International Journal of Mathematical Archive-4(4), 2013, 208-209
Available online through www.ijma.info ISSN 2229 – 5046

ALMOST CONTINUOUS MAPS

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(Received on: 12-03-13; Revised & Accepted on: 02-04-13)

ABSTRACT

This paper gives the conditions under which a closed map becomes almost continuous.

Key words and phrases: *Continuous, almost continuous, closed graph, fibers, closed fibers, compact fibers, H-closed topological space*

AMS Subject Classification Code: *54C08.*

INTRODUCTION

Study of almost continuous maps can be found in [3],[4],[5],[6],[7],[8],[9],[10].Using the characterizations of H-closed spaces L. Herrington and P.E.Long [4] have proved that if f: X $\rightarrow$ Y has strongly closed graph, where Y is H-closed , then f is weakly continuous(in fact almost continuous). Fuller [1] has proved that if f: X $\rightarrow$ Y has closed graph where Y is compact, then f is continuous. Garg and Goel [2] have proved that if f: X $\rightarrow$ Y is closed with compact fibers, where X is T_2, then f has closed graph. In this paper, we have given an analogue of the results of L. Herrington, P. E. Long and R. V. Fuller. The present paper is an endeavour to find the conditions under which a closed map becomes almost continuous.

Throughout, by a space we shall mean a topological space. No separation axioms are assumed and no map is assumed to be continuous or onto unless mentioned explicitly; cl(A) will denote the closure of the subset A in the space X. If X and Y are topological spaces, we say that a mapping f: X $\rightarrow$Y has a closed graph if G(f) = {(x, f(x)): x$\in$X} is a closed subset of X x Y. A function f:X $\rightarrow$ Y is said to be continuous at x $\in$ X if for each open set V containing f(x) , there exists an open set U containing x such that f(U) $\subset$V. A function f: X$\rightarrow$ Y is said to be almost continuous if $f^{-1}(V)$ is open for every regular open subset V of Y equivalently, $f^{-1}(F)$ is closed for every regular closed subset F of Y. A space X is said to be H-closed if for every open cover $\{U_\alpha : \alpha\in\Delta\}$ there exists a finite subfamily $\{U_{\alpha i} : i = 1,2,\ldots,n\}$ such that $\cup\{cl(U_{\alpha i}): 1\le i \le n\} = X$. If f: X$\rightarrow$Y is a map, then the fibers are the sets $f^{-1}(y)$ where y$\in$Y. The primary purpose of this paper is to give the conditions under which a closed map becomes almost continuous when the range space is H-closed.

MAIN RESULTS

The following lemma will be used in the proof of our theorem.

Lemma: A regular closed subset of H-closed space is closed.

Theorem 1: Let f: X $\rightarrow$ Y be closed with compact (closed) fibers where X is T_2 (regular) and Y is H-closed. Then f is almost continuous.

Proof: We prove the non-parenthesis part. The proof of parenthesis part is similar.

Let K be a regular closed subset of Y and let $x\notin f^{-1}(K)$. Then $x\notin f^{-1}(y)$, y$\in$K. Since X is T_2, there exist open sets U_x and U_y containing x and $f^{-1}(y)$ respectively such that $U_x \cap U_y =\emptyset$. Since f is closed, there exists an open set V_y

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containing y such that $f^{-1}(V_y) \subset U_y$. Now K being a regular closed subset of H-closed space is H-closed by above lemma, Therefore $\{V_y\}$, $y \in K$ being an open cover of K, has a finite subfamily$\{V_{yi}$:i=1…n$\}$ such that $K \subset \bigcup_{i=1}^{n} cl(V_{y_i})$ and

so $f^{-1}(K) \subset G_y = \bigcup_{i=1}^{n} clU_{y_i}$ as $f^{-1}(V_y) \subset U_y \Rightarrow V_y \subset f(U_y)$

$\Rightarrow cl(V_y) \subset \text{cl } f(U_y) \subset f\left(cl(U_y)\right) \Rightarrow f^{-1}(cl(V_y) \subset cl(U_y)$ .Then $U = \bigcap_{i=1}^{n} (U_{x_1})$ is an open set containing x and $U \cap f^{-1}(K) = \emptyset$ implying thereby that $x \notin cl(f^{-1}(K))$. Hence f is almost continuous.

The following example shows that the condition of H-closed space on the range space can not be dropped.

Example: Let X be the closed unit interval with the usual subspace topology and let Y be the closed unit interval using the usual open sets together with the set $\left\{\frac{1}{2}\right\}$ as a subbasis. The identity map i: $X \rightarrow Y$ is closed with closed as well as compact fibers but not weakly continuous and hence not almost continuous. Here the domain space X is T_2 as well as regular but Y is not H-closed.

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Source of support: Nil, Conflict of interest: None Declared