

ON REGULAR \wedge GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce a new class of sets called $r^{\wedge}g$ - closed sets in topological spaces and to study their properties. Further, we define and study $r^{\wedge}g$ - open sets and $r^{\wedge}g$ - continuity.

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Key Words: $r^{\wedge}g$ - closed sets, $r^{\wedge}g$ - open sets, $r^{\wedge}g$ continuous.

1. INTRODUCTION

In 1970, Levine [9] introduced the concept of generalized closed set in the topological spaces and a class of topological spaces called $T_{1/2}$ spaces. Extensive research on generalizing closedness was done in recent years by many mathematicians. In 1990, S.P. Arya and T.M. Nour [2] define generalized semi-open sets, generalized semi-closed sets. In 1993, N.Palaniappan and K. Chandrasekhara Rao [15] introduced regular generalized closed (rg-closed) sets. In 2000, A. Pushpalatha[16] introduced a new class of closed sets called weakly closed (w- closed) sets. In 2007, S.S. Benchalli and R. S. Wali[3] introduced the class of set called regular w-closed (rw-closed) sets in topological spaces. Recently SanjayMishra and VarunJoshi [17] introduced the concept of regular generalized weakly (rgw-closed) sets.

In this paper, we introduce a new class of sets called regular \wedge generalised - closed sets (briefly $r^{\wedge}g$ -closed sets) and we study their basic properties. We recall the following definitions, which will be used often throughout this paper.

2. PRELIMINARIES

Throughout this paper, X, Y, Z denote the topological spaces $(X, \tau), (Y, \sigma)$ and (Z, η) respectively, on which no separation axioms are assumed.

Definition 2.1: A subset A of a space X is called

- (1) a preopen set if $A \subseteq \text{int}(\text{cl}(A))$ and a preclosed set if $\text{cl}(\text{int}(A)) \subseteq A$.
- (2) a semi-open set if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
- (3) an α -open set if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and a α -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
- (4) a semi-preopen set ($=\beta$ -open) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and a semi-preclosed set (β -closed) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

The semi-closure (resp. α -closure) of a subset A of (X, τ) is denoted by $\text{scl}(A)$ (resp. $\alpha\text{cl}(A)$ and $\text{spcl}(A)$) and is the intersection of all semi-closed (resp. α -closed and semi-preclosed) sets containing A .

Definition 2.2: A subset A of X is called

1. a generalized closed (briefly g-closed) [9] set iff $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
2. Strongly generalized closed (briefly g^* - closed)[20] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X .
3. a regular open [18] set if $A = \text{int}(\text{cl}(A))$ and regular closed[18] set if $A = \text{cl}(\text{int}(A))$.
4. a semi generalized closed (briefly sg - closed)[4] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X .
5. a generalized semi closed (briefly gs - closed)[2] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
6. a generalized semi-pre closed (briefly gsp - closed)[5] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
7. a regular generalized closed (briefly rg - closed)[15] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
8. a generalized preclosed (briefly gp - closed) [10] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
9. a generalized pre regular closed (briefly gpr - closed)[7] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

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10. a weakly closed (briefly w – closed)[16] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X .
11. a regular weakly closed (briefly rw – closed)[3] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen in X .
12. a weakly generalized semi closed (briefly wg – closed) [13] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
13. a regular weakly generalized semi closed (briefly rwg – closed)[13] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
14. a regular generalized weakly semi closed (briefly rgw – closed)[17] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X .
15. a Mildly generalized closed (briefly mildly g closed) [12] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X .
16. a semi weakly generalized closed (briefly swg – closed)[13] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X .
17. a semi weakly g^* closed (briefly swg* closed)[12] if $\text{gcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X .

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3: A map $f: X \rightarrow Y$ is said to be

1. a continuous function [1] if $f^{-1}(V)$ is closed in X for every closed set V in Y .
2. a pr continuous [1] if $f^{-1}(V)$ is pr closed in X for every closed set V in Y .
3. a rg - continuous [4] if $f^{-1}(V)$ is $r\omega$ - closed in X for every closed set V in Y .
4. a sg -continuous [1] if $f^{-1}(V)$ is rg closed in X for every closed set V in Y .
5. a gs -continuous [1] if $f^{-1}(V)$ is gs closed in X for every closed set V in Y .
6. a rw-continuous [13] if $f^{-1}(V)$ is rw- closed in X for every closed set V in Y .
7. a rwg-continuous [13] if $f^{-1}(V)$ is rwg- closed in X for every closed set V in Y .
8. a rgw-continuous [13] if $f^{-1}(V)$ is rgw- closed in X for every closed set V in Y .
9. a swg –continuous [13] if $f^{-1}(V)$ is swg- closed in X for every closed set V in Y .

Theorem 2.3: Every regular open set in X is regular semiopen but not conversely.

Theorem 2.4: Every regular semiopen set in X is semiopen but not conversely.

Theorem 2.5: [6] If A is regular semiopen in X , then X/A is also regular semiopen.

Theorem 2.6: [6] In a space X , the regular closed sets, regular open sets and clopen sets are regular semiopen.

3. Regular \wedge Generalized Closed Sets ($r^{\wedge}g$ - closed sets)

Definition 3.1: A subset A of (X, τ) is called a regular \wedge generalized closed (briefly $r^{\wedge}g$ closed) if $\text{gcl}(A) \subset U$, whenever $A \subset U$ and U is regular open in X .

We denote the family of all $r^{\wedge}g$ closed sets in space X by $R^{\wedge}GC(X)$.

Theorem 3.2: Every closed set of a topological space (X, τ) is $r^{\wedge}g$ closed set.

Proof: Let $A \subset X$ be a closed set and $A \subset U$ where U be regular open. Since A is closed and every closed set is gclosed, $\text{gcl}(A) \subset \text{cl}(A) = A \subset U$. Hence A is an $r^{\wedge}g$ closed set.

Remark 3.3: The converse of the above theorem need not be true as seen in the following example.

Example 3.4: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{a, b\}$ then A is an $r^{\wedge}g$ closed set but it is not a closed set.

Theorem 3.5: Every gclosed set is $r^{\wedge}g$ closed.

Proof: Let A be a gclosed set. Let $A \subset U$ where U is regular open. Since every regular open set is open and A is gclosed, $\text{cl}(A) \subset U$. Every closed set is gclosed therefore $\text{gcl}(A) \subset \text{cl}(A) \subset U$. Hence A is $r^{\wedge}g$ closed.

Remark 3.6: The converse of the above theorem need not be true as seen in the following example.

Example 3.7: In example 3.4, $A = \{a, b\}$ is gclosed set but it is not a closed set.

Theorem 3.8: Every regular generalized closed set is $r^{\wedge}g$ closed.

Proof: Let A be regular generalized closed. Let $A \subset U$ and U be regular open. Then $\text{cl}(A) \subset U$, since A is rg closed. Every closed set is gclosed therefore $\text{gcl}(A) \subset \text{cl}(A) \subset U$. Hence A is $r^{\wedge}g$ closed.

Remark 3.9: The converse of the above theorem need not be true as seen in the following example.

Example 3.10: Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{a\}$. Then A is $r^{\wedge}g$ closed but not rg closed.

Theorem 3.11: Every g^* closed set is $r^{\wedge}g$ closed.

Proof: Let A be g^* closed in (X, τ) . Let $A \subset U$ where U is regular open. Since every regular open set is g open and A is g^* closed, $cl(A) \subset U$. Every closed set is g closed, then $gcl(A) \subset cl(A) \subset U$. Hence A is $r^{\wedge}g$ closed.

Remark 3.12: The converse of the above theorem need not be true as seen in the following example.

Example 3.13: In example 3.4, the set $A = \{a, b\}$ is $r^{\wedge}g$ closed but not g^* closed.

Theorem 3.14:

Every $r^{\wedge}g$ closed set is rwg closed.

Every $r^{\wedge}g$ closed set is rgw closed.

Every $r^{\wedge}g$ closed set is pr -closed.

Proof: Straight forward.

Remark 3.15: The converse of the above theorem need not be true as seen in the following examples.

Example 3.16:

- Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Let $A = \{d\}$, then A is rwg closed but not $r^{\wedge}g$ closed set.
- Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Let $B = \{d\}$, then B is rgw closed but not $r^{\wedge}g$ closed set.
- Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$. Let $B = \{d\}$, then B is pr closed but not $r^{\wedge}g$ closed.

Remark 3.16: $r^{\wedge}g$ closed sets and semi closed sets are independent to each other as seen from the following examples.

Example 3.17:

* Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{b\}$, A is semiclosed but not $r^{\wedge}g$ closed.

* Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$, the subset $\{a, c\}$ in X is $r^{\wedge}g$ closed but not semiclosed.

Remark 3.18: $r^{\wedge}g$ closed sets and preclosed sets are independent to each other as seen from the following examples.

Example 3.19:* Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{a, b\}$, then A is $r^{\wedge}g$ closed but not preclosed.

* Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Let $A = \{a\}$, then A is preclosed but not an $r^{\wedge}g$ closed set.

Remark 3.20: $r^{\wedge}g$ closed sets and semi-preclosed sets are independent to each other as seen from the following example.

Example 3.21:* Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. The subset $\{a\}$ is semi- preclosed but not $r^{\wedge}g$ closed and the subset $\{a, b\}$ is $r^{\wedge}g$ closed but not semi- preclosed.

Remark 3.22: $r^{\wedge}g$ closed sets and wg closed sets are independent to each other as seen from the following examples.

Example 3.23:

* Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Let $A = \{d\}$, then A is wg closed but not an $r^{\wedge}g$ closed set in X .

* Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$, the subset $\{a, c\}$ is an $r^{\wedge}g$ closed set but not a wg closed in X .

Remark 3.24: The concepts of $r^{\wedge}g$ closed sets and gs closed sets are independent of each other as seen from the following examples.

Example 3.25:

* Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Let $B = \{a\}$, then B is gs closed but it is not an $r^{\wedge}g$ closed set.

* Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{c\}, \{d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, the subset $\{a, d\}$ is an $r^{\wedge}g$ closed set but not gs closed set.

Remark 3.26: The concepts of $r^{\wedge}g$ closed sets and sg closed sets are independent of each other as seen from the following example.

Example 3.26: Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. The subset $\{a\}$ is sg closed but not $r^{\wedge}g$ closed and the subset $\{a, c\}$ is $r^{\wedge}g$ closed but not sg closed.

Remark 3.27: The concepts of $r^{\wedge}g$ closed sets and α closed sets are independent of each other as seen from the following examples.

Example 3.28:

* Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Let $A = \{d\}$, then A is α closed set but not an $r^{\wedge}g$ closed set.

* Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. The subset $\{a, b\}$ is $r^{\wedge}g$ closed set but it is not an α closed set.

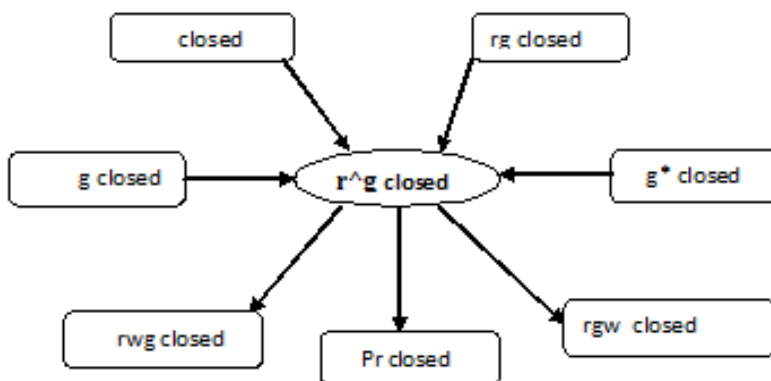
Remark 3.29: $r^{\wedge}g$ closed sets and swg closed sets are independent to each other as seen from the following examples.

Example 3.30: Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. The subset $\{d\}$ is swg closed set but not an $r^{\wedge}g$ closed set and the subset $\{a, c\}$ is $r^{\wedge}g$ closed set but not swg closed.

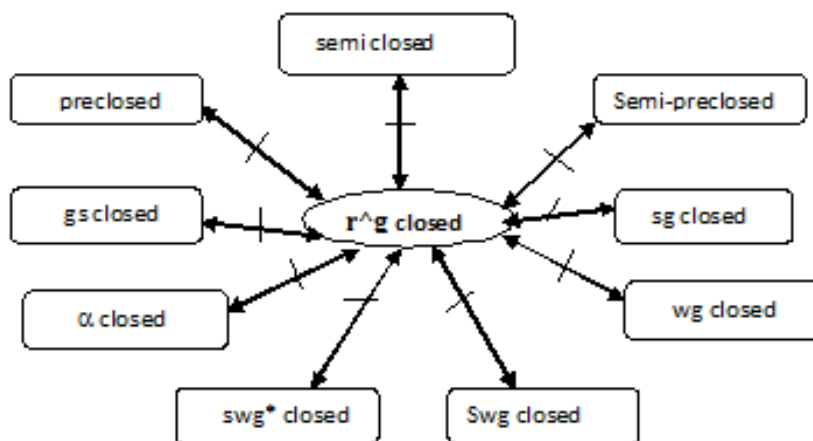
Remark 3.31: $r^{\wedge}g$ closed sets and swg^* closed sets are independent to each other as seen from the following examples.

Example 3.32: Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Let $A = \{d\}$, $B = \{a, c, d\}$, then A is swg^* closed set but not an $r^{\wedge}g$ closed set and B is an $r^{\wedge}g$ closed set but not an swg^* closed set.

Remark 3.33: The above discussions are shown in the following diagram.



Remark 3.34: The following is the diagrammatic representation of independent concepts of the sets with $r^{\wedge}g$ closed sets.



Theorem 3.35: Let A be an $r^{\wedge}g$ closed set in a topological space X . Then $gcl(A) - A$ contains no non-empty regular closed set in X .

Proof: Let F be a regular closed set such that $F \subset \text{gcl}(A) - A$. Then $F \subset X - A$ implies $A \subset X - F$. Since A is $r^{\wedge}g$ closed and $X - F$ is regular open, then $\text{gcl}(A) \subset X - F$. That is $F \subset X - \text{gcl}(A)$. Hence $F \subset \text{gcl}(A) \cap (X - \text{gcl}(A)) = \emptyset$. Thus $F = \emptyset$, whence $\text{gcl}(A) - A$ does not contain nonempty regular closed set.

Remark 3.36: The converse of the above theorem need not be true, that means if $\text{gcl}(A) - A$ contains no nonempty regular closed set, then A need not to be an $r^{\wedge}g$ closed as seen in the following example.

Example 3.37: Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{b\}$. $\text{gcl}(A) - A = \{d\}$, it does not contain non-empty regular closed set in X . But $A = \{b\}$ is not an $r^{\wedge}g$ closed set.

Theorem 3.38: The finite union of two $r^{\wedge}g$ closed sets are $r^{\wedge}g$ closed.

Proof: Assume that A and B are $r^{\wedge}g$ closed sets in X . Let $A \cup B \subset U$ where U is regular open. Then $A \subset U$ and $B \subset U$. Since A and B are $r^{\wedge}g$ closed, $\text{gcl}(A) \subset U$ and $\text{gcl}(B) \subset U$. Then $\text{gcl}(A \cup B) = \text{gcl}(A) \cup \text{gcl}(B) \subset U$. Hence $A \cup B$ is $r^{\wedge}g$ closed.

Remark 3.39: The intersection of two $r^{\wedge}g$ closed set in X need not be an $r^{\wedge}g$ closed set as seen in the following example.

Example 3.40: $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. If $A = \{a, b\}$ and $B = \{a, c\}$. Then A and B are $r^{\wedge}g$ closed sets. But $A \cap B = \{a\}$ is not an $r^{\wedge}g$ closed set.

Theorem 3.41: In a topological space X , if $\text{RO}(X) = \{X, \emptyset\}$, then every subset of X is an $r^{\wedge}g$ closed set.

Proof: Let X be a topological space and $\text{RO}(X) = \{X, \emptyset\}$. Let A be any arbitrary subset of X . Suppose $A = \emptyset$, then \emptyset is an $r^{\wedge}g$ closed set in X . If $A \neq \emptyset$, then X is the only set containing A and so $\text{gcl}(A) \subset X$. Hence A is $r^{\wedge}g$ closed. Thus every subset of X is $r^{\wedge}g$ closed.

Remark 3.42: The converse of the above theorem need not be true as seen in the following example.

Example 3.43: Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. All the subsets of (X, τ) are $r^{\wedge}g$ closed sets, but $\text{RO}(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Theorem 3.44: Let A be an $r^{\wedge}g$ closed set in the topological space (X, τ) . Then A is gclosed iff $\text{gcl}(A) - A$ is regular closed.

Proof: Necessity: Let A be gclosed then $\text{gcl}(A) = A$ and so $\text{gcl}(A) - A = \emptyset$ which is regular closed.

Sufficiency: Suppose $\text{gcl}(A) - A$ is regular closed. Then $\text{gcl}(A) - A = \emptyset$, by theorem 3.37. That is $\text{gcl}(A) = A$. Hence A is gclosed.

Theorem 3.45: If A is an $r^{\wedge}g$ closed subset of X such that $A \subset B \subset \text{gcl}(A)$, then B is an $r^{\wedge}g$ closed set.

Proof: Let $B \subset U$ where U is regular open. Then $A \subset B$ implies $A \subset U$. Since A is $r^{\wedge}g$ closed, $\text{gcl}(A) \subset U$. By hypothesis $\text{gcl}(B) \subset \text{gcl}(\text{gcl}(A)) = \text{gcl}(A) \subset U$. Hence B is $r^{\wedge}g$ closed.

Remark 3.46: The converse of the above theorem need not be true as seen in the following example.

Example 3.47: Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{c\}$ and $B = \{a, c\}$. Then A and B are $r^{\wedge}g$ closed sets. But $A \subset B$ is not a subset of $\text{gcl}(A)$.

Theorem 3.48: Let (X, τ) be a topological space, then for $x \in X$, the set $X \setminus \{x\}$ is either $r^{\wedge}g$ closed set or regular open set.

Proof: If $X \setminus \{x\}$ is not a regular open set, then X is the only regular open set containing $X \setminus \{x\}$. This implies that $\text{gcl}\{X \setminus \{x\}\} \subset X$. Hence $X \setminus \{x\}$ is $r^{\wedge}g$ closed set.

4. Regular \wedge Generalized Open Set:

Definition 4.1: A set $A \subset X$ is called regular \wedge generalized open ($r^{\wedge}g$ open) set if and only if its complement is regular \wedge generalized closed.

The collection of all $r^{\wedge}g$ open sets is denoted by $R^{\wedge}GO(X)$.

Remark 4.2: $gcl(X - A) = X - gint(A)$

Theorem 4.3: $A \subset X$ is $r^{\wedge}g$ open iff $F \subseteq gint(A)$, whenever F is regular closed and $F \subset A$.

Proof: Necessity: Let A be $r^{\wedge}g$ closed and $F \subset A$. Then $X - A \subset X - F$ where $X - F$ is regular open and $r^{\wedge}g$ closedness of $X - A$ implies $gcl(X - A) \subset X - F$. By remark 4.2, $X - gint(A) \subset X - F$. Therefore $F \subset gint(A)$.

Sufficiency: Suppose F is regular closed and $F \subset A$ then $F \subset gint(A)$. Let $X - A \subset U$, where U is regular open. Then $X - U \subset A$, where $X - U$ is regular closed. By hypothesis, $X - U \subset gint(A)$. Then $X - gint(A) \subset U$. By remark 4.2, $gcl(X - A) \subset U$. Hence $X - A$ is $r^{\wedge}g$ closed and A is $r^{\wedge}g$ open.

Theorem 4.4: If $gint(A) \subset B \subset A$ and if A is $r^{\wedge}g$ open then B is $r^{\wedge}g$ open.

Proof: Given $gint(A) \subset B \subset A$, then $X - A \subset X - B \subset gcl(X - A)$. Since A is $r^{\wedge}g$ open, $X - A$ is $r^{\wedge}g$ closed. This implies $X - B$ is $r^{\wedge}g$ closed. Hence B is $r^{\wedge}g$ open.

Remark 4.5: For any $A \subset X$, $gint(gcl(A) - A) = \phi$.

Theorem 4.6: If $A \subset X$ is $r^{\wedge}g$ closed, then $gcl(A) - A$ is $r^{\wedge}g$ open.

Proof: Let A be an $r^{\wedge}g$ closed and let F be a regular closed set such that $F \subset gcl(A) - A$. Then by theorem 3.37, $F = \phi$. So $F \subset gint(gcl(A) - A)$. By theorem 4.3, $gcl(A) - A$ is $r^{\wedge}g$ open.

Remark 4.7: The converse of the above theorem need not be true as seen in the following example.

Example 4.8: Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{b\}$, then $gcl(A) - A = \{c\}$ which is $r^{\wedge}g$ open in X but A is not $r^{\wedge}g$ closed in X .

5. $r^{\wedge}g$ Continuous and $r^{\wedge}g$ Irresolute Functions:

Definition 5.1: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $r^{\wedge}g$ continuous if every $f^{-1}(V)$ is $r^{\wedge}g$ closed in X for every closed set V of Y .

Definition 5.2: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $r^{\wedge}g$ irresolute if every $f^{-1}(V)$ is $r^{\wedge}g$ closed in X for every $r^{\wedge}g$ closed set V of Y .

Example 5.3: Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, d\}, \{a, c, d\}\}$. $Y = \{a, b, c, d\}$, $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a$, $f(b)=b$, $f(c)=c$, $f(d)=d$. Here the inverse image of the closed sets in Y are $r^{\wedge}g$ closed sets in X . Hence f is $r^{\wedge}g$ continuous.

Example 5.4: Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, b\}, \{c\}, \{a, b, c\}\}$ and $Y = X$, $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a$, $f(b)=b$, $f(c)=c$, $f(d)=d$. The inverse image of every $r^{\wedge}g$ closed set in Y is $r^{\wedge}g$ closed set in X . Hence f is $r^{\wedge}g$ irresolute.

Remark 5.5: Every $r^{\wedge}g$ irresolute function is $r^{\wedge}g$ continuous but the converse is not true as seen in the following example.

Example 5.6: In example 5.3, f is $r^{\wedge}g$ continuous but not $r^{\wedge}g$ irresolute.

Remark 5.7: Every continuous function is $r^{\wedge}g$ continuous. But the converse is not true as seen in the following example.

Example 5.8: * Let $X = \{a, b, c, d\}$. $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $Y = X$, $\sigma = \{Y, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ the identity mapping. Then f is $r^{\wedge}g$ continuous but not continuous.

Remark 5.9: Every $r^{\wedge}g$ continuous mapping is pr continuous, rgw continuous, rwg continuous. But the converse need not be true as seen in the following example.

Example 5.10: * Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$, $Y = X$. $\sigma = \{Y, \phi, \{a, b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a$, $f(b)=b$, $f(c)=d$, $f(d)=c$. Then f is pr continuous, rgw continuous, rwg continuous, but not $r^{\wedge}g$ continuous.

Remark 5.11: $r^{\wedge}g$ continuity and rw continuity are independent concepts as seen in the following example.

Example 5.12:

* Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}, Y = X, \sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ the identity mapping. Then f is rw continuous but not $r^{\wedge}g$ continuous.

* Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}, Y = X, \sigma = \{Y, \emptyset, \{b\}, \{b, c\}, \{a, d\}, \{a, b, d\}\}$. Define $g: (X, \tau) \rightarrow (Y, \sigma)$, the identity mapping. Here g is $r^{\wedge}g$ continuous but not rw continuous.

Remark 5.13: sg continuity, gs continuity, swg continuity are independent concepts with $r^{\wedge}g$ continuity as seen in the following example.

Example 5.14:

* Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}, Y = X, \sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$, by $f(a)=a, f(b)=c, f(c)=b, f(d)=d$. Then f is sg continuous, gs continuous and swg continuous but not $r^{\wedge}g$ continuous.

* Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}, Y = X, \sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ the identity mapping. Then g is $r^{\wedge}g$ continuous but not sg continuous, gs continuous and swg continuous.

Remark 5.15: $r^{\wedge}g$ continuity and wg continuity are independent concepts as seen in the following example.

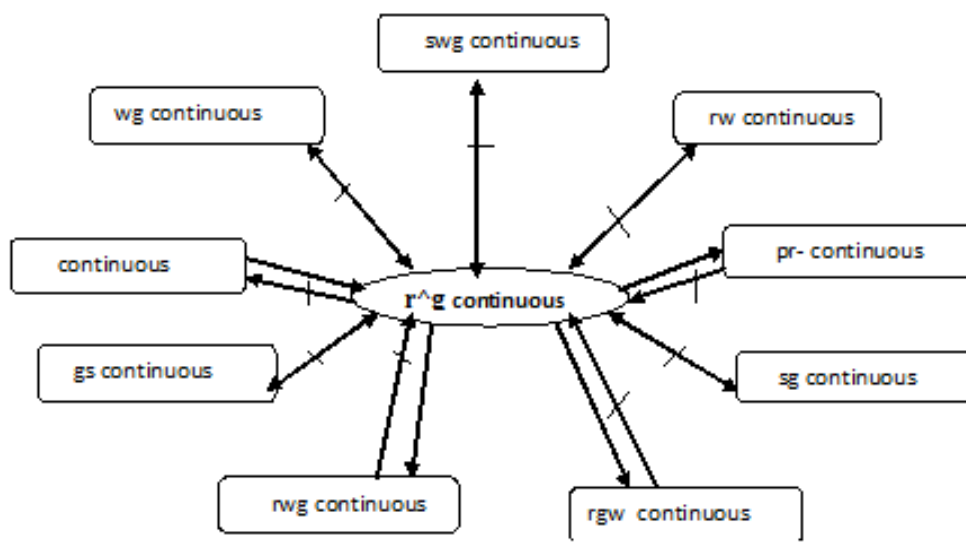
Example 5.16:

* Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}, Y = \{a, b, c, d\}, \sigma = \{Y, \emptyset, \{b\}, \{b, c\}, \{a, d\}, \{a, b, d\}\}$.

Define $f: (X, \tau) \rightarrow (Y, \sigma)$, the identity mapping, then f is $r^{\wedge}g$ continuous but not wg continuous.

* Let $X = Y = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}, \sigma = \{Y, \emptyset, \{a, b, c\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$, the identity mapping, then f is wg continuous but not $r^{\wedge}g$ continuous.

The above discussions are implicated as shown below.



Theorem 5.17: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

- (i) $(g \circ f)$ is $r^{\wedge}g$ -continuous if g is continuous and f is $r^{\wedge}g$ -continuous
- (ii) $(g \circ f)$ is $r^{\wedge}g$ -irresolute, if g is $r^{\wedge}g$ -irresolute and f is $r^{\wedge}g$ -irresolute.
- (iii) $(g \circ f)$ is $r^{\wedge}g$ continuous if g is $r^{\wedge}g$ continuous and f is $r^{\wedge}g$ -irresolute.

Proof:

- (i) Let V be any closed set in (Z, η) . Then $g^{-1}(V)$ is closed in (Y, σ) , since g is continuous. By hypothesis, $f^{-1}(g^{-1}(V))$ is $r^{\wedge}g$ closed in (X, τ) . Hence $g \circ f$ is $r^{\wedge}g$ continuous.
- (ii) Let V be $r^{\wedge}g$ closed set in (Z, η) . Since g is $r^{\wedge}g$ irresolute, $g^{-1}(V)$ is $r^{\wedge}g$ closed in (Y, σ) . As f is $r^{\wedge}g$ irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $r^{\wedge}g$ closed in (X, τ) . Hence $g \circ f$ is $r^{\wedge}g$ irresolute.

- (iii) Let V be closed in (Z, η) . Since g is $r\wedge g$ continuous. $g^{-1}(V)$ is $r\wedge g$ closed in (Y, σ) . As f is $r\wedge g$ irresolute, $f^{-1}g^{-1}(V) = (g \circ f)^{-1}(V)$ is $r\wedge g$ closed in (X, τ) . Hence $(g \circ f)$ is $r\wedge g$ continuous.

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