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UNSTEADY MHD HELE-SHAW FLOW AND HEAT TRANSFER OF RIVLIN-ERICKSEN FLUID THROUGH AN INCLINED CHANNEL WITH MOVING BOUNDARIES IN OPPOSITE DIRECTIONS

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ABSTRACT

The aim of the present investigation is to study the magneto hydrodynamic unsteady Hele-Shaw flow of a visco-elastic (Rivlin-Ericksen) fluid through an inclined channel between two parallel flat plates under the influence of magnetic field with heat transfer including heat generating sources, when the boundaries are moving in opposite directions. It is assumed that the flow is slow and fully developed. The constitutive equations for continuity, motion and energy of visco-elastic liquid are obtained and perturbation method is applied for solution. The velocity of the fluid increases when elastic parameter R_c increases and velocity decreases with increasing magnetic parameter M, source parameter S and Froude number F_r. Also, the temperature increases with the increase in Prandtl number P_r and decreases with the increase in source parameter S.

Key words: *MHD, Hele-Shaw flow, Heat transfer, Rivlin-Ericksen fluid, moving boundaries.*

2000 AMS Subject Classification: *76W05, 76D27, 80A20, 76A10, 74G10.*

INTRODUCTION

The Hele-Shaw flows of Newtonian and non-Newtonian fluids have attained much attention in recent years. Also, several studies have been made to investigate the flow of non-Newtonian fluids. Further, due to technological advances new materials of industrial importance have been developed whose rheological properties can not be adequately characterized by the classical Newtonian model but in recent years, especially with the emergence of polymers, it has been found that there are fluids which show a distinct deviation from Newtonian hypothesis. Such fluids are non-Newtonian fluids. In most of the investigation of visco-elastic fluids, the flow has been considered slow and the parameters characterizing the elastic properties of the fluids have been assumed small.

The unsteady Hele-Shaw flow of a non-Newtonian fluid and of a visco-elastic fluid through porous media respectively has been studied by GUPTA *et. al.* [5]. SINGH and SHARMA [12] have discussed the unsteady Hele-Shaw flow of a viscous fluid between two parallel porous walls in the presence of magnetic field applied perpendicular to the flow. KUMAR and SINGH [6] have studied on MHD Hele-Shaw flow of an elasticoviscous fluid through porous media. The heat transfer in the flow of a conducting fluid between two non-conducting porous disks-one rotating and the other at rest, in the presence of a transverse uniform magnetic field, the lower disk being adiabatic (given by SCHLICHTING [8]) was studied by BHATTACHARJEE and BORKAKATI [1]. SINGH and SINGH [11] discussed the laminar flow and heat transfer of an incompressible, electrically conducting second order Rivlin-Ericksen liquid in porous medium down a parallel plate channel inclined at an angle θ to the horizon in the presence of uniform transverse magnetic field. The MHD Hele-Shaw flow of dusty elasticoviscous fluid through porous media was discussed by VARSHNEY [13]. RATHOD and SHRIKANTH [7] have studied the unsteady MHD flow of Rivlin-Ericksen incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field. The magneto hydrodynamic unsteady flow of a visco-elastic liquid (Rivlin-Ericksen) near a porous wall suddenly set in motion with the heat transfer including heat generating sources or heat absorbing sinks has been studied by DATTA *et. al.* [4]. CHAKRABORTY and BORKAKATI [3] have investigated the laminar convection flow of an electrically conducting second order visco-elastic fluid in porous medium down an inclined parallel plate channel in the presence of uniform transverse magnetic field. The unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal parallel plates, the lower plate being a stretching sheet and upper being porous was investigated by SHARMA and KUMAR [9]. Recently, BODOSA and BORKAKATI [2] have studied MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel with heat sources or sinks.

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In the present paper, we study the unsteady MHD Hele-Shaw flow of Rivlin-Ericksen fluid through an inclined channel between two parallel flat plates under the influence of magnetic field with heat transfer including heat generating sources, when the boundaries are moving in opposite directions with velocity decreasing exponentially with time. Perturbation method is applied for solution. The effects of magnetic parameter M, source parameter S, elastic parameter R_c and Froude number F_r on the velocity distribution are discussed graphically. Also, the effects of Prandtl number P_r and source parameter S on the temperature distribution are discussed with the help of graphs.

MATHEMATICAL FORMULATION

Let us consider the two-dimensional Hele-Shaw flow of incompressible electrically conducting Rivlin-Ericksen fluid through an inclined channel between two parallel flat plates which are at a distance $2h$ apart under the influence of a uniform transverse magnetic field with heat transfer. We assume that the x'- axis is along a straight line midway between the two plates, the y'- axis perpendicular to it. A magnetic field of uniform strength B_0 is assumed to be applied in the y'- direction. Let u' be the velocity component along the direction of x'- axis.

To form the governing equations of the problem, we make the following assumptions-

1. The flow is assumed to be slow and unsteady.
2. The plates are infinitely long, so that the fluid velocity u' is a function of y' and t' only.
3. The temperature is uniform within the fluid particles and the buoyancy force is considered in the equation of motion of the fluid.
4. The flow between the plates is fully developed.
5. The conductivity of the fluid is assumed to be very small so that the induced magnetic field is neglected.
6. The Hall effect and viscous dissipation are assumed to be neglected.
7. Only electro-magnetic body force (Lorentz force) is considered.
8. Initially i.e. at time $t = 0$, the plates and the fluid are at zero temperature (i.e. $T = 0$) and there is no flow with in the channel. At time $t > 0$, the temperature of the plate $y = h$ changes to $\frac{\partial T}{\partial y} = 0$ and the temperature of the plate $y = -h$ changes according to $T = T_0 + (T_w - T_0)e^{-nt}$ where T_w and T_0 are the temperature of the plates and n is a non-negative real number, denoting the decay factor.
9. The velocities of both boundaries decrease exponentially with time.

Under the above assumptions, the governing equations of the problem are

$$\frac{\partial u'}{\partial x'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} + \frac{k_0}{\rho}\frac{\partial^3 u'}{\partial t'\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' + g\sin\theta + g\beta(T' - T_0) \tag{2}$$

$$\frac{\partial T'}{\partial t'} = -\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_0) \tag{3}$$

where

ρ = density of the fluid

B_0= uniform magnetic field applied transversely to the plate

σ = electrical conductivity of the fluid

ν = kinematic viscosity

k = thermal conductivity

C_p= specific heat of the fluid

β = coefficient of thermal expansion

g = acceleration due to gravity

p' = pressure

k_0 = coefficient of the elasticity

η_0 = coefficient of viscosity

S' = coefficient of heat source

The boundary conditions of the problem are

$$\left.\begin{aligned} t' > 0: \; u' &= -u_0 e^{-n't'}, T' = T_0 + (T_w - T_0)e^{-n't'} && at \;\; y' = -h \\ : \; u' &= u_0 e^{-n't'}, \frac{\partial T'}{\partial y'} = 0 && at \;\; y' = +h \end{aligned}\right\} \tag{4}$$

Now we consider the following non-dimensional parameters as given by SHIH-I PAI [10]:

$$x = \frac{x' u_0}{\nu},\; y = \frac{y' u_0}{\nu},\; y = \frac{u'}{u_0},\; t = \frac{t' u_0^2}{\nu},\; M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},\; T = \frac{T' - T_0}{T_w - T_0},\; P_r = \frac{\mu C_p}{k},\; n = \frac{\nu n'}{u_0^2},$$

$$S = \frac{S' \nu}{u_0^2},\; G_r = \frac{\nu g \beta\,(T_w - T_0)}{u_0^3},\; p = \frac{p'}{\rho u_0^2},\; R_c = \frac{k_0 u_0^2}{\rho \nu^2},\; F_r = \frac{u_0^2}{gh},\; R_e = \frac{u_0 h}{\nu},\; \frac{h u_0}{\nu} = 1$$

Substituting the non-dimensional parameters in the equations (1) - (3), we get

$$\frac{\partial u}{\partial x} = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + R_c \frac{\partial^3 u}{\partial t \partial y^2} - Mu + \frac{\sin\theta}{F_r R_e} + G_r T \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + ST \tag{7}$$

where R_c is the elastic parameter, M is the magnetic field parameter, F_r is the Froude number, R_e is the Reynolds number, G_r is the Grashof number, P_r is the Prandtl's number and S is the source parameter.

The non-dimensional boundary conditions are

$$\left.\begin{aligned} t > 0: \;\; u &= -e^{-nt}, T = e^{-nt} && at \;\; y = -1 \\ : \;\; u &= e^{-nt}, \frac{\partial T}{\partial y} = 0 && at \;\; y = +1 \end{aligned}\right\} \tag{8}$$

SOLUTION OF THE PROBLEM

The equation (5) shows that u is a function of y and t only and constant. Also the equation (6) shows that the velocity u in independent of x and therefore u is a function of y and t only. Thus the term $\frac{\partial p}{\partial x}$ must be a constant of function of t only. Let

$$\frac{\partial p}{\partial x} = -h(t) \tag{9}$$

Then equation (6) becomes

$$\frac{\partial u}{\partial t} = h(t) + \frac{\partial^2 u}{\partial y^2} + R_c \frac{\partial^3 u}{\partial t \partial y^2} - Mu + \frac{\sin\theta}{F_r R_e} + G_r T \tag{10}$$

Now we assume the solution of the problem as

$$\left.\begin{aligned} u &= f(y)e^{-nt} \\ T &= g(y)e^{-nt} \\ h &= h_0 e^{-nt} \end{aligned}\right\} \tag{11}$$

The corresponding boundary conditions are given by

$$\left.\begin{aligned} f(-1) &= -1, && g(-1) = +1 \\ f(+1) &= +1, && g'(+1) = 0 \end{aligned}\right\} \tag{12}$$

Using equation (11) into equation (7) and (10), we obtain

$$(1 - nR_c)\frac{\partial^2 f}{\partial y^2} - (M - n)f = -h_0 - \frac{\sin\theta\, e^{nt}}{F_r R_e} - G_r g \tag{13}$$

$$\frac{\partial^2 g}{\partial y^2} + P_r(S + n)g = 0 \tag{14}$$

Now solving the equations (13) and (14) using the boundary conditions (12), we get

$$f(y) = \left[\frac{-M_2}{(M-n)\cosh\, b_1} - \frac{G_r}{2M_1 \cosh\, b_1}\left(1+\frac{1}{\cos\, 2a_1}\right)\right]\cosh b_1 y$$

$$+\left[\frac{1}{\sinh\, b_1} + \frac{G_r}{2M_1 \sinh\, b_1}\left(1-\frac{1}{\cos\, 2a_1}\right)\right]\sinh b_1 y + \frac{M_2}{(M-n)} + \frac{G_r \cos\, a_1(1-y)}{M_1 \cos\, 2a_1} \tag{15}$$

$$g(y) = \frac{\cos\, a_1(1-y)}{\cos\, 2a_1} \tag{16}$$

where

$$a_1 = \sqrt{P_r(S+n)}\,,\ b_1 = \sqrt{\frac{M-n}{1-nR_c}}\,,\quad M_1 = P_r(S+n)(1-nR_c) + M - n\,,\ M_2 = h_0 + \frac{\sin\, \theta e^{nt}}{F_r R_e}$$

Hence, using the equations (15) and (16) in equation (11), the solution is given by

$$u = \left[\frac{1}{\sinh\, b_1} + \frac{G_r}{2M_1 \sinh\, b_1}\left(1-\frac{1}{\cos\, 2a_1}\right)\right] e^{-nt}\sinh b_1 y$$

$$-\left[\frac{M_2}{(M-n)\cosh\, b_1} + \frac{G_r}{2M_1 \cosh\, b_1}\left(1+\frac{1}{\cos\, 2a_1}\right)\right] e^{-nt}\cosh b_1 y + \frac{M_2 e^{-nt}}{M-n} + \frac{G_r e^{-nt}\cos\, a_1(1-y)}{M_1 \cos\, 2a_1} \tag{17}$$

$$T = \frac{e^{-nt}\cos\, a_1(1-y)}{\cos\, 2a_1} \tag{18}$$

RESULTS AND DISCUSSION

The velocity distribution has been tabulated in tables 1-4 and plotted in figures 1-4 against y for fixed h_0=1.0, n=1.0, t=1.0, θ=30°, P_r=0.5, G_r=5.0, R_e=1.0 and different values of magnetic parameter M, source parameter S, elastic parameter Rc and Froude number F_r respectively.

Fig. 1 is obtained by plotting velocity distribution for fixed S=0.05, R_c = 0.1, F_r=3.0 and different values of magnetic parameter M. It is clear that the velocity u increases up to y=0.2 and then decreases up to y=1.0. It is also observed that the velocity u is maximum near y=0.2 and is minimum near y=-1.0. Also, comparison of graphs shows that the fluid velocity decreases continuously with increasing magnetic parameter M from 1.5 to 7.5.

Fig. 2 is drawn for the fluid velocity for fixed M=1.5, R_c = 0.1, F_r=3.0 and different values of source parameter S. Comparing the graphs for S=0.25 to 0.65 with the graph for S=0.05, it is concluded that the fluid velocity decreases as S increases from 0.05 to 0.65.

Again, fig. 3 depicts the velocity profiles for fixed M=1.5, S=0.05, F_r=3.0 and different values of elastic parameter R_c. It is found that the velocity profiles have the same characteristics as Fig. 1 when y increases from -1.0 to 1.0. Also, comparison of graphs shows that the fluid velocity increases as R_c increases from 0.1 to 0.4.

Fig. 4 is obtained by plotting the velocity distribution for fixed M=1.5, S=0.05, $R_{C=}$0.1 and different values of Froude number F_r. The graphs in this figure are coincident in nature. It is found from table-4 that the fluid velocity u gradually decreases when F_r increases from 3.0 to 12.0.

The temperature distribution has been tabulated in tables 5 and 6 and plotted in figures 5 and 6 against y for fixed n=1.0, t=1.0 and different values of Prandtl number P_r and source parameter S.

From fig. 5 for fixed S=0.05, the temperature slightly increases with the increase in y. It is also observed that when P_r increases from 0.0025 to 0.5, the temperature increases continuously.

Finally, fig. 6 is drawn for the temperature distribution for fixed P_r=0.5 and different values of S. It is found that the temperature decreases continuously with the increase in y except the graph for S=0.05 for which the temperature increases with the increase in y. On comparing the graphs for S=0.25 to 0.65 with the graph for S=0.05, it is observed that the temperature decreases with the increase in source parameter S.

Table 1: Values of Velocity for h_0 = 1.0, n = 1.0, t = 1.0, θ = 30°, P_r = 0.5, G_r = 5.0, R_e = 1.0, S = 0.05, R_c = 0.1, F_r = 3.0 and different values of M

y	M=1.5	M=2.5	M=3.5	M=7.5
-1.0	-0.368	-0.368	-0.368	-0.368
-0.5	3.2161	2.2691	1.7377	0.8762
0.0	5.1416	3.7249	2.9111	1.5337
0.5	4.4846	3.4049	2.7645	1.6107
1.0	0.3679	0.3679	0.3679	0.3679

Table 2: Values of Velocity for h_0 = 1.0, n = 1.0, t = 1.0, θ = 30°, P_r = 0.5, G_r = 5.0, R_e = 1.0, M = 1.5, R_c = 0.1, F_r = 3.0 and different values of S

y	S = 0.05	S = 0.25	S = 0.45	S = 0.65
-1.0	-0.368	-0.368	-0.368	-0.368
-0.5	3.2161	2.9446	2.7155	2.5196
0.0	5.1416	4.7265	4.3762	4.0767
0.5	4.4846	4.1348	3.8396	3.5871
1.0	0.3679	0.3679	0.3679	0.3679

Table 3: Values of Velocity for h_0 = 1.0, n = 1.0, t = 1.0, θ = 30°, P_r = 0.5, G_r = 5.0, R_e = 1.0, M = 1.5, S = 0.05, F_r = 3.0 and different values of R_c

y	R_c = 0.1	R_c = 0.2	R_c = 0.3	R_c = 0.4
-1.0	-0.368	-0.368	-0.368	-0.368
-0.5	3.2161	3.5495	3.9549	4.4587
0.0	5.1416	5.651	6.2722	7.0463
0.5	4.4846	4.9236	5.4629	6.1414
1.0	0.3679	0.3679	0.3679	0.3679

Table 4: Values of Velocity for h_0 = 1.0, n = 1.0, t = 1.0, θ = 30°, P_r = 0.5, G_r = 5.0, R_e = 1.0, M = 1.5, S = 0.05, R_c = 0.1 and different values of F_r

y	F_r = 3.0	F_r = 6.0	F_r = 9.0	F_r = 12.0
-1.0	-0.368	-0.368	-0.368	-0.368
-0.5	3.2161	3.1877	3.1782	3.1734
0.0	5.1416	5.1041	5.0915	5.0853
0.5	4.4846	4.4561	4.4466	4.4419
1.0	0.3679	0.3679	0.3679	0.3679

Table 5: Temperature Distribution for n = 1.0, t = 1.0, S = 0.05 and different values of P_r

y	P_r = 0.0025	P_r = 0.025	P_r = 0.25	P_r = 0.5
-1.0	0.3679	0.3679	0.3679	0.3679
-0.5	0.3687	0.3767	0.5092	1.4104
0.0	0.3693	0.383	0.6174	2.2698
0.5	0.3697	0.3868	0.6852	2.8346
1.0	0.3698	0.3881	0.7083	3.0313

Table 6: Temperature Distribution for n = 1.0, t = 1.0, P_r = 0.5 and different values of S

y	S = 0.05	S = 0.25	S = 0.45	S = 0.65
-1.0	0.3679	0.3679	0.3679	0.3679
-0.5	1.4104	-13.3568	-0.8080	-0.3127
0.0	2.2698	-25.0216	-1.8396	-0.9299
0.5	2.8346	-32.8273	-2.5428	-1.3586
1.0	3.0313	-35.5703	-2.7921	-1.5119

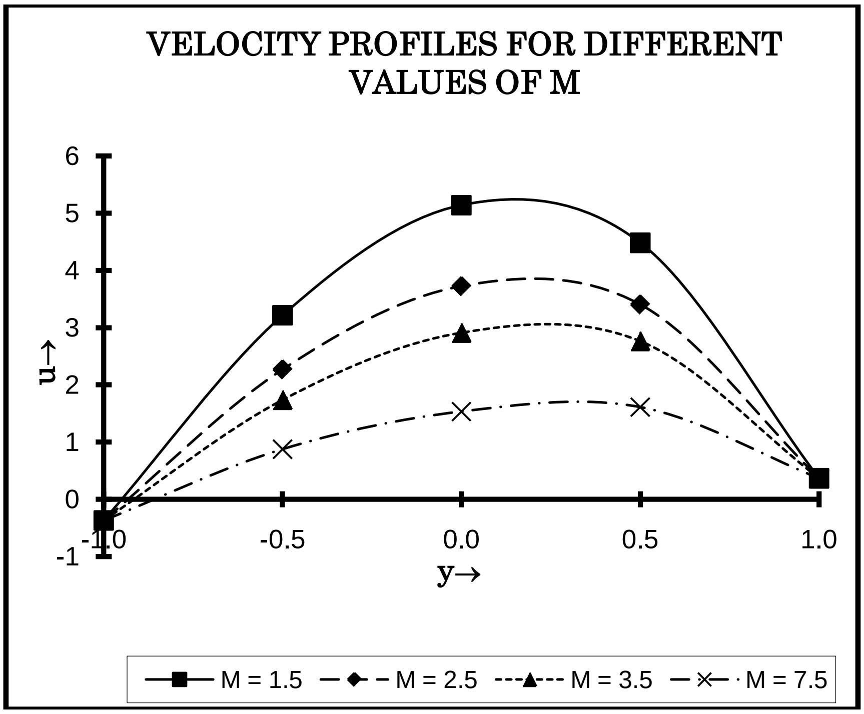

Fig. 1

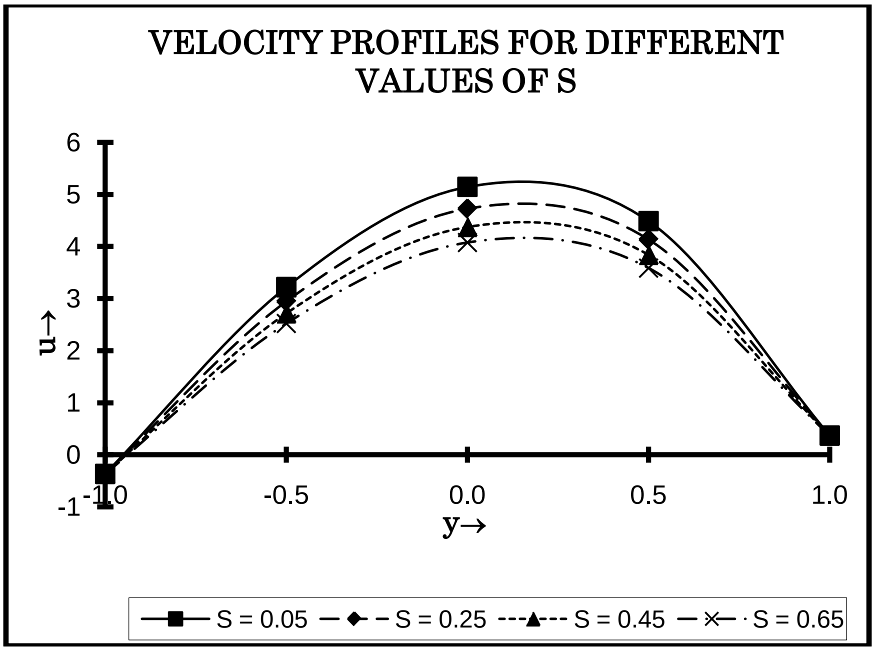

Fig. 2

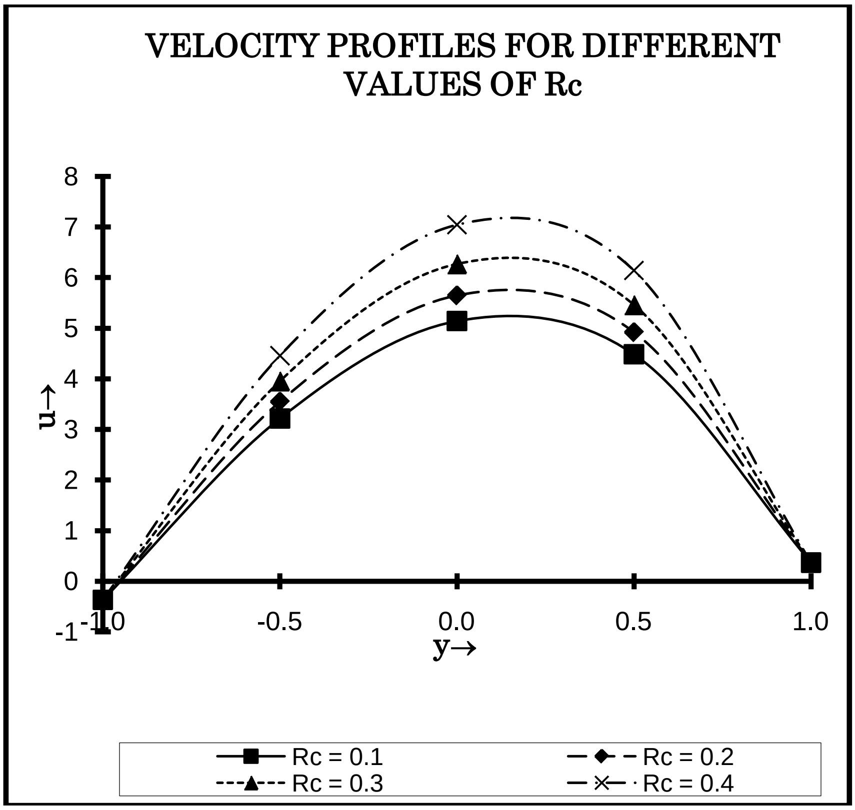
VELOCITY PROFILES FOR DIFFERENT VALUES OF Rc
8
7
6
5
4
3
2
1
0
-1
u→
-1.0
-0.5
0.0
0.5
1.0
y→
Rc = 0.1
Rc = 0.2
Rc = 0.3
Rc = 0.4

Fig. 3

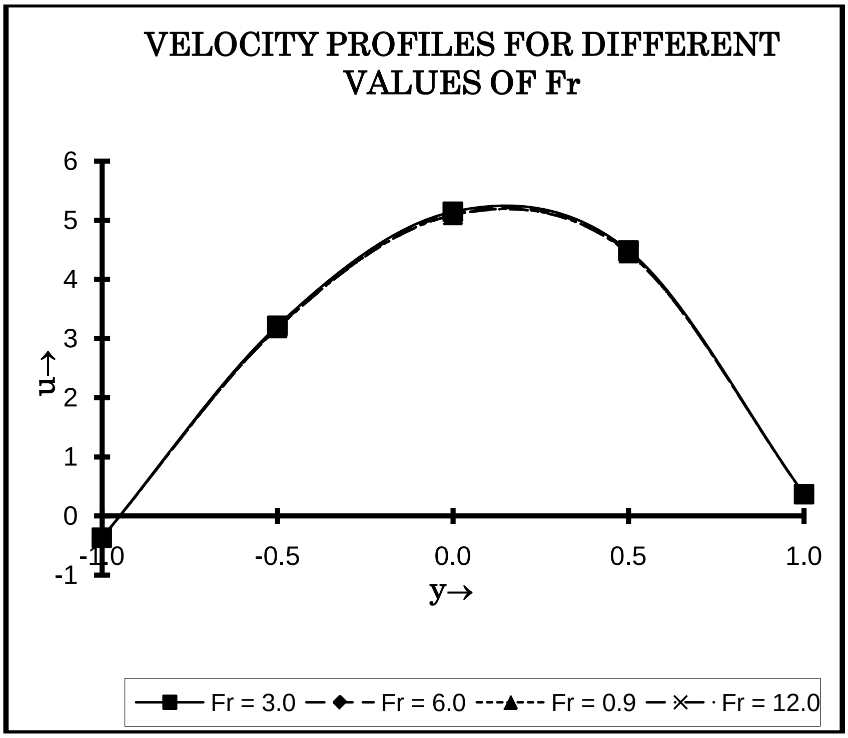
VELOCITY PROFILES FOR DIFFERENT VALUES OF Fr
6
5
4
3
2
1
0
-1
u→
-1.0
-0.5
0.0
0.5
1.0
y→
Fr = 3.0
Fr = 6.0
Fr = 0.9
Fr = 12.0

Fig. 4

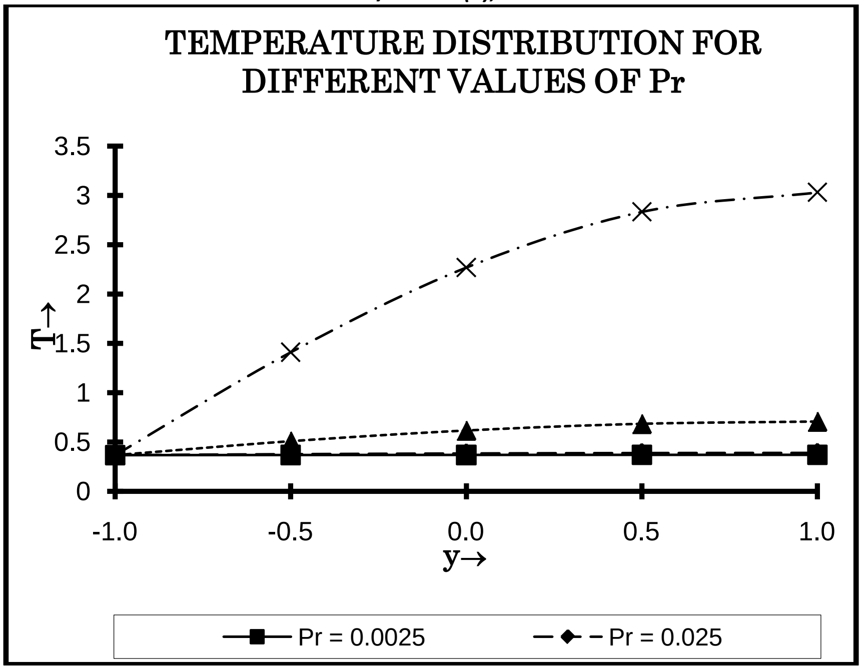

Fig. 5

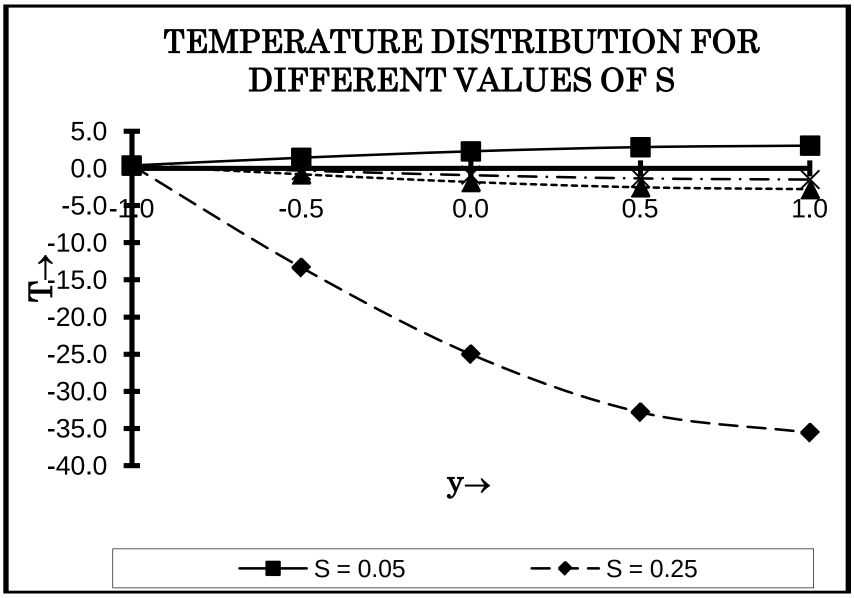

Fig. 6

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