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ON FUZZY b-I^+ SETS AND ITS CONTINUOUS FUNCTIONS

F. Nirmala Irudayam[1*] & I. Arockiarani[2]

[1]Department of Mathematics with Computer Applications, Nirmala College for Women, Coimbatore, India
[2]Department of Mathematics, Nirmala College for women, Coimbatore, India

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ABSTRACT

In this paper we propose the concept of a new class of set defined in simple extended fuzzy ideal topological space as fuzzy b-I^+-open sets. We also introduce other sets in this fuzzy simple extended ideal topological space. Furthermore we intend to define its continuity as fuzzy b-I^+-continuity and investigate some of its properties. Also we study these in relation to some other types of sets and functions.

1. INTRODUCTION

In 1963 Levine [11] introduced the concept of simple extension of a topology τ as τ (B) = $\{(B\cap O)\cup O' / B\notin\tau\}$. In general topology, the notion of ideal was introduced by Kuratowski [10], Vaidyanathaswamy [22], [23] and several other authors. Janković & Hamlet [9] have done an extensive study on the importance of ideal in general topology. Further Abd El-Monsef *et al* [2] investigated I- open sets and I-continuous functions. Casksu Guler and Aslim [4] have introduced the concept of bI-sets and bI-continuous functions and further research was done by Metin Akdag [16] on these sets. Nirmala and I. Arockiarani [19] have introduced the concept of bI-open sets in the light of simple extension topology. The fundamental concept of a fuzzy set was introduced by Zadeh [28]. Subsequently, Chang [6] defined the notion of fuzzy topology. An alternative definition of fuzzy topology was given by Lowen [12]. Yalvac [25] introduced the concepts of fuzzy set and function on fuzzy spaces. Malakar [15] introduced the concepts fuzzy semi-irresolute and strongly irresolute functions. Hatir & Jafari [8] and Nasef & Hatir [18] defined fuzzy semi-I-open set and fuzzy pre-I-open set via fuzzy ideal. Mahmoud [13] and Sarkar [21] presented some concepts in the ideal setting to the fuzzy setting and studied its properties. Sarkar [21] introduced the notions of fuzzy ideal and fuzzy local function in fuzzy set theory. Mahmoud [13], [14] investigated on the application of fuzzy set theory. Yuksel, Kara and Acikoz [27] defined fuzzy b-I-open sets and obtained several characterizations of fuzzy b-I-continuous functions.

Here, we introduce the concept of fuzzy b-I-open set and fuzzy b-I-open functions and study their properties in fuzzy simple extended ideal topological spaces.

2. PRELIMINARIES

Throughout this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by $A \le X$, with a membership function defined in the sense of Zadeh [28].

A subfamily τ of I^X is called a fuzzy topology due to Chang [6]. Moreover, the pair (X,τ) is a fuzzy topological space, on which no separation axioms are assumed unless explicitly stated. The fuzzy closure, the fuzzy interior and the fuzzy complement of any set in A in (X, τ) are denoted by Cl(A), Int(A) and 1_X -A, respectively. A fuzzy set which is a fuzzy point Wong [24] with support $x \in X$ and the value $\lambda\in (0, 1]$ will be denoted by x_λ . The value of a fuzzy set A for some $x\in X$ will be denoted by A(x).

Also, for a fuzzy point x_λ and a fuzzy set A we shall write $x_\lambda \in A$ to mean that $\lambda\le A(x)$. For any two fuzzy sets A and B in (X, τ), $A \le B$ if and only if $A(x) \le B(x)$ for all $x\in X$. Pao-ming & Ying-ming [20] defined a fuzzy set in (X, τ) to be quasi–coincident with a fuzzy set B, denoted by AqB, if there exists $x\in X$ such that $A(x)+B(x)>1$. Pao-ming & Ying-Ming [20], Chankraborty & Ahsanullah [5] called a fuzzy set V in (X, τ) to have a q-neigbourhood (q-nbd, for short) of a fuzzy point x_λ if and only if there exists a fuzzy open set U such that $x_\lambda qU \le V$. We will denote the set of all q-nbd of x_λ in (X,τ) by $N_q(x_\lambda)$.

***Corresponding author:** F. Nirmala Irudayam[1*]*
[1]Department of Mathematics with Computer Applications, Nirmala College for Women, Coimbatore, India

Definition 2.1: A fuzzy subset A of a fuzzy topological space (X,τ) is said to be
(i) fuzzy α-open set [3] if $A \leq Int(Cl(Int(A)))$
(ii) fuzzy pre-open set [3] if $A \leq Int(Cl(A))$
(iii) fuzzy semi-open set [1] if $A \leq Cl(Int(A))$,
(iv) fuzzy β-open set [7] if $A \leq Cl(Int(Cl(A)))$.
(v) fuzzy b-open[27] if $A \leq Cl(Int(A)) \vee Int(Cl(A))$.

A nonempty collection of fuzzy sets I of a set X is called a fuzzy ideal [14], [21] on X if and only if
(1) $A \in I$ and $B \leq A$, then $B \in I$ (heredity),
(2) If $A \in I$ and $B \in I$, then $A \vee B \in I$ (finite additivity).

The triple (X,τ, I) means fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ. For (X, τ, I) the fuzzy local function of $A \leq X$ with respect to τ and I is denoted by $A^*(\tau, I)$ (briefly A^*) [21]. The fuzzy local function $A^*(\tau, I)$ of A is the union of all fuzzy points x_λ such that if $U \in N_q(x_\lambda)$ and $E \in I$ then there is at least one $y \in X$ for which $U(y)+A(y)-1> E(y)$ [21]. Fuzzy closure operator of a fuzzy set A in (X,τ, I) is defined as $Cl^*(A)=A \vee A^*$ [21]. In (X, τ, I), the collection $\tau^*(I)$ means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta=\{U-E: U \in \tau, E \in I\}$ as a base [21].

Definition 2.2: A fuzzy subset A of a fuzzy ideal topological space (X,τ, I) is said to be
(i) fuzzy I-open [17] if $A \leq Int(A^*)$
(ii) fuzzy α-I-open set [26] if $A \leq Int(Cl^*(Int(A)))$
(iii) fuzzy pre-I-open set [18] if $A \leq Int(Cl^*(A))$
(iv) fuzzy semi-I-open set [8] if $A \leq Cl^*(Int(A))$
(v) fuzzy β-I-open set [26] if $A \leq Cl(Int(Cl^*(A))))$.
(vi) fuzzy b-I-open set [27] if $A \leq Cl^*(Int(A)) \vee Int(Cl^*(A))$.

The family of all fuzzy I-open (resp. fuzzy α-I-open, fuzzy pre-I-open, fuzzy semi-I-open, fuzzy β-I-open) sets is denoted by FIO(X) (resp. FαIO(X), FPIO(X), FSIO(X), FβIO(X)). The complement of a fuzzy I-open set (resp. fuzzy α-I-open set, fuzzy pre-I-open set, fuzzy semi-I-open set, fuzzy β-I-open set, fuzzy b-I-open set) is said to be fuzzy I-closed set (resp. fuzzy α-I-closed set, fuzzy pre-I-closed set, fuzzy semi-I-closed set, fuzzy β-I-closed set, fuzzy b-I-closed).

For any subset A of X, the interior of A is the same as the interior in usual topology and the closure of A is newly defined as a combination of the local function [30] in ideal topology and simple extension. In SEITS the new local function [30] is defined as $A^{+*}=\{x \in X/ U \cap A \notin I$ for each neighbourhood U of x in $\tau^+\}$ and $cl^{+*}(A) = A \cup A^{+*}$. Also $\tau^{+*} = \{V/ cl^{+*}(X\backslash V) = X\backslash V\}$, where $\tau^+ \subseteq \tau^{+*}$.

Definition 2.3: A subset A of X in SEITS (X,τ^+,I) is said to be
(1) αI^+ open [19] if $A \subseteq int(cl^{+*}(int(A)))$,
(2) semiI^+ open [19] if $A \subseteq cl^{+*}(int(A))$,
(3) pre I^+ open [19] if $A \subseteq int(cl^{+*}(A))$,
(4) βI^+ open [19] if $A \subseteq cl^{+*}(int(cl^{+*}(A)))$,
(5) bI^+open [19] if $A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int(A))$.

The class of all semiI^+ open (resp. preI^+-open , αI^+ open) sets in X are denoted by $SI^+O(X,\tau^+,I)$ (resp.$PI^+O(X,\tau^+,I)$, $\alpha I^+O(X,\tau^+,I)$).

Lemma 2.4: [27]: Let A and B be fuzzy subsets of a fuzzy ideal topological space (X, τ, I). Then we have
(i) If $A \leq B$, then $A^* \leq B^*$,
(ii) If $U \in \tau$, then $U \wedge A \leq (U \wedge A)^*$,
(iii) A^* is fuzzy closed in (X, τ, I) .

3. FUZZY b-I^+-OPEN SETS

Definition 3.1: A subset A of a fuzzy topological space (X, τ^+) is said to be

(i) fuzzy α^+ open if $A \leq Int(Cl^+(int(A)))$.

(iii) fuzzy pre^+ open if $A \leq Int(Cl^+(A))$.

(iv) fuzzy $semi^+$ open if $A \leq Cl^+(int(A))$.

(iv) b^+-open if $A \leq Cl^+(Int(A)) \vee Int(Cl^+(A))$.

(v) β^+-open if $A \leq Cl(Int(cl^+(A)))$

The family of all fuzzy α^+ open sets(resp. fuzzy pre^+ open, fuzzy $semi^+$ open ,fuzzy b^+-open, fuzzy β^+-open) sets in (X, τ^+) is denoted as $F\alpha^+O(X)$(resp. $FP^+O(X)$, $FS^+O(X)$, $FB^+O(X)$ and $F\beta^+O(X)$).

Definition 3.2: A subset A of a fuzzy simple extended ideal topological space (X, τ^+, I) is said to be

(i) fuzzy I^+ open if $A \leq int(A^{+*})$

(ii) fuzzy αI^+ open if $A \leq Int(Cl^{+*}(int(A)))$.

(iii) fuzzy $preI^+$ open if $A \leq Int(Cl^{+*}(A))$.

(iv) fuzzy $semiI^+$ open if $A \leq Cl^{+*}(int(A))$.

(v) fuzzy b-I^+-open if $A \leq Cl^{+*}(Int(A)) \vee Int(Cl^{+*}(A))$.

(vi) fuzzy βI^+-open if $A \leq Cl(Int(cl^{+*}(A)))$

The family of all fuzzy I^+ open sets(resp.fuzzy αI^+ open , fuzzy $preI^+$ open, fuzzy $semiI^+$ open, fuzzy b-I^+-open, fuzzy βI^+-open) sets in (X, τ^+, I) is denoted as $FI^+O(X)$ (resp. $F\alpha I^+O(X)$, $FPI^+O(X)$, $FSI^+O(X)$, $FbI^+O(X)$ and $F\beta I^+O(X)$).

Theorem 3.3: In a fuzzy simple extended topological space (X, τ^+), the following statements hold:

a) Every fuzzy open set is fuzzy b^+-open,
b) Every fuzzy α^+-open set is fuzzy b^+-open,
c) Every fuzzy $semi^+$-open set is fuzzy b^+-open,
d) Every fuzzy pre^+-open set is fuzzy b^+-open.
f) Every fuzzy b^+-open set is fuzzy β^+-open.

Proof: The proof is obvious.

However the converses need not be true.

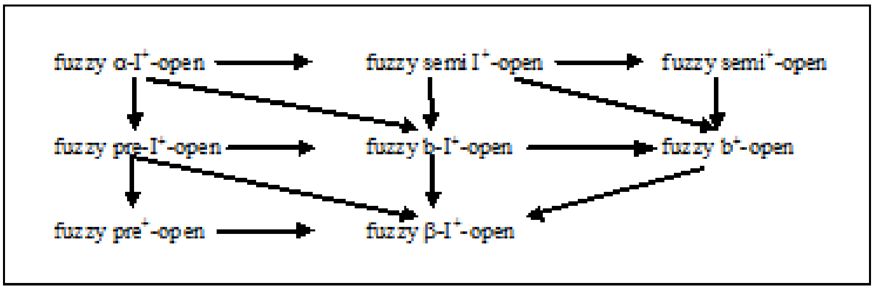
Theorem 3.4: In a fuzzy simple extended ideal topological space (X, τ^+, I), the following statements hold:

(i) Every fuzzy I^+-open set is fuzzy b-I^+-open,
(ii) Every fuzzy open set is fuzzy b-I^+-open,
(iii) Every fuzzy α-I^+-open set is fuzzy b-I^+-open,
(iv) Every fuzzy semi-I^+-open set is fuzzy b-I^+-open,
(v) Every fuzzy pre-I^+-open set is fuzzy b-I^+-open.
(vi) Every fuzzy bI^+-open set is fuzzy b^+-open.

Proof: The proof is obvious.

However the converses need not be true.

From the above theorems we have the following pictorial representation in FSEITS.



Definition 3.5: A fuzzy subset A of a FSEITS (X, τ+, I) is said to be fuzzy $*^+$-perfect if $A=A^{+*}$.

Theorem 3.6: For a fuzzy simple extended ideal topological space (X, τ+, I) and a fuzzy subset A of X, we have

a) If $I=\{0_X\}$ and $A\in FP^+O(X)$ then $A\in FbI^+O(X)$,

b) If $I=P(X)$ and $A\in FbI^+O(X)$, then $A=Int(A)$,

c) If $Int(A)=0_X$ and $A\in FbI^+O(X)$, then $A\in FPI^+O(X)$,

d) If A is fuzzy $*^+$-perfect and $A\in FbI^+O(X)$, then $A\in FSI^+O(X)$.

Proof:

(a) Let $I=\{0_X\}$, then we know that $A^{+*}=Cl^+(A)$. Let A be a fuzzy pre$^+$-open set. Therefore,

$A \le Int(Cl^+(A))=Int(A^{+*}) \le Int(Cl^{+*}(A)) \le Int(Cl^{+*}(A))\vee Cl^{+*}(Int(A))$. Hence A is fuzzy b-$I^+$-open.

(b) It is obvious that if $I=P(X)$, then $A^{+*}=0_X$ and $Cl^{+*}(A)=A$.

Let A be a fuzzy b-I^+-open set, then

$A \le Cl^{+*}(Int(A))\vee Int(Cl^{+*}(A))$

$= Int(A)\vee Int(A) = Int(A)$.

Hence A is fuzzy open.

(c) and (d) are obvious.

Theorem 3.7: The union of two fuzzy b-I^+open sets are fuzzy b-I^+open,in a fuzzy simple extended ideal topological space.

Proof: Consider A and B to be two fuzzy b-I^+open sets in (X, τ+, I),then

We have $A\vee B \le [Cl^{+*}(Int(A))\vee Int(Cl^{+*}(A))]\vee[Cl^{+*}(Int(B))\vee Int(Cl^{+*}(B))]$

$= [Cl^{+*}(Int(A))\vee Cl^{+*}(Int(B))]\vee[Int(Cl^{+*}(A))\vee Int(Cl^{+*}(B))]$

$\le [Cl^{+*}(Int(A)\vee Int(B))]\vee[Int(Cl^{+*}(A)\vee Cl^{+*}(B))]$

$\le Cl^{+*}(Int(A\vee B))\vee Int(Cl^{+*}(A\vee B))$

Hence the proof.

Lemma 3.8: Let A and B be fuzzy subsets of a fuzzy simple extended ideal topological space (X, τ+, I). Then we have

(i) If $A \le B$, then $A^{+*} \le B^{+*}$,

(ii) If $U\in\tau^+$, then $U\wedge A \le (U\wedge A)^{+*}$,

(iii) A^{+*} is fuzzy closed in (X, τ+, I).

Theorem 3.9: Let (X, τ+,I) be a fuzzy ideal topological space and A, B be fuzzy subsets of X. Then the following properties hold:

i) If $U_\alpha\in FbI^+O(X)$ for each $\alpha\in\Delta$, then $\vee\{U_\alpha : \alpha\in\Delta\}\in FbI^+O(X, \tau^+, I)$,

ii) If $A\in FbI^+O(X)$ and $B\in\tau^+$, then $A\wedge B\in FbI^+O(X, \tau^+, I)$

iii) If $A\in F\alpha I^+O(X)$ and $B\in FbI^+O(X)$, then $A\wedge B\in FbI^+O((X, \tau^+, I)$

Proof:

a) Consider $U_\alpha \in FbI^+O(X)$, then we have

$U_\alpha \leq Cl^{+*}(Int(U_\alpha))\vee Int(Cl^{+*}(U_\alpha))$ for each $\alpha\in\Delta$. By Lemma 3.8, we obtain

$$\bigvee_{\alpha\in\Delta} U_\alpha \leq \bigvee_{\alpha\in\Delta} [Cl^{+*}(Int(U_\alpha))\vee Int(Cl^{+*}(U_\alpha))]$$

$$= \bigvee_{\alpha\in\Delta} [(Int(U_\alpha)\vee(Int(U_\alpha))^{+*})\vee Cl^{+*}(Int(U_\alpha))]$$

$$\leq [\bigvee_{\alpha\in\Delta} (Int(U_\alpha))^{+*}]\vee Int(\bigvee_{\alpha\in\Delta} U_\alpha)\vee Int(\bigvee_{\alpha\in\Delta} (Cl^{+*}(U_\alpha)))$$

$$\leq (Int(\bigvee_{\alpha\in\Delta} U_\alpha))^{+*}\vee Int(\bigvee_{\alpha\in\Delta} U_\alpha)\vee Int(Cl^{+*}(\bigvee_{\alpha\in\Delta} U_\alpha))$$

$$= Cl^{+*}(Int(\bigvee_{\alpha\in\Delta} U_\alpha))\vee Int(Cl^{+*}(\bigvee_{\alpha\in\Delta} U_\alpha))$$

and hence $\bigvee_{\alpha\in\Delta} U_\alpha \in FbI^+O(X)$.

b) Let $A\in FbI^+O(X)$ and $B\in\tau^+$, then

$$A\wedge B \leq [Cl^{+*}(Int(A))\vee Int(Cl^{+*}(A))]]\wedge B$$
$$= [Cl^{+*}(Int(A))\wedge B]\vee[Int(Cl^{+*}(A))\wedge B]$$
$$= [(Int(A)\vee((Int(A))^{+*})\wedge B] \vee [Int(A\vee A^{+*})\wedge B]$$
$$= [(Int(A)\wedge B)\vee((Int(A))^{+*}\wedge B)]\vee[Int(A\vee A^{+*})\wedge Int(B)] \leq [(Int(A)\wedge B)\vee(Int(A)\wedge B)^{+*}]\vee Int[(A\vee A^{+*})\wedge B]$$
$$= [(Int(A)\wedge Int(B))\vee(Int(A)\wedge Int(B))^{+*}]\vee Int[(A \wedge B)\vee(A^{+*}\wedge B)]$$
$$\leq [Int(A\wedge B)\vee(Int(A\wedge B))^{+*}]\vee Int[(A\wedge B)\vee(A\wedge B)^{+*}]$$
$$= Cl^{+*}(Int(A\wedge B))\vee Int(Cl^{+*}(A\wedge B)).$$

Therefore $A\wedge B\in FbI^+O(X)$.

c) Proof is obvious.

Lemma 3.10: Let (X, τ^+, I) be a fuzzy simple extended ideal topological space and A, B be fuzzy subsets of X, such that $B \leq A$. Then $B^{+*}(\tau^+|A, I|A) = B^{+*}(\tau^+, I)\wedge A$

Theorem 3.11: Let (X, τ^+, I) be a fuzzy simple extended ideal topological space and if $U\in\tau^+$ and $A\in FbI^+O(X)$, then $U\wedge A \in FbI^+O(U, \tau^+|U, I|U)$.

Proof: We know that $Int_U(V) = Int(V)$ for any fuzzy subset V of U, since $U\in\tau^+$. Using this condition and Lemma 3.10, we have the result.

Definition 3.12: Let (X, τ^+, I) be a fuzzy simple extended ideal topological space. A fuzzy subset of X is called fuzzy b-I^+closed if its complement is fuzzy b-I^+open. The family of all fuzzy bI^+closed sets in $(X, \tau^+, I)$is denoted by $FbI^+C(X)$.

Theorem 3.13: Let A be a fuzzy subset of a fuzzy simple extended ideal topological space (X, τ^+, I) .Then A is fuzzy b-I-closed if $Cl^{+*}(Int(A)) \wedge Int(Cl^{+*}(A)) \leq A$.

Proof: Since $A \in FbI^+C(X)$, we have, $1_X - A \in FbI^+O(X)$.

Thus, $1_X - A \leq Cl^{+}*(Int(1_X - A)) \vee Int(Cl^{+}*(1_X - A))$

$\leq Cl(Int(1_X - A)) \vee Int(Cl(1_X - A))$

$= (1_X - (Int(Cl(A)))) \vee (1_X - (Cl(Int(A))))$

$\leq (1_X - Int(Cl^{+}*(A)))) \vee (1_X - (Cl^{+}*(Int(A))))$.

Hence we obtain $Cl^{+*}(Int(A)) \wedge Int(Cl^{+*}(A)) \leq A$.

Remark 3.14: For fuzzy subset A of (X, τ^+,I), we have $1_X - Int(Cl^{+}*(A)) \neq Cl^{+}*(Int(1_X - A))$

Corollary 3.15: Let A be a fuzzy subset in (X, τ^+, I) such that $1_X - Int(Cl^{+}*(A)) = Cl^{+}*(Int(1_X - A))$.

Then A is fuzzy b-I^+closed if and only if $Cl^{+*}(Int(A)) \wedge Int(Cl^{+*}(A)) \leq A$.

Corollary 3.16: Let (X, τ^+,I) be a fuzzy simple extended ideal topological space.

a) If $A \in FbI^+C(X)$ and $B \in \tau^+$, then $A \vee B \in FbI^+C(X)$,

b) If $A \in FbI^+C(X)$ and $B \in F\alpha I^+C(X)$, then $A \vee B \in FbI^+C(X)$.

Proof: It is clear from Theorem 3.9 and Definition 3.12.

4. FUZZY b-I^+CONTINUOUS FUNCTIONS

Definition 4.1: A function f :(X, τ^+, I) → (Y, σ) is called fuzzy b-I^+continuous if the inverse image of each fuzzy open set in Y is fuzzy b-I^+open in (X, τ^+, I).

Theorem 4.2: A function f :(X, τ^+, I) →(Y, σ) is fuzzy b-I^+ continuous if and only if for each fuzzy point x_α in X and each fuzzy open set A≤Y containing $f(x_\alpha)$, there exists $B \in FbI^+O(X)$ containing x_α such that f(B)≤A.

Proof: Necessity: Let $x_\alpha \in X$ and A be any fuzzy open set in Y containing $f(x_\alpha)$. Let us consider $B=f^{-1}(V)$, then since the function f is fuzzy b-I^+ continuous and by Definition 4.1, we have B is fuzzy b-I^+open set containing x_α and f(B) ≤A.
Sufficiency:

Let A be any fuzzy open set in Y containing $f(x_\alpha)$. Then by hypothesis there exists B x_α fuzzy b-I^+open such that $f(B\, x_\alpha) \leq A \Rightarrow B\, x_\alpha \leq f^{-1}(A)$.

Let $\vee B\, x_\alpha = f^{-1}(A)$. Therefore $f^{-1}(A)$ is fuzzy b-I^+open by Theorem 3.9 (a). Hence f is fuzzy b-I^+continuous.

Remark 4.3: Every fuzzy continuous function is fuzzy b-I continuous.

Definition 4.4: A function f: (X, τ^+,I) →(Y,σ) is called fuzzy b^+continuous if the inverse image of each fuzzy open set in Y is fuzzy b^+open in (X, τ^+, I)

Remark 4.5: Every fuzzy b-I^+continuous function is fuzzy b^+continuous.

Theorem 4.6: A function f: (X, τ^+,I) →(Y, σ) is fuzzy b-I^+continuous if the graph function g:X →XxY of f is fuzzy b-I^+continuous.

Proof: Let A be a fuzzy open set in Y. Then 1_X xV is fuzzy open set in XxY. Since g is fuzzy b-I^+continuous

$g^{-1}(1_X \times V) \in FbI^{+}O(X)$.

Thus $f^{-1}(V) = 1_X \wedge f^{-1}(V) = g^{-1}(1 \times V)$, $f^{-1}(V) \in FbI^{+}O(X)$.

Hence f is fuzzy b-I^+continuous.

Theorem 4.7: Let f: $(X, \tau^+, I) \to (Y, \sigma)$ be a fuzzy b-I-continuous function and $U \in \tau^+$. Then the restriction $f|_U$ is fuzzy b-I^+continuous.

Proof: Let $V \in \sigma$, then $f^{-1}(V)$ is fuzzy b-I^+open in X since f is fuzzy b-I^+continuous. By Theorem 3.9(b), $f^{-1}(V) \wedge U$ is fuzzy b-I^+open in U. Hence $(f|U)^{-1}(V) = U \wedge f^{-1}(V)$ is fuzzy b-I^+open in U. Therefore we have that f |U is fuzzy b-I^+continuous.

Theorem 4.8: If f: $(X, \tau^+, I) \to (Y, \sigma)$ is fuzzy b-I^+continuous and $g:(Y, \sigma^+, J) \to (Z, \phi)$ is fuzzy continuous, then $g o f$: $(X, \tau^+, I) \to (Z, \phi)$ is fuzzy b-I^+continuous.

Proof: Let W be any fuzzy open set in Z. Since g is fuzzy continuous, $g^{-1}(W)$ is fuzzy open in Y. Since f is fuzzy b-I^+continuous, $f^{-1}(g^{-1}(W))$ is fuzzy b-I^+open in X. Hence g o f is fuzzy b-I^+continuous.

Definition 4.9: A function f: $(X, \tau^+, I) \to (Y, \sigma)$ is called fuzzy b-I^+ irresolute if the inverse image of each fuzzy b^+open set of Y is fuzzy b-I^+open in (X, τ^+, I).

Remark 4.10: Every fuzzy b-I^+ irresolute function is fuzzy b-I^+continuous.

Definition 4.11: A function f: $(X, \tau^+, I) \to (Y, \sigma)$ is called fuzzy b^+irresolute if the inverse image of each fuzzy b^+open set of Y is fuzzy b^+open in (X, τ^+, I).

Theorem 4.12: If f: $(X, \tau^+, I) \to (Y, \sigma)$ is fuzzy b-I^+irresolute and g :$(Y, \sigma^+, J) \to (Z, \phi)$ is fuzzy b^+irresolute, then $g o f$:$(X, \tau^+, I) \to (Z, \phi)$ is fuzzy b-I^+irresolute.

Proof: Consider W to be any fuzzy b^+open set in Z. Since g is fuzzy b^+irresolute, $g^{-1}(W)$ is fuzzy b^+open in Y. Since f is fuzzy b-I^+irresolute, $f^{-1}(g^{-1}(W))$ is fuzzy b-I^+open in X. Hence $g o f$ is fuzzy b-I^+irresolute.

Theorem 4.13: Let f: $(X, \tau^+, I) \to (Y, \sigma)$ be a function, then the following statements are equivalent:

i) f is fuzzy b-I^+irresolute,

ii) For each fuzzy point x_λ in X and each fuzzy b^+open set V in Y containing $f(x_\lambda)$, there exists a fuzzy b-I^+open set U containing x_λ such that $f(U) \leq V$,

iii) $f^{-1}(V) \leq Cl^{+*}(Int(f^{-1}(V))) \vee Int(Cl^{+*}(f^{-1}(V)))$ for every fuzzy b^+open set V in Y,

iv) $f^{-1}(F)$ is fuzzy b-I^+closed in X for every fuzzy b^+ closed set F in Y.

Proof:

(i)⇒ (ii): Let $x_\lambda \in X$ and V be any fuzzy b^+open set in Y containing $f(x_\lambda)$. By assumption, $f^{-1}(V)$ is fuzzy b-I^+open in X. Set $U=f^{-1}(V)$, then U is a fuzzy b-I^+open in X containing x_λ such that $f(U) \leq V$.

(ii)⇒ (iii): Let V be any fuzzy b^+open set in Y and $x_\lambda \in f^{-1}(V)$. By (ii) there exists a fuzzy b-I^+open set U of X containing x_λ such that $f(U) \leq V$. Thus we obtain,

$$\begin{aligned} x_\lambda \in U &\leq Cl^{+*}(Int(U)) \vee Int(Cl^{+*}(U)) \\ &\leq Cl^{+*}(Int(f^{-1}(V))) \vee Int(Cl^{+*}(f^{-1}(V))) \end{aligned}$$

and hence

$x_\lambda \in Cl^{+*}(Int(f^{-1}(V))) \vee Int(Cl^{*}(f^{-1}(V)))$. This shows that for every fuzzy b$^+$open set V of Y,

$f^{-1}(V) \leq Cl^{+*}(Int(f^{-1}(V))) \vee Int(Cl^{*}(f^{-1}(V)))$ holds.

(iii)⇒ (iv): Let F be any fuzzy b$^+$closed subset of Y and $V=1_Y$ -F. Then V is fuzzy b$^+$open in Y. By (iii), $f^{-1}(V) \leq Cl^{+*}(Int(f^{-1}(V))) \vee Int(Cl^{+*}(f^{-1}(V)))$.

This shows that $f^{-1}(F)= 1_X - f^{-1}(V)$ is fuzzy b-I$^+$closed in X.

(iv)⇒ (i): Let V be any fuzzy b$^+$open set in Y and $F=1_Y$ -V. Then by iv),

$f^{-1}(F) = 1_X - f^{-1}(V)$ is fuzzy b-I$^+$ closed in X.

Hence $f^{-1}(V)$ is fuzzy b-I$^+$open in X and f is fuzzy b-I$^+$ irresolute.

5. FUZZY b-I$^+$OPEN AND FUZZY b-I$^+$CLOSED FUNCTIONS

Definition 5.1: A function f: $(X, \tau^+) \to (Y, \sigma^+, J)$ is called fuzzy b-I$^+$open (resp. fuzzy b-I$^+$closed) if the image of each open (resp. closed) set of X is fuzzy b-I$^+$open (resp. fuzzy b-I$^+$closed) set in (Y, σ^+, J).

Definition 5.2: The fuzzy closure of a fuzzy set A is the smallest fuzzy b-I$^+$ closed set containing A and denoted as f b-I$^+$cl(A)

Remark 5.3: If a fuzzy set is b-I$^+$closed then f b-I$^+$cl(A)=A.

Definition 5.4: A function f: $(X, \tau^+) \to (Y, \sigma^+)$ is called fuzzy b$^+$-open (resp. fuzzy b$^+$-closed) if the image of each open (resp. closed) set of X is fuzzy b$^+$-open (resp. fuzzy b$^+$-closed) set in (Y, σ^+, J).

Theorem 5.5:
(i) Every fuzzy open function is a fuzzy b-I$^+$open function.
(ii) Every fuzzy b-I open function is a fuzzy b-I$^+$open function.
(iii) Every fuzzy b- I$^+$-open function is fuzzy b$^+$-open function.

Proof: Obvious.

Theorem 5.6: If a function f: $(X, \tau^+) \to (Y, \sigma^+, J)$ is a fuzzy b-I$^+$open function ,then for each $x_\lambda \in X$ and each fuzzy open set U containing x_λ ,there exists a fuzzy open b-I$^+$open set W containing $f(x_\lambda)$ such that $W \leq f(U)$.

Proof: Suppose let us assume that $x_\lambda \in X$ and U be any open set containing x_λ. Since f is fuzzy b-I$^+$open, $f(U) \in$ FbI$^+$O(Y).Let W=f(U),then $f(x_\lambda) \in W$ where W is fuzzy b-I$^+$open such that $W \leq f(U)$.

Theorem 5.7: Let f: $(X, \tau^+) \to (Y, \sigma^+, J)$ be a fuzzy b-I$^+$open function. Suppose $W \leq Y$ and $F \leq X$ is fuzzy closed set containing $f^{-1}(W)$,then there exists a fuzzy b-I$^+$ closed set $H \leq Y$ containing W such that $f^{-1}(H) \leq F$.

Proof: Let F be a fuzzy closed set in X. Then $G=1_X - F$ is fuzzy open in X.Since f is fuzzy bI$^+$ open function, f(G) is fuzzy bI$^+$ open in Y. Hence $H=1_Y - f(G)$ is fuzzy bI$^+$ closed in Y and $f^{-1}(H)= f^{-1}(1_Y - f(G))= 1_X - f^{-1}(f(G)) \leq 1_X - G = F$.

Theorem 5.8: For a bijective function f: $(X, \tau^+, I) \to (Y, \sigma^+, J)$,the following statements are equivalent:

(i) $f^{-1}: (Y, \sigma^+, J) \to (X, \tau^+)$ is fuzzy b-I$^+$ continuous,
(ii) f is fuzzy b-I$^+$ open,
(iii) f is fuzzy b-I$^+$ closed.

Proof: Obvious.

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