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ON REGULAR ^ GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce a new class of sets called r^g - closed sets in topological spaces and to study their properties. Further, we define and study r^g - open sets and r^g - continuity.

Mathematics Subject Classification: *54A05.*

Key Words: *r^g - closed sets, r^g - open sets, r^g continuous.*

1. INTRODUCTION

In 1970, Levine [9] introduced the concept of generalized closed set in the topological spaces and a class of topological spaces called $T_{1/2}$ spaces. Extensive research on generalizing closedness was done in recent years by many mathematicians. In 1990, S.P. Arya and T.M. Nour [2] define generalized semi-open sets, generalized semi-closed sets. In 1993, N.Palaniappan and K. Chandrasekhara Rao [15] introduced regular generalized closed (rg-closed) sets. In 2000, A. Pushpalatha[16] introduced a new class of closed sets called weakly closed(w- closed) sets .In 2007, S.S. Benchalli and R. S. Wali[3] introduced the class of set called regular w-closed(rw-closed) sets in topological spaces. Recently SanjayMishra and VarunJoshi [17] introduced the concept of regular generalized weakly (rgw-closed) sets.

In this paper, we introduce a new class of sets called regular ^ generalised - closed sets (briefly r^g -closed sets) and we study their basic properties. We recall the following definitions, which will be used often throughout this paper.

2. PRELIMINARIES

Throughout this paper, X, Y, Z denote the topological spaces (X, τ),(Y, σ) and (Z, η) respectively, on which no separation axioms are assumed.

Definition 2.1: A subset A of a space X is called
(1) a preopen set if $A \subseteq int(cl(A))$ and a preclosed set if $cl(int(A)) \subseteq A$.
(2) a semi-open set if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
(3) an α-open set if $A \subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))) \subseteq A$.
(4) a semi-preopen set (=β-open) if $A \subseteq cl(int(cl(A)))$ and a semi-preclosed set (β-closed) if $int(cl(int(A))) \subseteq A$.

The semi-closure (resp. α-closure) of a subset A of (X,τ) is denoted by scl(A) (resp. αcl(A) and spcl(A))and is the intersection of all semi-closed (resp. α-closed and semi-preclosed) sets containing **A.**

Definition 2.2: A subset A of X is called
1. a generalized closed (briefly g-closed) [9] set iff $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
2. Strongly generalized closed (briefly g* closed)[20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X
3. a regular open [18] set if A = int(cl(A)) and regular closed[18] set if A = cl(int(A)) .
4. a semi generalized closed (briefly sg – closed)[4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.
5. a generalized semi closed (briefly gs – closed)[2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
6. a generalized semi-pre closed (briefly gsp – closed)[5] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
7. a regular generalized closed (briefly rg – closed)[15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
8. a generalized preclosed (briefly gp – closed) [10]if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
9. a generalized pre regular closed (briefly gpr – closed)[7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

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10. a weakly closed (briefly w – closed)[16] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semiopen in X.
11. a regular weakly closed (briefly rw – closed)[3] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semiopen in X.
12. a weakly generalized semi closed (briefly wg – closed) [13] if cl(int(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
13. a regular weakly generalized semi closed (briefly rwg – closed)[13] if cl(int(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
14. a regular generalized weakly semi closed (briefly rgw – closed)[17] if cl(int(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi-open in X.
15. a Mildly generalized closed (briefly mildly g closed)[12]if cl(int(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
16. a semi weakly generalized closed (briefly swg – closed)[13] if cl(int(A) $\subseteq$ U whenever A $\subseteq$ U and U is semiopen in X.
17. a semi weakly g* closed (briefly swg* closed)[12] if gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semiopen in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3: A map f: X $\to$Y is said to be
1. a continuous function[1]if f^{-1}(V) is closed in X for every closed set V in Y.
2. a pr continuous [1] if f^{-1}(V) is pr closed in X for every closed set V in Y.
3. a rg - continuous [4] if f^{-1} (V) is rω- closed in X for every closed set V in Y.
4. a sg -continuous [1] if f^{-1}(V) is rg closed in X for every closed set V in Y.
5. a gs -continuous [1] if f^{-1}(V) is gs closed in X for every closed set V in Y.
6. a rw-continuous[13] if f^{-1}(V) is rw- closed in X for every closed set V in Y.
7. a rwg-continuous[13] if f^{-1}(V) is rwg- closed in X for every closed set V in Y.
8. a rgw-continuous[13] if f^{-1}(V) is rgw- closed in X for every closed set V in Y.
9. a swg –continuous[13] if f^{-1}(V) is swg- closed in X for every closed set V in Y.

Theorem 2.3: Every regular open set in X is regular semiopen but not conversely.

Theorem 2.4: Every regular semiopen set in X is semiopen but not conversely.

Theorem 2.5: [6] If A is regular semiopen in X, then X/A is also regular semiopen.

Theorem 2.6: [6] In a space X, the regular closed sets, regular open sets and clopen sets are regular semiopen.

3. Regular ^ Generalized Closed Sets (r^g - *closed sets)*

Definition 3.1: A subset A of (X,τ) is called a regular ^generalized closed **(briefly r^g closed)** if gcl(A) $\subset$U, whenever A$\subset$ U and U is regular open in X.

We denote the family of all r^g closed sets in space X by R^GC(X).

Theorem 3.2: Every closed set of a topological space (X,τ) is r^g closed set.

Proof: Let A$\subset$ X be a closed set and A $\subset$ U where U be regular open. Since A is closed and every closed set is gclosed, gcl(A) $\subset$ cl(A) = A$\subset$ U. Hence A is an r^g closed set.

Remark 3.3: The converse of the above theorem need not be true as seen in the following example.

Example 3.4: Let X = {a, b, c}, τ = {X,φ, {a},{b},{a, b}}. Let A = {a, b} then A is an r^g closed set but it is not a closed set.

Theorem 3.5: Every gclosed set is r^g closed.

Proof: Let A be a gclosed set. Let A $\subset$ U where U is regular open. Since every regular open set is open and A is gclosed, cl(A) $\subset$ U. Every closed set is gclosed therefore gcl(A) $\subset$ cl(A)$\subset$ U. Hence A is r^g closed.

Remark 3.6: The converse of the above theorem need not be true as seen in the following example.

Example 3.7: In example 3.4, A = {a, b} is gclosed set but it is not a closed set.

Theorem 3.8: Every regular generalized closed set is r^g closed.

Proof: Let A be regular generalized closed. Let A $\subset$ U and U be regular open. Then cl(A) $\subset$ U, since A is rg closed. Every closed set is gclosed therefore gcl(A) $\subset$ cl(A) $\subset$ U. Hence A is r^g closed.

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Remark 3.9: The converse of the above theorem need not be true as seen in the following example.

Example 3.10: Let X = {a, b, c, d, e} τ = {X,φ, {a},{b},{c},{a, b},{a, c},{b, c}{a, b, c}}. Let A = {a}. Then A is r^g closed but not rg closed.

Theorem 3.11: Every g*closed set is r^g closed.

Proof: Let A be g*closed in (X,τ). Let A ⊂ U where U is regular open. Since every regular open set is gopen and A is g*closed, cl(A) ⊂ U. Every closed set is gclosed, then gcl(A) ⊂ cl(A) ⊂ U. Hence A is r^g closed.

Remark 3.12: The converse of the above theorem need not be true as seen in the following example.

Example 3.13: In example 3.4, the set A = {a, b} is r^g closed but not g* closed.

Theorem 3.14:
Every r^g closed set is rwg closed.
Every r^g closed set is rgw closed.
Every r^g closed set is pr-closed.

Proof: Straight forward.

Remark 3.15: The converse of the above theorem need not be true as seen in the following examples.

Example 3.16:
- Let X = {a, b, c, d}, τ = {X,φ, {a},{c},{a, c},{c, d},{a, c, d}}. Let A = {d}, then A is rwg closed but not r^g closed set.
- Let X = {a, b, c, d}, τ = {X,φ,{a},{c},{a, c},{c, d},{a, c, d}}. Let B = {d}, then B is rgw closed but not r^g closed set.
- Let X = {a, b, c, d}, τ = {X, φ,{a},{c, d},{a, c, d}}. Let B = {d}, then B is pr closed but not r^g closed.

Remark 3.16: r^g closed sets and semi closed sets are independent to each other as seen from the following examples.

Example 3.17:
***** Let X = {a, b, c},τ ={X, φ,{a},{b},{a, b}}. Let A = {b}, A is semiclosed but not r^g closed.
***** Let X = {a, b, c}, τ = {X,φ, {a},{c},{a, c}}, the subset {a, c} in X is r^g closed but not semiclosed.

Remark 3.18: r^g closed sets and preclosed sets are independent to each other as seen from the following examples.

Example 3.19:* Let X = {a, b, c} and τ = {X,φ, {a},{b},{a, b}}. Let A = {a, b}, then A is r^g closed but not preclosed.
***** Let X = {a, b, c, d, e}, τ = {X,φ, {a, b},{c. d},{a, b, c, d}}. Let A = {a}, then A is preclosed but not an r^g closed set.

Remark 3.20: r^g closed sets and semi-preclosed sets are independent to each other as seen from the following example.

Example 3.21:* Let X = {a, b, c}, τ= {X,φ, {a},{b},{a, b}}. The subset {a} is semi- preclosed but not r^g closed and the subset {a, b} is r^g closed but not semi- preclosed.

Remark 3.22: r^g closed sets and wg closed sets are independent to each other as seen from the following examples.

Example 3.23:
***** Let X = {a, b, c, d,}, τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}. Let A = {d}, then A is wg closed but not an r^g closed set in X.
***** Let X = {a, b, c, d,}, τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}, the subset{a, c} is an r^g closed set but not a wg closed in X.

Remark 3.24: The concepts of r^g closed sets and gs closed sets are independent of each other as seen from the following examples.

Example 3.25:
* Let X = {a, b, c, d},τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}. Let B = {a}, then B is gs closed but it is not an r^g closed set.
* Let X = {a, b, c, d}, τ = {X,φ,{c},{d},{b, d},{c, d},{a, c, d},{b, c, d}}, the subset {a, d} is an r^g closed set but not gs closed set.

Remark 3.26: The concepts of r^g closed sets and sg closed sets are independent of each other as seen from the following example.

Example 3.26: Let X = {a, b, c, d},τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}. The subset {a} is sg closed but not r^g closed and the subset {a, c} is r^g closed but not sg closed.

Remark 3.27: The concepts of r^g closed sets and α closed sets are independent of each other as seen from the following examples.

Example 3.28:
* Let X = {a, b, c, d}, τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}. Let A = {d}, then A is α closed set but not an r^g closed set.
* Let X = {a, b, c}, τ = {X,φ, {a},{b},{a, b}}. The subset {a, b} is r^g closed set but it is not an α closed set.

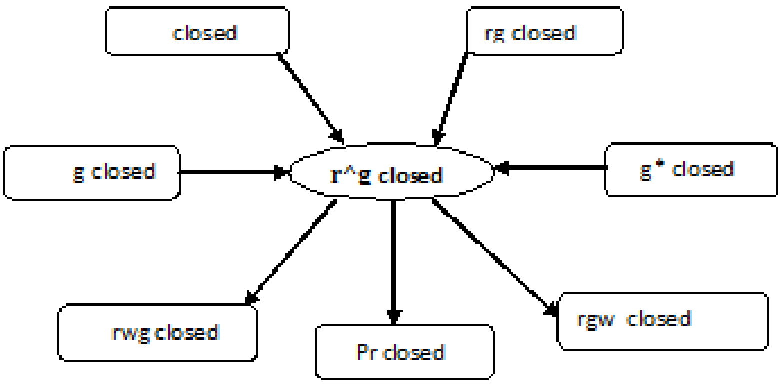
Remark 3.29: r^g closed sets and swg closed sets are independent to each other as seen from the following examples.

Example 3.30: Let X = {a, b, c, d}, τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}. The subset {d} is swg closed set but not an r^g closed set and the subset {a, c} is r^g closed set but not swg closed.

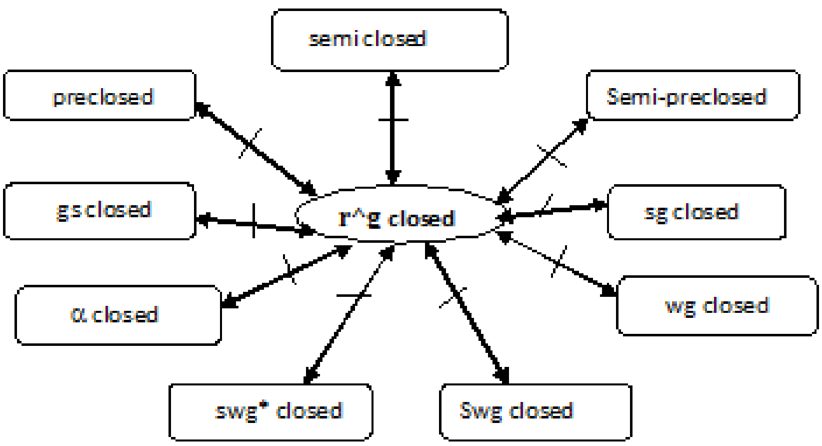
Remark 3.31: r^g closed sets and swg* closed sets are independent to each other as seen from the following examples.

Example 3.32: Let X = {a, b, c, d}, τ = {X,φ, {a},{c},{a, c}{c, d}{a, c, d}}. Let A = {d}, B = {a, c, d}, then A is swg* closed set but not an r^g closed set and B is an r^g closed set but not an swg* closed set.

Remark 3.33: The above discussions are shown in the following diagram.



Remark 3.34: The following is the diagrammatic representation of independent concepts of the sets with r^g closed sets.



Theorem 3.35: Let A be an r^g closed set in a topological space X. Then gcl(A) – A contains no non-empty regular closed set in X.

Proof: Let F be a regular closed set such that F ⊂ gcl(A) – A. Then F⊂ X-A implies A⊂ X-F. Since A is r^g closed and X-F is regular open, then gcl(A) ⊂ X-F. That is F ⊂ X-gcl(A). Hence F⊂ gcl(A)∩(X – gcl(A)) = φ. Thus F =φ, whence gcl(A)-A does not contain nonempty regular closed set.

Remark 3.36: The converse of the above theorem need not be true , that means if gcl(A)-A contains no nonempty regular closed set , then A need not to be an r^g closed as seen in the following example.

Example 3.37: Let X = {a, b, c, d}, τ = {X,φ, {a},{b, c},{a, b, c}}. Let A = {b}. gcl(A) – A = {d}, it does not contain non-empty regular closed set in X. But A = {b} is not an r^g closed set.

Theorem 3.38: The finite union of two r^g closed sets are r^g closed.

Proof: Assume that A and B are r^g closed sets in X. Let A∪B ⊂ U where U is regular open. Then A⊂U and B⊂U. Since A and B are r^g closed, gcl(A) ⊂ U and gcl(B) ⊂ U. Then gcl(A∪B) = gcl(A) ∪ gcl(B) ⊂ U. Hence AUB is r^g closed.

Remark 3.39: The intersection of two r^g closed set in X need not be an r^g closed set as seen in the following example.

Example 3.40: X = {a, b, c}, τ = {X,φ,{a},{c},{a, c}}. If A = {a, b} and B= {a, c}. Then A and B are r^g closed sets. But A∩B = {a} is not an r^g closed set.

Theorem 3.41: In a topological space X, if RO(X) = {X,φ}, then every subset of X is an r^g closed set.

Proof: Let X be a topological space and RO(X) = {X,φ}. Let A be any arbitrary subset of X. Suppose A =φ, then φ is an r^g closed set in X. If A ≠ φ, then X is the only set containing A and so gcl(A) ⊂ X. Hence A is r^g closed. Thus every subset of X is r^g closed.

Remark 3.42: The converse of the above theorem need not be true as seen in the following example.

Example 3.43: Let X = {a, b, c, d, e}, τ = {X,φ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}}. All the subsets of (X,τ) are r^g closed sets, but RO(X) = {X,φ, {a},{b},{c},{a, b},{a, c},{b, c}}.

Theorem 3.44: Let A be an r^g closed set in the topological space (X,τ). Then A is gclosed iff gcl(A)-A is regular closed .

Proof: Necessity: Let A be gclosed then gcl(A) = A and so gcl(A) – A = φ which is regular closed.

Sufficiency: Suppose gcl(A) – A is regular closed. Then gcl(A) – A = φ, by theorem 3.37. That is gcl(A) = A. Hence A is gclosed.

Theorem 3.45: If A is an r^g closed subset of X such that A⊂ B⊂ gcl(A), then B is an r^g closed set.

Proof: Let B ⊂ U where U is regular open. Then A ⊂ B implies A⊂U. Since A is r^g closed, gcl(A) ⊂ U. By hypothesis gcl(B) ⊂ gcl(gcl(A)) = gcl(A) ⊂ U. Hence B is r^g closed.

Remark 3.46: The converse of the above theorem need not be true as seen in the following example.

Example 3.47: Let X = {a, b, c, d}, τ = {X,φ, {a},{b},{a, b}{a, b, c}}. Let A = {c} and B = {a, c}. Then A and B are r^g closed sets. But A ⊂ B is not a subset of gcl(A).

Theorem 3.48: Let (X,τ) be a topological space, then for x ∈ X, the set X\{x} is either r^g closed set or regular open set.

Proof: If X \{x} is not a regular open set, then X is the only regular open set containing X\{x}.This implies that gcl{X\{x}} ⊂ X . Hence X\{x} is r^g closed set.

4. Regular ^ Generalized Open Set:

Definition 4.1: A set A ⊂ X is called regular ^ generalized open (r^g open) set if and only if its compliment is regular ^ generalized closed.

The collection of all r^g open sets is denoted by R^GO(X).

Remark 4.2: gcl(X –A) = X – gint(A)

Theorem 4.3: $A \subset X$ is r^g open iff $F \subseteq$ gint(A) , whenever F is regular closed and $F \subset A$.

Proof: Necessity: Let A be r^g closed and $F \subset A$. Then $X – A \subset X – F$ where X – F is regular open and r^g closedness of X – A implies gcl(X – A) $\subset$X – F. By remark 4.2, X – gintA $\subset$ X-F. Therefore $F \subset$ gint(A).

Sufficiency: Suppose F is regular closed and $F \subset A$ then $F \subset$ gint(A). Let X-A $\subset$ U, where U is regular open. Then $X – U \subset A$, where X – U is regular closed. By hypothesis, X – U $\subset$ gintA. Then X – gint(A) $\subset$ U. By remark 4.2, gcl(X – A) $\subset$ U. Hence X – A is r^g closed and A is r^g open.

Theorem 4.4: If gintA $\subset B \subset A$ and if A is r^g open then B is r^g open.

Proof: Given gintA$\subset B \subset A$, then $X – A \subset X – B \subset$ gcl(X – A). Since A is r^g open, X –A is r^g closed. This implies X – B is r^g closed. Hence B is r^g open.

Remark 4.5: For any $A \subset X$, gint(gcl(A) – A) = φ.

Theorem 4.6: If $A \subset X$ is r^g closed, then gcl(A) – A is r^g open.

Proof: Let A be an r^g closed and let F be a regular closed set such that $F \subset$ gcl(A) – A. Then by theorem 3.37, F = φ . So $F \subset$ gint(gcl(A) – A). By theorem 4.3, gcl(A) – A is r^g open.

Remark 4.7: The converse of the above theorem need not be true as seen in the following example.

Example 4.8: Let X = {a, b, c}, τ = {φ, X,{a},{b},{a, b}}.Let A = {b}, then gcl(A) – A = {c} which is r^g open in X but A is not r^g closed in X.

5. r^g Continuous and r^g Irresolute Functions:

Definition 5.1: A function f : $(X,\tau) \to (Y,\sigma)$ is called r^g continuous if every $f^{-1}(V)$ is r^g closed in X for every closed set V of Y.

Definition 5.2: A function f: $(X,\tau) \to (Y,\sigma)$ is called r^g irresolute if every $f^{-1}(V)$ is r^g closed in X for every r^g closed set V of Y.

Example 5.3: Let X = {a, b, c, d}, τ = {X,φ,{a},{c},{d},{a, c}{c, d},{a, d},{a, c, d}}. Y = {a, b, c, d}, σ = {Y,φ,{a},{c}, {a, c}, {c, d},{a, c, d}}. Define f :$(X,\tau) \to (Y,\sigma)$ by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Here the inverse image of the closed sets in Y are r^g closed sets in X. Hence f is r^g continuous.

Example 5.4: Let X={a, b, c, d}, τ = {φ,X,{a, b},{c},{a, b, c}} and Y = X, σ = {Y,φ,{a},{c},{a, c},{a, b},{a, b, c}}. Define f:$(X,\tau) \to (Y,\sigma)$ by f(a)=a, f(b)=b, f(c)=c, f(d)=d. The inverse image of every r^g closed set in Y is r^g closed set in X. Hence f is r^g irresolute.

Remark 5.5: Every r^g irresolute function is r^g continuous but the converse is not true as seen in the following example.

Example 5.6: In example 5.3, f is r^g continuous but not r^g irresolute.

Remark 5.7: Every continuous function is r^g continuous. But the converse is not true as seen in the following example.

Example 5.8:*Let X= {a, b, c, d}. τ = {X,φ,{a},{b},{a, b},{a, b, c}} and Y = X, σ = {Y,φ,{a},{c},{d},{a, c},{a, d}, {a, c, d}}. Define f :$(X,\tau) \to (Y,\sigma)$ the identity mapping. Then f is r^g continuous but not continuous.

Remark 5.9: Every r^g continuous mapping is pr continuous, rgw continuous, rwg continuous. But the converse need not be true as seen in the following example.

Example 5.10: * Let X = {a, b, c, d},τ = {X,φ,{a},{c, d},{a, c, d}},Y=X. σ={Y,φ,{a, b, c}}.Let f: $(X,\tau) \to (Y,\sigma)$ by f(a) =a, f(b) = b, f(c) = d, f(d) = c. Then f is pr continuous, rgw continuous, rwg continuous, but not r^g continuous.

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Remark 5.11: r^g continuity and rw continuity are independent concepts as seen in the following example.

Example 5.12:
***** Let X = {a, b, c, d},τ = {X, φ,{a},{c},{a, b},{a, c},{a, b, c}},Y=X. σ={Y,φ, {a},{b}, {a, b},{a, b, c}}. Let f: (X,τ) →(Y,σ) the identity mapping. Then f is rw continuous but not r^g continuous.

* Let X= {a, b, c, d}τ={X,φ,{a},{b}, {a, b} {a, b, c}}. Y =X. σ = {Y,φ, {b},{b, c},{a, d}, {a, b, d}} Define g:(X,τ) →(Y,σ), the identity mapping. Here g is r^g continuous but not rw continuous.

Remark 5.13: sg continuity, gs continuity, swg continuity are independent concepts with r^g continuity as seen in the following example.

Example 5.14:
* Let X = {a, b, c, d},τ={X,φ,{a},{c},{a, c},{c, d},{a, c, d},Y=X,σ= {Y,φ,{a},{b}, {a, b} {a, b, c}}. Define f:(X,τ) →(Y,σ), by f(a)=a, f(b)=c, f(c)=b, f(d)=d. Then f is sg continuous, gs continuous and swg continuous but not r^g continuous.

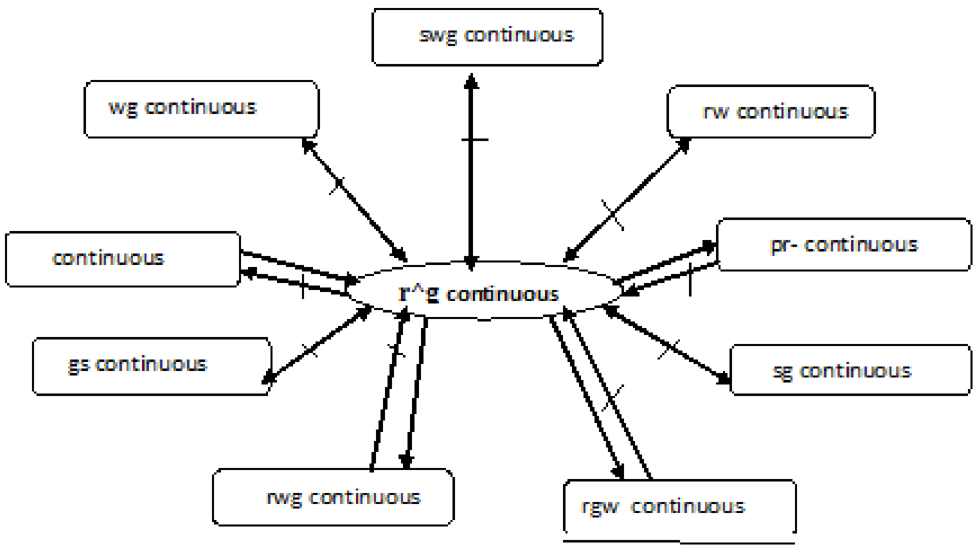
***** Let X = {a, b, c, d},τ = {X,φ,{a},{c},{a, b},{a, c},{a, b, c}},Y=X. σ={Y,φ, {a},{b}, {a, b},{a, b, c}}.Let g: (X,τ) →(Y,σ) the identity mapping. Then g is r^g continuous but not sg continuous, gs continuous and swg continuos.

Remark 5.15: r^g continuity and wg continuity are independent concepts as seen in the following example.

Example 5.16:
* Let X = {a, b, c, d} , τ = {X,φ,{a},{b},{a, b},{a, b, c}}. Y = {a, b, c, d}, σ = {Y,φ,{b}, {b, c}, {a, d}, {a, b, d}}.

Define f: (X,τ) → (Y,σ), the identity mapping, then f is r^g continuous but not wg continuous.
* Let X = Y = {a, b, c, d} τ = {X,φ,{a},{c, d},{a, c, d}}, σ = {Y,φ,{a, b, c}}. Define f : (X,τ) → (Y,σ), the identity mapping, then f is wg continuous but not r^g continuous.

The above discussions are implicated as shown below.



Theorem 5.17: Let f: (X, τ) →(Y, σ) and g:(Y, σ) →(Z, η) be any two functions. Then
(i) (gof) is r^g-continuous if g is continuous and f is r^g -continuous
(ii) (gof) is r^g -irresolute, if g is r^g –irresolute and f is r^g -irresolute.
(iii) (gof) is r^g continuous if g is r^g continuous and f is r^g -irresolute.

Proof:
(i) Let V be any closed set in (Z,η). Then $g^{-1}(V)$ is closed in (Y,σ), since g is continuous. By hypothesis, $f^{-1}(g^{-1}(V))$ is r^g closed in (X,τ). Hence gof is r^g continuous.

(ii) Let V be r^g closed set in (Z,η).Since g is r^g irresolute, $g^{-1}(V)$ is r^g closed in (Y,σ). As f is r^g irresolute,$f^{-1}g^{-1}(V)= (gof)^{-1}(V)$ is r^g closed in (X,τ).Hence gof is r^g irresolute.

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(iii) Let V be closed in (Z,η).Since g is r^g continuous. $g^{-1}(V)$ is r^g closed in (Y,σ). As f is r^g irresolute, $f^{-1}g^{-1}(V) = (gof)^{-1}(V)$ is r^g closed in (X,τ).Hence (gof) is r^g continuous.

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