $P_n A K_{1,m}$ vertex of P_m is obtained by attaching $K_{1,m}$ to each vertex of P_n.

Definition 1.5: The graph $C_n A K_1$ is called a crown.

Definition 1.6: Comb is a graph obtained by joining a single pendant edge to each vertex of a path.

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We shall make frequent reference to the following results.

Theorem 1.7 [2]**:** Any path is a Harmonic mean graph.

Theorem 1.8 [2]: Any cycle is a Harmonic mean graph.

Theorem 1.9 [2]**:** Combs are Harmonic mean graphs.

Theorem 1.10 [3]: Crowns are harmonic mean graphs.

2. MAIN RESULTS

Theorem 2.1: $C_n AK_1 \cup P_n AK_1$) is a Harmonic mean graph.

Proof: Let G = $C_n AK_1 \cup P_n AK_1$ be the given graph.

Let $u_{1,}$ $u_{2,}$ …..u_m, u_{m+1},….u_n and $v_1, v_2 \dots v_m$, $v_{m+1} \dots\dots v_n$ are the vertices of G.

Define a function f: V(G) $\rightarrow$ {1,2….q+1}

by $f(u_i) = 2i$, $1 \le i \le m$

$f(u_i) = 2i-1$, $m+1 \le i \le n$

$f(v_i) = 2i-1$, $1 \le i \le m$

$f(v_i) = 2i$, $m+1 \le i \le n$

Edges are labeled with

$f(u_1u_2) = 2$

$f(u_iu_{i+1}) = 2i+1$, $2 \le i \le m-1$.

$f(u_mu_1) = 4$

$f(u_iu_{i+1}) = 2i$, $m+1 \le i \le n-1$

$f(u_iv_i) = 2i-1$, $1 \le i \le 2$

$f(u_iv_i) = 2i$, $3 \le i \le m$

$f(u_iv_i) = 2i-1$, $m+1 \le i \le n$

Hence G = $C_n AK_1 \cup P_n AK_1$ is a harmonic mean graph.

Example 2.2: A Harmonic mean labeling $C_8 AK_1 \cup P_5 AK_1$ is given below.

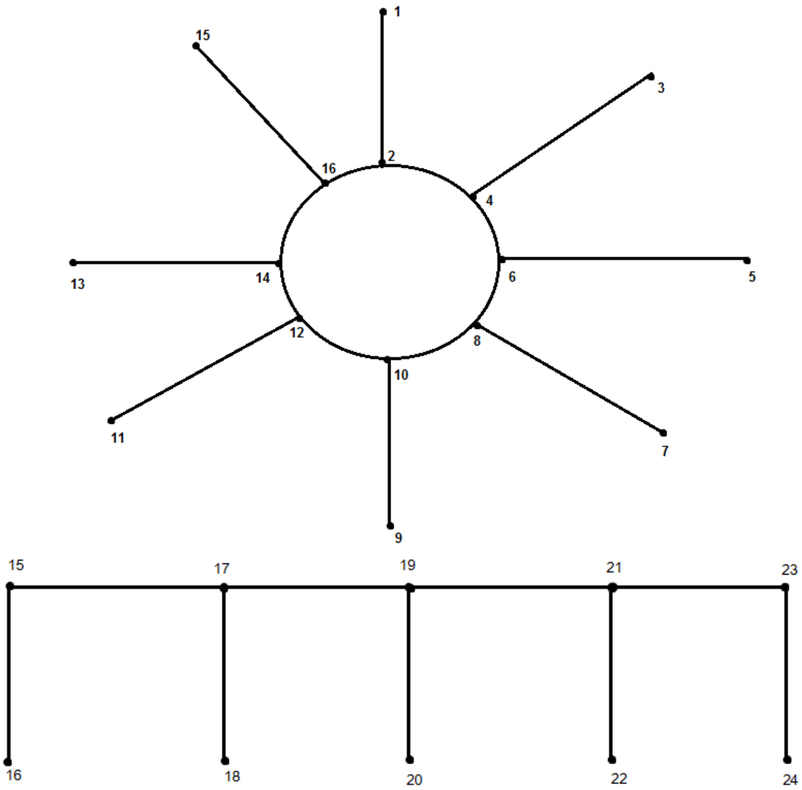

Figure: 1

Next we have

Theorem 2.3: A graph obtained by attaching $K_{1,2}$ at each pendant vertex of a comb is a Harmonic mean graph.

Proof: Let G_1 be a comb and G be the graph obtained by attaching $K_{1,2}$ at each pendant vertex of G_1

Let its vertices be u_i, v_i, v_{i1}, v_{i2}, $1 \le i \le n$

Define a function f: V (G) $\rightarrow$ {1, 2…..q+1}

by $f(u_i) = 4i-2,\ 1 \le i \le n$

$f(v_1) = 3$

$f(v_i) = 4i-3,\ 2 \le i \le n$

$f(v_{11}) = 1$

$f(v_{i1}) = 4i-1,\ 2 \le i \le n$

$f(v_{i2}) = 4i,\ 1 \le i \le n$

Edges are labeled with

$f(u_i u_{i+1}) = 4i,\ 1 \le i \le n-1$

$f(u_1 v_1) = 2$

$f(u_i v_i) = 4i-3,\ 2 \le i \le n$

$f(v_1 v_{11}) = 1$

$f(v_i\ v_{i1}) = 4i-2,\ 2 \le i \le n$

$f(v_i v_{i2}) = 4i-1,\ 1 \le i \le n$

Hence f provide a Harmonic mean labeling for G.

Example 2.4: A Harmonic mean labeling for G is given below

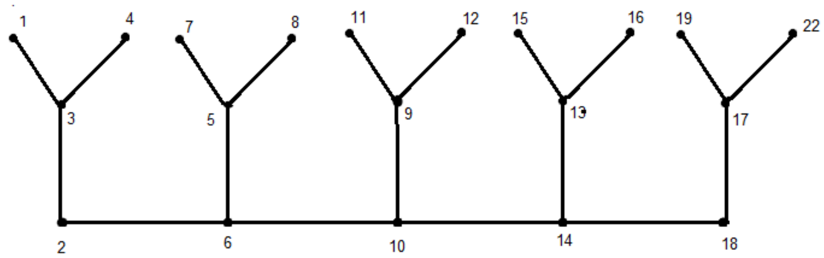

Figure: 2

The same argument gives the following.

Theorem 2.5: A graph obtained by attaching $K_{1,\,2}$ at each pendant vertex of the crown is a Harmonic men graphs.

Proof: Let G_1 be the Crown and G be the graph obtained by attaching $K_{1,\,2}$ at each pendant vertex of the crown. Let it vertices be u_i, v_i, v_{i1}, v_{i2}, $1 \le i \le n$

Define a function f:V(G)$\rightarrow${1,2….q+1}

by $f(u_1) = 2$

$f(u_i) = 4i-2,\ 2 \le i \le n$

$f(v_1) = 4$

$f(v_i) = 4i-2,\ 2 \le i \le n$

$f(v_{11}) = 1$

$f(v_{i1}) = 4i,\ 2 \le i \le n$

$f(v_{i2}) = 4i+1,\ 1 \le i \le n$

Edges are labeled with

$f(u_1 u_2) = 3$

$f(u_i u_{i+1}) = 4i+1,\ 2 \le i \le n-1$

$f(u_n u_1) = 4$

$f(u_i v_i) = 4i-2,\ 1 \le i \le n$

$f(v_1 v_{11}) = 1$

$f(v_i v_{i1}) = 4i-1,\ 2 \le i \le n$

$f(v_1\ v_{12}) = 5$

$f(v_i v_{i2}) = 4i,\ 2 \le i \le n$

Example 2.6: The following figure shows the Harmonic mean labeling behaviour for G.

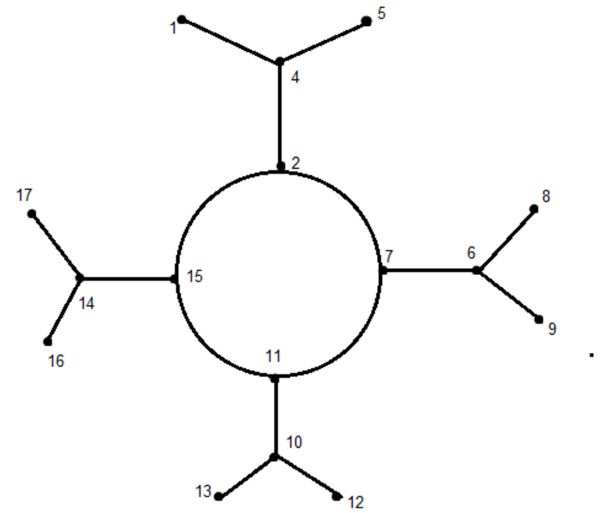

Figure: 3

Now we prove the following

Theorem 2.7: A Graph obtained by attaching a triangle at each pendant vertex of a comb is a Harmonic mean graph.

Proof: Let G^1 be a comb and G be a graph obtained by attaching a triangle at each pendant vertex of a comb. Let its vertices be u_i, v_i, u_{i1}, $u_{i2,}$ $1 \le i \le n$.

Define a function f: V (G) $\rightarrow$ {1, 2…..q+1}
by $f(u_i) = 5i\text{-}2$, $1 \le i \le n$
$f(v_i) = 5i\text{-}1$, $1 \le i \le n$
$f(v_{i1}) = 5i\text{-}4$, $1 \le i \le n$
$f(v_{i2}) = 5i\text{-}3$, $1 \le i \le n$
Edges are labeled with
$f(u_iu_{i+1}) = 5i$, $1 \le i \le n\text{-}1$
$f(u_iv_i) = 5i\text{-}1$, $1 \le i \le n$
$f(v_i\, v_{i1}) = 5i\text{-}3$, $1 \le i \le n$
$f(v_iv_{i2}) = 5i\text{-}2$, $1 \le i \le n$
$f(v_{i1}v_{i2}) = 5i\text{-}4$, $1 \le i \le n$

Hence f provide a Harmonic mean labeling for G.

Example 2.8: Harmonic mean labeling pattem of G is shown below.

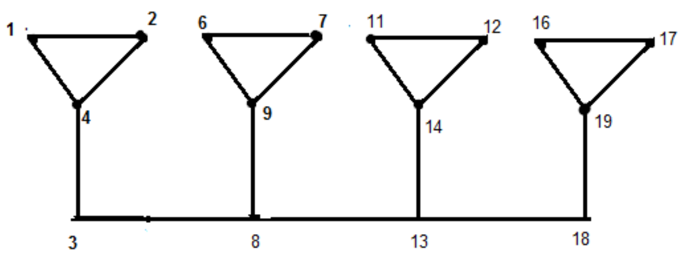

Figure: 4

Consequently we have the following

Theorem 2.9: A graph obtained by attaching a triangle at each pendant vertex of a crown is Harmonic mean graph.

Proof: Let G_1 be a crown and G be a graph obtained by attaching a triangle at each pendant vertex of a crown.

Let the vertices of the graph be u_i, v_i v_{i1}, v_{i2}, $1 \le i \le n$

Define a function f: V(G) $\rightarrow$ {1,2……….q+1}
by $f(u_1) = 2$

$f(u_i) = 4i-1,\ 2 \leq i \leq n$
$f(v_1) = 4$
$f(v_i) = 4i-2,\ 2 \leq i \leq n$
$f(v_{11}) = 1$
$f(v_{i1}) = 4i,\ 2 \leq i \leq n$
$f(v_{i2}) = 4i+1,\ 1 \leq i \leq n$

Edges are labeled with
$f(u_1u_2) = 3$
$f(u_i u_{i+1}) = 4i+1,\ 2 \leq i \leq n-1,$
$f(u_nu_1) = 6$
$f(u_iv_i) = 4i-2,\ 1 \leq i \leq n$
$f(v_1v_{11}) = 1$
$f(v_i v_{i1}) = 5i-2\ 2 \leq i \leq n.$
$f(v_1v_{12}) = 3$
$f(v_iv_{i2}) = 5i-1,\ 2 \leq i \leq n$
$f(v_{11}v_{12}) = 2$
$f(v_{i1}v_{i2}) = 5i-3,\ 2 \leq i \leq n$

Thus f provide a Harmonic mean labeling for G.

Example 2.10: The Harmonic mean labeling pattern of G is shown in the following figure

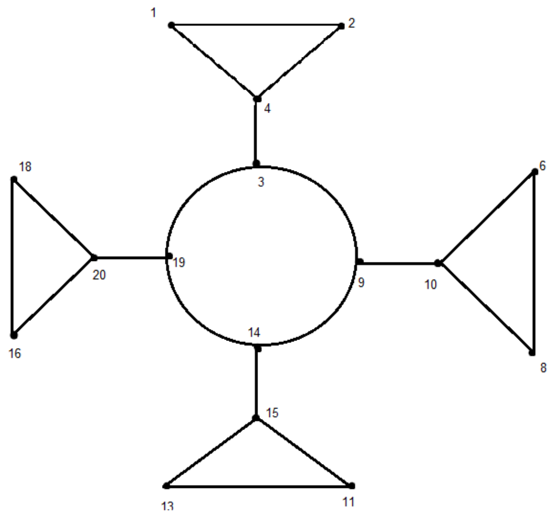

Figure: 5

Next we have

Definition 2.11: Let v be a vertex of a graph G. Then the duplication of v is a graph G(v) obtained from G by adding a new vertex v^1 with $N(v^1) = N(v)$

Definition 2.12: Let e = uv be an edge of G. Then the duplication of an edge e = uv is a graph G(uv) obtained by adding a new edge u'v' s.t
$N(u') = N(u) \cup \{v'\} - \{v\}$ and
$N(v') = N(v) \cup \{u'\} - \{u\}$

Now we prove the following

Theorem 2.13: The graph obtained by duplicating each edge in a cycle C_n is a Harmonic mean graph.

Proof: Let G be a graph obtained by duplicating each edge in a cycle C_n.

Let $u_1, u_2 \ldots\ldots\ldots\ldots\ldots\ldots u_nu_1$ be the given cycle.

Let $e_1, e_2 \ldots\ldots\ldots e_n$ are the edges obtained by duplication.

Define a function $f: V(G) \rightarrow \{1,2\ldots\ldots q+1\}$

by $f(u_i) = 3i$, $1 \le i \le n$

$f(v_1) = 1$

$f(v_i) = 3i-1$, $2 \le i \le n$

Edges are labeled with $f(u_1u_2) = 4$

$f(u_iu_{i+1}) = 3i+2$, $2 \le i \le n-1$

$f(u_nu_1) = 6$

$f(u_iv_i) = 3i-1$, $1 \le i \le 2$

$f(u_iv_i) = 3i$, $3 \le i \le n$

$f(u_1v_2) = 3$

$f(u_iv_{i+1}) = 3i+1$, $2 \le i \le n-1$

$f(u_nv_1) = 1$

Hence G is a Harmonic mean graph.

Example 2.14: The Harmonic mean labeling pattern of G is given below

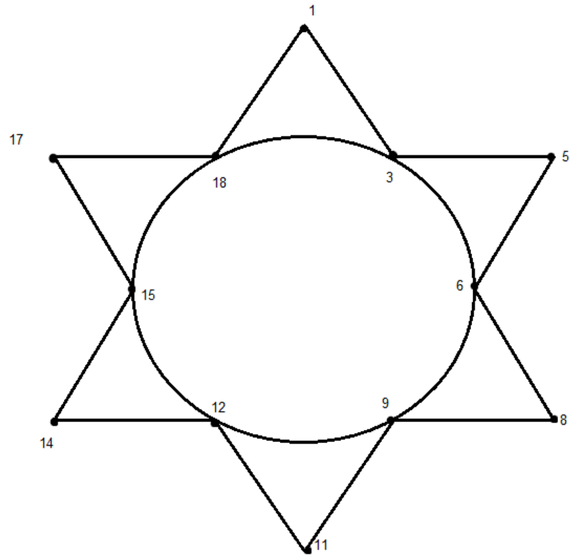

Figure: 6

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