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BIANCHI TYPE-VI_0 DARK ENERGY MODEL WITH VARIABLE EoS PARAMETER IN A SAEZ-BALLESTER SCALAR-TENSOR THEORY OF GRAVITATION

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ABSTRACT

Bianchi Type-VI_0 cosmological model is investigated in a Saez-Ballester scalar-tensor theory of gravitation. Three different time-dependent skewness parameters along spatial directions are introduced to represent the deviation of pressure from isotropy. Exact solutions of the field equations are obtained by assuming a special law of variation for the mean Hubble's parameter, which yields a constant value of deceleration parameter. Some physical and geometrical properties of the model are discussed.

Keywords: *Bianchi type-VI_0 model, Dark energy, Scalar-tensor theory.*

1. INTRODUCTION

The discovery of the accelerating expansion of the universe is realized as a major breakthrough in modern cosmology and astrophysics. The present accelerating phase of the universe is confirmed by the cosmological observations of the type Ia supernovae (SN Ia) [13, 20]. This acceleration, popularly known as dark energy, is characterized by the negative pressure and the positive energy density and it violates the strong energy condition. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests 73% content of the universe in the form of dark energy, 23% in the form of dark matter and the rest 4% in the form of the usual baryonic matter as well as radiation.

Conventionally, the dark energy models are characterized by an equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ which is not necessarily constant, where ρ is the energy density and p is the pressure of the fluid. In recent years, many authors [1, 2, 3, 14, 15, 16, 23, 27] have obtained dark energy models in general relativity with variable EoS parameter.

In the last few decades, several modifications of Einstein's theory of gravitation have been developed. Among them scalar-tensor theories proposed by Brans and Dicke [6], Saez and Ballester [22], Nordtvedt [12], Wagoner [28], Ross [21], Dunn [7], Barber [4] are important. Saez and Ballester [22] developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. Inspite of the dimensionless character of the scalar field, an anti-gravity regime appears. This theory also suggests a possible way to solve the missing-matter problem in non-flat FRW cosmologies.

The field equations given by Saez and Ballester [22] for the combined scalar and tensor field are

$$R_{ij} - \frac{1}{2} g_{ij} R - w\emptyset^n \left(\emptyset_{,i}\emptyset_{,j} - \frac{1}{2} g_{ij}\, \emptyset_{,k}\emptyset^{,k}\right) = -T^i_{ij} \tag{1}$$

where w is dimensionless coupling constant and n is an arbitrary constant.

The scalar field satisfies the equation

$$2\emptyset^n \emptyset^i_{;i} + n\emptyset^{n-1}\emptyset_{,k}\emptyset^{,k} = 0 \tag{2}$$

It can be easily proved that

$$T^{ij}_{;j} = 0 \tag{3}$$

which are the consequences of the field equations. $T^{ij}_{;j} = 0$ is the energy-momentum tensor of matter. Here comma and semi-colon designate partial and covariant differentiation respectively with respect to cosmic time t.

Cosmological models in the Saez-Ballester scalar-tensor theory of gravitation have been studied by Singh and Agrawal [25], Ram and Tiwari [17], Singh and Ram [24], Mohanty and Sahu [9], Tripathi *et al.* [26], Reddy *et al.* [19] and many. Recently, Rao *et al.* [18], Naidu *et al.* [10, 11] have studied Bianchi type models in Saez-Ballester scalar-tensor

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theory of gravitation with variable EoS parameter. Motivated by the above works, in the present paper, we have examined Bianchi type-VI$_0$ dark energy model with variable EoS parameter in the framework Saez-Ballester scalar-tensor theory of gravitation.

2. METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and totally anisotropic Bianchi type-VI$_0$ line element, given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2 \tag{4}$$

where the metric potentials A, B and C are functions of t alone.

The energy momentum tensor of fluid is defined as

$$T_i^j = diag\,[T_0^0, T_1^1, T_2^2, T_3^3] \tag{5}$$

We can parametrize the energy momentum tensor given in (5) as follows :

$$\begin{aligned} T_i^j &= diag\,[\,\rho, -p_x, -p_y, -p_z\,] \\ &= diag\,[\,1, -\omega_x, -\omega_y, -\omega_z\,]\rho \\ &= diag\,[\,1, -\omega, -(\omega+\gamma), -(\omega+\delta)\,]\rho \end{aligned} \tag{6}$$

where ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and $\omega_x, \omega_y, \omega_z$ are the directional EoS parameters along x, y and z axes respectively ; $\omega(t) = \frac{p}{\rho}$ is the deviation free EoS parameter of the fluid. The deviation from isotropy is parametrized by setting $\omega_x = \omega$ and then introducing skewness parameters γ and δ which are the deviations from ω, respectively along the y and z axes .

The Saez-Ballester field equations (1)-(3) for the metric (4) with the help of (6) lead to the following system of equations

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} + \frac{w}{2}\,\phi^n\dot{\phi}^2 = \rho \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} - \frac{w}{2}\phi^n\dot{\phi}^2 = -\omega\rho \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} - \frac{w}{2}\phi^n\dot{\phi}^2 = -(\omega+\gamma)\rho \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} - w\phi^n\dot{\phi}^2 = -(\omega+\delta)\rho \tag{10}$$

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0 \tag{11}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{12}$$

Here, and also in what follows, an overhead dot designates ordinary differentiation with respect to t .

Integrating equation (11), we obtain

$$B = lC \tag{13}$$

where l is a constant of integration which can be taken as unity without any loss of generality so that

$$B = C \tag{14}$$

The average scale factor S and the spatial volume V are given by

$$S = (AB^2)^{\frac{1}{3}} \tag{15}$$

$$V = S^3 = AB^2 \tag{16}$$

The generalized mean Hubble's parameter H is defined as

$$H = \frac{\dot{S}}{S} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{17}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of x, y and z axes respectively.

The expansion scalar θ, shear scalar σ^2 and the mean anisotropy parameter A_m are given by

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{18}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{19}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{20}$$

where $\Delta H_i = H_i - H \ (i = 1, 2, 3)$

3. SOLUTIONS OF THE FIELD EQUATIONS

The field equations (7)-(10) and (12) are a system of five independent equations with seven unknown parameters $A,\ B = C,\ \rho,\ \omega,\ \gamma,\ \delta$ and $\emptyset$. Two additional constraints are required to obtain explicit solutions of these field equations.

We apply the special law of variation for Hubble's parameter proposed by Bermann [5] which yields a constant value of deceleration parameter. The constant deceleration parameter q is given by

$$q = -\frac{S\ddot{S}}{(\dot{S})^2} = \text{constant} \tag{21}$$

Here we take the constant as negative so that it represents an accelerating model of the universe.

From equation (21), we get

$$S = (at + b)^{\frac{1}{1+q}} \tag{22}$$

where $a \neq 0$ and b are constants of integration. This equation implies that the condition of expansion is $1 + q > 0$.

We assume that the shear scalar σ is proportional to expansion scalar θ which gives

$$A = B^m \tag{23}$$

where m is a constant.

From equations (22) and (23), we obtain

$$A = (at + b)^{\frac{3m}{(m+2)(1+q)}} \tag{24}$$

$$B = C = (at + b)^{\frac{3}{(m+2)(1+q)}} \tag{25}$$

Hence the metric (4) takes the form

$$ds^2 = -dt^2 + (at + b)^{\frac{6m}{(m+2)(1+q)}}dx^2 + e^{2x}(at + b)^{\frac{6}{(m+2)(1+q)}}dy^2 + e^{-2x}(at + b)^{\frac{6}{(m+2)(1+q)}}dz^2 \tag{26}$$

4. PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

Equation (26) represents Bianchi type-VI$_0$ dark energy model with variable EoS parameter in Saez-Ballester scalar-tensor theory of gravitation. The model has point type singularity at $t = -b/a$ when $m > 0$ and $m + 2 > 0$ and $-2 < m < 0$. The model has cigar type singularity at $t = -b/a$ when $m < 0$ and $m + 2 > 0$, $m > 0$ and $m + 2 < 0$, $m < 0$ and $m + 2 < 0$ (MacCalum [8]).

The spatial volume

$$V = (at + b)^{\frac{3}{1+q}} \tag{27}$$

Hubble's parameter

$$H = \frac{a}{(1+q)(at+b)} \tag{28}$$

The expansion scalar

$$\theta = \frac{3a}{(1+q)(at+b)} \tag{29}$$

Shear scalar

$$\sigma^2 = \frac{3(m-1)^2a^2}{(m+2)^2(1+q)^2(at+b)^2} \tag{30}$$

The mean anisotropy parameter

$$A_m = \frac{(2m+1)^2+9}{3(m+2)^2} \tag{31}$$

Also, we have

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} \neq 0, m > 1 \tag{32}$$

The scalar field

$$\emptyset = \left[\emptyset_0\left(\frac{n+2}{2}\right)(at+b)^{\frac{q-2}{1+q}}\right]^{\frac{2}{n+2}} \tag{33}$$

The energy density

$$\rho = \frac{9a^2(2m+1)}{(m+2)^2(1+q)^2(at+b)^2} - \frac{1}{(at+b)^{\frac{6m}{(m+2)(1+q)}}} + \frac{wk_1^2}{2(at+b)^{\frac{6}{1+q}}} \tag{34}$$

The EoS parameter of the fluid

$$\omega = \frac{1}{\rho}\left[\frac{wk_1^2}{2(at+b)^{\frac{6}{1+q}}} - \frac{27a^2-6a^2(m+2)(1+q)}{(m+2)^2(1+q)^2(at+b)^2} - \frac{1}{(at+b)^{\frac{6m}{(m+2)(1+q)}}}\right] \tag{35}$$

Skewness parameters

$$\gamma = \frac{1}{\rho}\left[\frac{3a^2\{(m-1)(m+2)(1+q)-3(m^2+m-2)\}}{(m+2)^2(1+q)^2(at+b)^2} + \frac{2}{(at+b)^{\frac{6m}{(m+2)(1+q)}}}\right] \tag{36}$$

and

$$\delta = \frac{1}{\rho}\left[\frac{3a^2\{(m-1)(m+2)(1+q)-3(m^2+m-2)\}}{(m+2)^2(1+q)^2(at+b)^2} + \frac{2}{(at+b)^{\frac{6m}{(m+2)(1+q)}}} + \frac{wk_1^2}{2(at+b)^{\frac{6}{1+q}}}\right] \tag{37}$$

We observe that the spatial volume is zero at $t = -b/a$ and it increases as t increases. This implies that the universe starts evolving with zero volume at $t = -b/a$ and expands with cosmic time t which is a big-bang scenario. The parameters $H, \theta, \sigma^2, \rho, \omega, \gamma$ and δ diverge at $t = -b/a$ and they become zero for large values of t. Since $\frac{\sigma^2}{\theta^2} \neq 0$ for all values of m except for $m = 1$, hence the model is anisotropic except for $m = 1$. The scalar field in the model vanishes for $t = -b/a$, and it is infinitely large for large values of t. Also since $1 + q > 0$, we obtain an accelerating model of the universe.

5. CONCLUSION

A Bianchi type-VI$_0$ dark energy model with variable EoS parameter is investigated in Saez-Ballester scalar-tensor theory of gravitation.We have obtained the solutions of the field equations with the help of variation law for Hubble's parameter given by Bermann. The spatial scale factors and volume of this model vanish at $t = -b/a$. The EoS parameter and the skewness parameter appear to be time-dependent. The present model gives an accelerating cosmological model of the universe.

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