

## ON GROUPS OF GEOMETRIC FIGURES

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### ABSTRACT

In this paper we introduce the basic of definitions of groups for geometric figures; we describe some of geometric figures by using the properties of groups. The classification of some geometric figures by using the properties of some algebraic structures.

Key words: group, geometric figures.

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### INTRODUCTION AND BACKGROUND

The introduction of coordinates by Rene Descartes and Concurrent developments of algebra marked a new stage for geometry, since geometric figures, could now be represented analytically.

Any group is a set A together with a binary operation \* on A, such that the following axioms are satisfied:

- 1) Closure: for all a, b in A, a\*b in A.
- 2) Associative: for all a, b, c in A  $a^*(b^*c) = (a^*b)*c$ .
- 3) There is an element e in A such that  $e^a = a^e = a$  for all a in A.
- 4) For all a in A, there is an element b in A with the property that a\*b=b\*a=e (the element b is an inverse of a with respect to \*)

A group A is abelian if a\*b=b\*a for all a, b in A.

We call a group A is cyclic if there is some element a in A such that  $A = \{a^n, a \text{ in } A, n \text{ integer}\}$ , a is called generator of A.

**Definition(1)**: An isomorphism of a group A with a group B is a one to one function f mapping from A onto B such that for all a and b in A,

f(ab) = f(a) f(b)

The group A and B are then isomorphic. The usual notation is  $A \approx B$  [2].

Theorem (2): Any infinite cyclic group A is isomorphic to the group Z of integers under addition [2].

**Definition** (3): A path or arc in a space X is a continuous map of some closed interval into X. The images of the end points of the interval are called the end points of the path, and the path is said to join it's the end points [3].

Definition (4): A space X is called path wise connected if any two points of X can be joined by a path [3].

A path wise connected space is connected, but the converse is not true.

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**Definition** (5): A path is called closed or loop, if the initial and terminal points are the same. The loop is said to be based at the common end point.

The set of all loops based at any point x of X is a group; this group is called the fundamental group and denoted by  $\pi$  (X, x) [3].

**Proposition** (1): If X is path wise connected then the fundamental group of X denoted by  $\pi$  (X) [3].

**Definition (6):** The n-dimension manifold is a Hausdorff space (space satisfies the  $T_2$  separation axiom) such that each point has an open neighborhood homeomorphic to the open n-dimensional disc  $U^n = \{x \text{ in } \mathbb{R}^n : |x| < 1\}$ . where n is positive integer [3].

**Definition** (7): A graph is a finite set of points in space, called the vertices of the graph, some pairs of vertices being joined by arcs, called the edges of the graph [1].

**Definition (8):** A connected graph is a graph that is one piece, whereas one which splits into several pieces is disconnected [3].

**Definition (9):** A connected graph which contains no loops is called a tree [1].

**Definition** (10): The n-simplex is the simplest geometric figure determined by a collection of n+1 point in Euclidean space  $R^n$ . 0-simplex is a point, 1-simplex is the closed segment with end-points, 2-simplex is the triangle with three vertices and 3-simplex is the solid tetrahedron with four vertices [5].

**Definition** (11): An n-face of a simplex is a subset of the set of vertices of the simplex with order n+1.the faces of an n-simplex with dimension less than n are called its proper faces[5].

**Definition** (12): A simplicial complex K is a finite set of simplices satisfying the following conditions:

1) If s  $\in$  K and t < s (t is a face of s) then t  $\in$  K.

2) Intersection condition: if  $s \in K$  and  $t \in K$  then  $s \cap t$  is either empty or else a face both of s and of t.

The dimension of K is the largest dimension of any simplex in K [1].

**Definition** (13): Let  $S_n = (v^0 ... v^n)$  be an n-simplex, an orientation for  $s_n$  is a collection of ordering for the vertices consisting of a particular ordering and all even permutation of it [1].

**Definition** (14): The boundary of  $s_n$  is defined as the (n-1)-chain of K over Z given by :  $\partial(s_n) = s_{n-1} 0 + s_{n-1} 1 + \ldots + s_{n-1} n$ . where  $s_{n-1}$  i is an (n-1)-face of  $s_n[5]$ .

We call an n-chain a cycle if its boundary is zero, and denote the set of n-cycles of K over Z by A<sub>n</sub> [5].

**Definition(15):** The quotient group  $H_n = A_n / B_n$  is the n-dimensional homology group of the complex K over Z, where  $A_n$  is the set of n-cycles of K over Z and  $B_n$  is the boundary of n-cycles ( $B_n$  is a subgroup of  $A_n$ ).

When the cycles are 1-cycles, is called the first homology group and are 2-cycles is called the second homology group [1].

**Notation:**  $H_n(k)$  is measure the number of independent n-dimensions of holes in k, where  $0 \le n \le \dim k$ .

#### THE MAIN RESULTS

Aiming to our study, we will introduce the geometric figure G.

Definition: The geometric figure G is a subset from the space X, where G lies in X. Consider geometric figure (1)



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The fundamental group describe this figure is the fundamental group  $\pi$  (e<sub>0</sub>) for any number of vertices, where  $\pi$  (e<sub>0</sub>) is the trivial group with one element but if we have only v<sub>0</sub>, then we can construct a chain group from v<sub>0</sub> it is  $\lambda_0 v_0$ ,

If we have  $\{v_0, v_1\}$ , then  $\lambda_0 v_0 + \lambda_1 v_1$  form 0-chain of generators  $v_0, v_1$ , also for  $\{v_0, v_1, \dots, v_n\}$ , then  $\lambda_0 v_0 + \lambda_1 v_1 + \dots + \lambda_n v_n$  is a 0-chain of n+1 generators.



Fig. - 2

The fundamental group for Fig. (2) is  $\pi$  (Z) at S<sup>1</sup>, since the fundamental group of the circle is isomorphic to (Z,+) the additive group of integers. And  $\pi$  (e<sub>0</sub>) at both P and C, which C a path-connected space with a trivial fundamental group.



Fig. - 3

The fundamental group for Fig. (3) is  $\pi$  (e<sub>0</sub>) for all figures P,C,L and Y.



The fundamental group for Fig. (4) Is  $\pi$  (Z) at S<sup>1</sup> and H<sub>2</sub>(S<sup>2</sup>)  $\approx$  Z, where H<sub>2</sub> is second Homology group for the sphere.



Fig. - 5

The fundamental group for Fig. (5) Are  $\pi$  (e<sub>0</sub>) at T<sub>1</sub>, where T<sub>1</sub> is a tree and  $\pi$  (Z) at T<sub>2</sub>.Also can form a 1-chain for T<sub>1</sub> and T<sub>2</sub>.





The fundamental group for Fig. (6) is  $\pi$  (K)  $\approx$  Z ,where K is cylinder and  $\pi$  (T)  $\approx$  Z+Z ,which T is the tours and T =S<sup>1</sup>×S<sup>1</sup>.





The fundamental group for Fig. (7) is  $H_2$  (M) for all figures W, which is the regular cube for 2-dimantion, X which is the regular tetrahedron and  $S^2$ 

which is a sphere.



Fig. - 8

In Fig. (8) the tape  $M^0$  is a simple connected region,  $\pi (M^0) \approx 0$ , and  $M^1$  is a double connected region, then  $\pi (M^1) \approx Z$ .



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In Fig. (9)  $M^2$  is a triple connected region of two holes in a tape,  $\pi$  ( $M^2$ ) = Z+Z, and  $M^3$  is multiple connected regions,  $\pi$  ( $M^3$ ) = Z + Z + Z, so on,



Fig.- 10

Proposition: The number of holes describes the connectedness of the manifolds.

**Proof:** in Fig. (8), a tape  $M^0$  is a pathwise connected then it's a simple connected.

The tape  $M^1$  is contains one hole, then the hole is divide the boundary of the tape into two boundaries, so is called double connected.

In Fig. (9),  $M^2$  is contains two holes then the boundary of tape is divide into three boundaries.

It's called of triple boundaries.

If the tape contains more than two holes then is called multiple connected. See Fig. (10).





Proposition: The limit of the increasing the holes in the tape will give n filters, which is a 1-graph. See Fig. (11)

**Proof:** the tape M<sup>n</sup> in Fig. (11) is contains a finite numbers of n holes,

Consider the tape is a new type of a graph called a finite net tape graph, which is 1-dimension graph.

Then the limit of net tape graph is a filter graph of 1-dimensions graph.

**Proposition:** the algebra of this type in following figure will be

 $\pi (M^{\infty}) = Z + Z + Z + \dots \infty.$ 



We have one hole  $h_1$ , then there are 1-cycle  $c_1$  round  $h_1$ :

Any circle  $c_2$  can be shrinked until to be a point except  $c_1$  then  $\pi$  (M<sup>1</sup>)  $\approx$  Z

If there are two holes  $h_1,h_2$  see Fig.(13) ,we obtain  $c_1,c_2$  are two cycles round two holes  $h_1,h_2$  respectively,  $\pi$  (M<sup>2</sup>)  $\approx$  Z+Z, and by the increasing of holes in the tape then the fundamental group of M<sup> $\infty$ </sup> is Z+Z+... $\infty$ .





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