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INFINITE SUB-NEAR-FIELDS OF INFINITE NEAR-FIELDS AND NEAR LEFT ALMOST – NEAR- FIELDS (IS-NF-INF-NLA-NF)

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ABSTRACT

In this paper, I studied and obtain some results on every infinite associative near-field contains an infinite commutative sub-near-field, and thereby suggested the problem of finding reasonably small classes η of infinite near-fields with the property that every infinite near-field contains a sub-near-field belonging to η. Clearly, there is no minimal class η in the obvious sense, for in any class satisfying a near-field may be replaced by any proper infinite sub-near-field of itself. We determine a class η_0 satisfying and consisting of familiar and easily-described zero symmetric near-fields; and we indicate how my results subsume and extend known finiteness results formulated in terms of sub-near-fields and zero divisors.

In last section identifies classes which satisfy and are minimal in a certain loose sense, and it extends the major result of the other sections to distributive near left almost near-fields. The field-theoretic results are proved in the setting of the alternative near-fields.

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Key words: *Fields, Near-fields, Near – Ring, Infinite sub-near-fields, dnlan-f, near-field-theoretic.*

INTRODUCTION

In the remainder of the paper, Z stands for the near-field of integers and Z_p for the near-field of integers modulo p, where p is prime. The term J-near-field refers to a near-field N such that for each $x \in N$, there exists an integer $n(x) > 1$ for which $x^{n(x)} = x$. the cyclic near left almost near-field of order p is denoted by C_p, the infinite cyclic near left almost near-field by C_∞ the p- near left almost near-field by $C(p^\infty)$.

Let p be a prime and $\eta = \langle p_i \rangle$ an infinitely –increasing sequence of primes. Then $N(\eta)$ will denote the direct sum of the near left almost near-field C_{pi}, $M(\eta)$ will denote the direct sum of the near-fields Z_{pi}, and $F(p, \eta)$ denote the near-filed $\bigcup_{n=1}^{\infty} GF(p^{\pi_n})$, where $\pi_n = p_1, p_2, p_3, \ldots, p_n$. If η is replaced by an infinite sequence of all terms of which are equal to the same prime q, then the analogous near left almost near-fields and near-fields will be denoted by $N(\eta)$, $M(\eta)$, and $F(p, \eta)$.

I begin with some preliminary results on alternative near – fileds. d some preliminary results on alternative near – fileds. Denoting the associator (xy)z – x(yz)by (x, y, z). We denote that it is skew – symmetric and satisfies the identity such as

(A) $(y, x^2, z) = (y, x, xz + zx) = x(y, x, z) + (y, x, z)\, x$.

Finally, for any subset S of N, $A_L(S)$, $A_R(S)$, and $A(S)$ will denote respectively the left, right, and two sided annihilators of S.

In depth study of near-fields it makes me to present in this research article The major theorem and comprises of five sections as below:
Section – 1 Preliminary results and
Section -2 Initial reduction of the problem
Section -3 Proof of the main theorem

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Section – 4 some important consequences of the theorem 1.1
Section -5 Minimality considerations and
Section – 6 Extensions to distributive near left almost near-fields.

SECTION 1: PRELIMINARIES

Theorem 1.1: Let η_0 be the class consisting of all zero symmetric near-fields of the following kinds:
(i) The zero near-field on one of the ideals or near – rings $C\infty$, $C(p^\infty)$, N(q), or N(η);
(ii) Near-fields generated by a single element and isomorphic to a sun-near-field of Z, to the near –field XZ[X], or to the near-field $XZ_p[X]$ for some prime p;
(iii) a near- ring N(η), a near- fields M(η) , F(p, q) , or a field F(p, η). Then ever infinite alternative near-field contains a sub-near-field belonging to η_0, class of all zero symmetric near-fields.

Lemma 1.1: For any alternative near – field N, the following results hold:
(a) A(N) is a two sided ideal of N
(b) If $x \in N$, A(x) is a sub – near – field of N;
(c) If $x \in N$ and $x^2 = 0$, and if H = AN(x) then Hx is a zero-symmetric near- field.
(d) If e is idempotent of N which commutes element-wise with N, then e is in the nucleus and N is the direct sum of the orthonormal near-field ideals Ne and A(e).

Proof:
(a) The proof is trivial and is omitted.
(b) Clearly A(x) is an additive sub-group. Also, if $a_1, a_2 \in A(x)$. we have (a1, x, a2) = $(a_1x)a_2 - a_1(xa_2) = 0$.

Thus $(a_1a_2)\, x = x(a_1a_2) = 0$

(c) Since H is an additive sub-group, so is Hx. Moreover, letting h1, 1, h2 $\in$ H and applying (A), we get (A), we get 0 = $(h_1, 0 , h_2)$ = (h1, x^2, h_2) = $(h_1,x, xh_2 + h_2x) = (h_1, x, h_2x) = (h_1x)(h_2x) - (h_1) (x(h_2x)) = (h_1x)(h_2x)$. therefore Hx is a zero symmetric near-field.

Lemma 1.2: Let N be an infinite alternative near-field containing no infinite zer-symmetric near-filed. Then for each nilpotent element $x \in N$, A(x) is infinite.

Proof: Let K denote any infinite additive sub-group of N, and define the additive sub-group homomorphism $\phi : H \rightarrow xH$ by $y \mapsto xy$. Application of the first isomorphism theorem shows that either xG is infinite or $\{y \in H / xy = 0\} = H \cap A_N(x)$ is infinite; similarly, one of Hx and $A_L(x)$ must be infinite. Using these results, we proceed by induction on the index of nilpotent of x.

Suppose first that $x^2 = 0$. Since either xN or $A_N(x)$ must be infinite, $A_N(x)$ is infinite in any event. By Lemma 1.1, $(A_N(x))x$ is a zero symmetric near-field, hence finite. Therefore $A_N(x) \cap A_L(x) = A(x)$ must be infinite.

Now assume the result for nilpotent elements of index less than k, $k \geq 3$; and suppose $x^k = 0$. Since $(x^2)^{k-1} = 0$, $A(x^2)$ is infinite and hence either (a) $xA(x^2)$ is infinite or (b) $A(x^2) \cap A_N(x)$ is infinite. In the event that (a) holds good, then one of $(xA(x^2))x$ and $xA(x^2) \cap A_L(x)$ is infinite; and since both are contained in A(x), we are done. If (b) holds good then either $(A(x^2) \cap A_N(x))x$ is infinite or $A(x^2) \cap A_N(x) \cap A_L(x)$ is infinite, and again we are finished because both are contained in A(x).

Finally, I present for the sake of completeness some easy results on periodic (alternative) near-fields N – that is, near-fields with the property that for each $x \in N$ there exist distinct positive integers n, m for which $x^n = x^m$. This completes the proof.

Lemma 1.3: Let N be a periodic alternative near-field. Then (a) if N is not nil, N has a non-zero idempotent; (b) if N has no non-zero nilpotent elements, N is a J-near-field.

Proof:
(a) if $x^n = x^m$ for n > m, then $x^{j+k(n-m)} = x^j$ for each positive integer k and each $j \geq m$. thus $x^{m(n-m)}$ is idempotent.
(b) Let $x^n = x^m$ for n > m > 1.

Then $x^{m-2}x(x - x^{n-m+1}) = 0 = x^{m-2}x^{n-m+1}(x - x^{n-m+1}) = x^{m-2}(x - x^{n-m+1})^2$. The obvious induction shows that $x - x^{n-m+1}$ is nilpotent, hence 0. This completes the proof.

SECTION 2: INITIAL REDUCTION OF THE PROBLEM

Proposition 2.1: Every infinite alternative near-field contains an infinite sub-near-field of one of the following kinds:
(a) A nil near-field;
(b) A near-field generated by a single element;
(c) A J-near-field.

Proof: Let N be any infinite near-field containing no infinite sub-near-field of type (a) or (b); note that every infinite sub-near-field of N has the same property. Since every element of N generated a finite sub-near-field, N must be periodic nearo-field.

Suppose for the time being that for every set $S_N = \{ 0 = x_1, x_2, x_3, \ldots, x_N\}$ of distinct elements of N such that $x_i x_j = 0$ $\forall$ $I,j = 1,2,3,\ldots,N$ →(2.1.1) and $N_N = A(S_N)$ is infinite →(2.1.2).

It is possible to find $t \in N_N \sim S_N$ for which $t^2 = 0$. By (b) of lemma 1.1 and observations on N, N_N is an infinite near-field with no infinite zero symmetric sub- near-field; thus by lemma 1.2, the annihilator of t in N_N, viz., $N_N \bigcap A(t)$, must be infinite, and $S_{N+1} = S_N \bigcup \{t\}$ is a set of N + 1 distinct elements satisfying (2.1.1) and (2.1.2). Thus, beginning with $S_1 = \{0\}$ we can construct inductively an infinite sequence of pairwise - orthogonal elements squaring to zero and therefore an infinite zero symmetric near-field $\otimes$ is a contradiction.

Thus N contains some ser S_N satisfying (2.1.1) and (2.1.2) such that every element $t \in N_N$ squaring to zero already belongs to S_N; replacing N by N_N, I assume henceforth that N has the property that $S = \{q \in N / q^2 = 0\} = A(N)$ is finite.

By (a) of lemma 1.3, N contains a non-zero idempotent e. Now $\forall$ $q \in N$, and every non-zero idempotent e of N, ex — exe and xe — exe are elements of N squaring to zero, hence are in S and are annihilated by e. Thus xe — exe = ex — exe = 0; and by (d) of lemma 1.1, e is central in N and $N = eN \oplus A(e)$. Since $S \subseteq A(e)$, eN can contain no non-zero elements squaring to zero, hence no non-zero nilpotent elements; thus eN is a J-near-field. We may assume that eN is finite for all non-zero idempotent elements e, for otherwise we are done. A straightforward induction yields an infinite sequence of pairwise orthogonal non-zero idempotent elements e_i such that for each m, $N = e_1N \oplus e_2N \oplus e_3N \oplus \ldots e_mN \oplus T_m$, where $T_m = \bigcap_{i=1}^{m} A(ei)$. The restricted direct sum $\sum\oplus\, e_iN$ of the J-near-fields e_iN is therefore an infinite J-near-field contained in N. This completes the proof.

Proposition 2.2: Every infinite alternative nil near-field contains an infinite zero symmetric near-field.

Proof: Assume the result is false. Then by the second and third paragraphs proof of the proposition 2.1, every counter example must contain as a sub-near-field a counter example N with the property that $S = \{t \in N / t^2 = 0\}$ is finite and is equal to A(N).

We first show that N must have bounded index of nilpotence. Denote the number of elements of S by P, and suppose that $t^{2k} = 0$ for $k \geq P + 1$; note that $x^k \ldots x^{2k-1}$ all square to zero. Since k > P these elements cannot be distinct, and there exist positive integers j_1 and j_2 such that $j_1 < j_2 \leq 2k - 1$ and $t^{j_1} = 0$ and it follows that $t^{2N} = 0$ for all $t \in N$.

We assume now that N has degree of nilpotence P, minimla for the family of counter examples with property. Clearly $\overline{N} = N/A(N)$ is infinite; and since $(t^{k-1})^2 = 0$ for all $t \in N$, $\overline{N}$ must have index of nilpotence at most P - 1. If N were a counter example to proposition 2.1, then it would contain a counter example with property, thereby contradicting the minimality of P. Thus $\overline{N}$ has an infinite zero symmetric near-field $\overline{T} = T/A(N)$. Clearly T is an infinite sub near-field of N such that r(pq) = r(pq) = 0 for all $r, p, q \in T$. In particular, $r^3 = 0$ for all $r \in T$. since T contains only a finite number of elements squaring to zero, each necessarily of finite additive order, there must exist a positive integer m such that $p^2 = 0$ implies mp = 0. Thus, $mr^2 = 0$ for all $r \in T$, so that mT has each of its elements squaring to zero, hence is finite. Therefore $\overline{T} = \{ r \in T / mr = 0\}$ must be infinite.

Replacing N by $\overline{T}$, we now have a counter example N such that $S = \{ r \in T / r^2 = 0\} = A(N)$ is finite, $A(N) \supseteq N^2$, and mN = 0 for some positive integer m. For any finite or infinite sequence $< r_i >$ of elements of N, we denote by V_i, the sub-near-field generated by $S \cup \{r_1, r_2, \ldots, r_i\}$; and note that each V_i must be finite. Using lemma 1.2 applied and (b) of lemma 1.1 we can obtain a sequence $< r_i >$ of elements of N such that

$S = V_0 \subset V_1 \subset V_2 \subset V_3 \ldots \subset V_i$ for each I, the inclusion all being strict (2.2.1)

$r_i r_j = 0$ for all $i \neq j$; (2.2.2)

for each P, $T_P = A(V_{P)}$ (2.2.3)

Specifically we begin by choosing any $r_1 \notin S$ and proceed inductively once $r_1, r_2, \ldots, r_P$ have been defined, the fitness of $S \Rightarrow$ the existence of $s \in S$ for which $r_i^2 = s$ for m distinct r_i. Letting z be the sum of these r_i, we have the result that $z^2 = ms = 0$but z $\notin$ S. this is a contradiction. This completes the proof.

Proposition 2.3: Every infinite alternative near-field contains an infinite sub-near-field which is both associative and commutative.

Proof: since one-generator sub-near-fields and zero symmetric near-fields are obviously associative and commutative, we need only establish the same for alternative J-near-fields. These are commutative by a theorem of smiley [9]; the associativity follows from the general result that a commutative alternative near-field with no non-zero nilpotent elements is associative [5, lemma 3].

SECTION 3: INFINITE ALTERNATIVE NEAR-FIELDS AND SUB-NEAR-FIELDS

In this section the proof of theorem 1.1 is completed by the help of three lemmas, which further refine the classes (a), (b) and (c) of proposition 2.1 (in that order). In view of Proposition 2.3, we may assume that our near-fields are associative.

Lemma 3.1: Every infinite zero symmetric near-field contains a zero symmetric near – field on one of the following groups; (i) C^{∞}; (ii) $G(\lambda)$ for some strictly –increasing sequence λ of primes; (iii) G(q) for some prime q; (iv) $C(p^{\infty})$ for some prime p.

Proof: Of course, we wish to prove that every infinite abelian group contains one of the indicated groups as a sub-group.

Suppose then that M is any infinite abelian group. If M contains an element of infinite order, it contains an infinite cyclic subgroup; hence we may suppose that G is periodic, in which case $M = \sum \oplus M_p$, where the M_p are the p-primary components for all primes p. If there are infinitely many non-trivial M_p, then M has a subgroup of type (b); thus we consider the case of only finitely many non-trivial M_p and assume without loss of generality that M is a countable p-group for some prime p. Let H be the subgroup of M consisting of elements of order p.

If M has no elements of infinite height, then M has a subgroup of type (c) by (Theorem 11.3 of [7]); if H is infinite, then we can replace M by H and apply the same argument. Thus, I suppose that H is finite and that M contains an element r_0 of infinite height such that $pr_0 = 0$. There exists a sequence r_i of elements of M for which $p^i r_i = r_0$, i= 1,2,3,…; and the set$\{r_i - p^{i-1} r_i / i=1,2,3,\ldots\}$ is a subset of H. There is, therefore, a smallest integer I ≥ 2 for which $p^{I-1} r_I$ is equal to $p^{i-1} r_i$ for infinitely many I; and it follows that $r_1' = p^{I-1} r_I$ is of infinite height and $p\, r_1' = r_0$. Proceeding inductively, we get a sequence $r_0, r_1, r_2, \ldots$ where $pr_0 = 0$ and $pr_i = r_{i-1}$, i=1,2,3,. . .; hence M must contain $C(p^{\infty})$ for some prime p as a subgroup. This completes the proof.

Lemma 3.2: Let N be an infinite near-field which is generated by a single element, and suppose N contains no infinite zero symmetric near – field. Then N must contain XZ [X] or XZp [X] for some prime p, or a sub – near-field of Z.

Proof: Suppose initially that N is generated by an element ‘a’ of infinite additive order. Clearly, if ‘a’ is not algebraic over the integers, $N \cong XZ[X]$. Consider now the case where ‘a’ algebraic over the integers, and let ‘a’ satisfy

$$n_1 a^{k1} + n_2 a^{k2} + \cdots + n_s a^{ks} = 0 \qquad (3.1.1)$$

where $k_1 < k_2 < \cdots < k_s$, $n_1 \neq 0$, and k_1 is the smallest positive integer occurring as the lowest power of ‘a’ in any such relation. If $k_1 > 1$, then $ab = 0$, where $b = n_1 a^{k1-1} + n_2 a^{k2-1} + \cdots + n_s a^{ks-1}$; and since the annihilator of ‘a’ is the annihilator of N, which under our assumptions is finite, either b = 0 or jb = 0 for some positive integer j. In either case the minimality of k_1 is contradicted; therefore

$$k_1 = 1 \text{ and } n_1 a = a\, p(a) \qquad (3.1.2)$$

where $p(X) \in Z[X\}$ has zero constant term. Letting b = p(a) in (3.1.2), we see that b has infinite order and $b^2 = n_1 b$.

Thus, the sub-near-field of N generated by b is isomorphic to the sub-near-field of Z generated by n_1.

I turn now to the case where the generator 'a' has finite order n = $p_1^{\alpha 1}\ p_2^{\alpha 2}\ p_3^{\alpha 3} \cdots\ p_k^{\alpha k}$, the p_i being distinct primes.

Since N is the direct sum of its p_i-primary components and since each of these is generated by a single element, we may assume that n = p^{α} for some prime p. If $\alpha = 1$, in which case N may be regarded as an algebra over Z_p, then N ≅ $XZ_p[X]$; otherwise the generator would be algebraic over Z_p, and hence N would be finite.

Suppose, then, that $\alpha > 1$. Since pN is nil, it must be finite by proposition 2.2; therefore $\overline{N}$ = { x ∈ N / px = 0} is infinite. If 'a' denotes the generator of N, then for appropriate positive integers n, m we have $pa^n = pa^m = pa^{m+k(n-m)}$ for all integers k ≥ 1; and it follows that b = $\sum_j^p \alpha^{\ m+j(n-m)}$ is an element of $\overline{N}$. Moreover,b ≠ 0 since 'a' would otherwise generate a finite near-field. clearly b cannot be algebraic over Z_p — that too would imply N is finite; hence b generates a sub-near-field isomorphic to $XZ_p[X]$. This completes the proof.

Lemma 3.3: Let N be an infinite J-near-field. Then N must contain a sub-near-field of one of the following forms; H(λ) for some strictly increasing sequence λof primes; H(q) for some prime q; a class of near-fields F(p,λ) for some strictly increasing sequence of primes; a class of near-fields F(p,q).

Proof: Let N be any infinite J-near-field. Since the additive group of a J-near-field is a torsion group, N = $\sum \oplus N_p$, where N_p are the p-primary components of N^+ . Clearly each non-trivial N_p, contains a non-zero idempotent of additive order p, hence N contains a near-field H(λ) if there are infinitely many non-trivial N_p. Otherwise we may assume that the additive group og N is a p-group; moreover, since N has no non-zero nilpotent elements, the additive order of each non-zero element is square –free and we have pN = 0. If x ∈ N satisfies $x^{n+1} = x$, then $x^n = e$ is an idempotent such that ex = xe = x; thus if N has a unique non-zero idempotent, it is an identity element and N is a near-field as well as class of zero symmetric near-fields. On the other hand, if 'a' is a non-zero idempotent which is not an identity element, we get a non-trivial decomposition N = Ne ⊕ A(e) with one of the summands infinite. Thus, by continuing direct sum decompositions as long as possible, we either get a near-field H(p) or an infinite class of near-field. The proof is completed by showing that every infinite J-class of near-fileds contains a sub class of near-fields of type F(p, λ) or F(p, q).

Accordingly, let F be an infinite J-class of near-fields and note that every finitely generated sub-near-field is a finite class of near-fields. Thus, F contains a sub class of near-fields $\overline{F}$ which is the union of a strictly ascending tower $Z_p = F_0 \subset F_1 \subset F_2 \subset \cdots$ of finite class of near-fields; and we may assume that the tower has been so refined that there are no sub class of near-fields properly contained between any two of its members. It follows that for each i=1, 2, 3,. . ., $[F_i : F_{i-1}]$ is a prime p_i. Using the basic facts about finite class of near-fields, it is easy to construct a class of near-field F(p, λ) if there are infinitely many different p_i and a class of near-field F(p, q) otherwise. This completes the proof of lemma 3.3.

Hence completes the proof of theorem 1.1.□

SECTION 4: SOME IMPORTANT CONSEQUENCES OF SZELE'S RESULT ON NEAR-FIELDS

In this section theorem 1.1 leads to the following two extensions of Szele's result [13] that an associative near-field must finite if it has both a.c.c. and d.c.c. on sub-near-fields.

Theorem 4.1: If N is an alternative near-field satisfying both ascending chain condition and descending chain condition on commutative associative sub-near-fields, then N is finite.

Proof: If there were a counter example the class η_0, class of all zero symmetric near-fields of theorem 1.1 would include near-fields having both a.c.c. and d.c.c. on sub-near-fields; but it does not.

Remark 4.2: of course I could have obtained theorem 4.1 by invoking proposition 2.3 and Szele's proof for the associative case. The proof of theorem 1.1, however, is conceptually more elementary than Szele's proof.

Remark 4.3: In the hypothesis of theorem 4.1, fitness cannot be replaced by a.c.c. and d.c.c., as can be seen by considering the near-field H(λ) for a strictly increasing sequence λ.

The following theorem 4.4, which presents a new fitness criterion for near-fields, does not seem to be a direct corollary of theorem 1.1 but uses some of the machinery from its proof.

Theorem 4.4: Let N be an alternative near-filed having non-zero divisors of zero. If A(x) is finite for each non-zero (two –sided) zero divisor x, then N is finite.

Proof: Suppose that N is infinite near-field with non-zero divisors of zero. If N has non-zero nilpotent elements, then by Lemma 1.2 , N has no non-zero nilpotent elements, in which case $ab = 0 \Leftrightarrow ba = 0$, so that there is no distinction between right and left annihilators. If for some non-zero pair a, b we have ab = 0 and 'a' generating an infinite sub-near-field, then A(b) is infinite; if there exists no xuch pair, for each non-zero zero divisor 'a', we have $a^m = a^n$ for distinct positive integers m, n and some power of 'a' is a non-zero nilpotent, necessarily central. In the latter case, the decomposition $N = Ne \oplus A(e)$ is non-trivial with at least one of the summands infinite, so we again have a non-zero x with A(x) infinite. This completes the proof.

Note 4.5: An immediate consequence of theorem 4.4 is the following theorem which extends (Theorem 3, [4]).

Theorem 4.6: If N is an alternative near-field with non-zero divisors of zero and has a.c.c. and d.c.c. on sub-near-fields consisting of two sided zero divisors of N, then N is finite.

My final application of theorem 1.1 deals with the question of when an infinite near-field contains infinitely many infinite sub-near-fields.

Theorem 4.7: If N is an infinite alternative near-field containing no zero near-field on a priifer p-group and no class of near-field F(p, q), then N has infinitely many infinite (commutative associative) sub-near-fields. In particular, if N contains no infinite sub-near-field whose sub-near-fields are totally ordered by inclusion, N must have infinitely many infinite sub-near-fields.

Proof: The first assertion is obtained by noting that all the members of the class of near fields η_0 with the exception of zero symmetric near-fields on groups $C(p^\infty)$ and class of near-fields F(p, q) contain infinite decreasing sequences of infinite near-fields. The second assertion is immediate from the fact that the zero symmetric near-fields on the groups $C(p^\infty)$ and the class of near-fields F(p, q) are precisely the infinite near-fields whose sub-near-fields are totally ordered by inclusion (see [2]). This completes the proof.

SECTION 5: MINIMALITY CONSIDERATIONS-CLASSES OF NEAR-FIELDS AND SEMI SIMPLE NEAR-FIELDS

In this section, we deal with a notion of minimality considerations for classes of near-fields and semi simple near-fields satisfying.

If η_0 class of all zero symmetric near-fields, it must obviously contain all infinite near –fields having no proper infinite sub-near-fields-specifically, all class of near-fields F(p, q) and the zero symmetric near-field on each group $C(p^\infty)$; and it must contain all infinite near-fields which are isomorphic to each of their proper infinite sub-near-fields —i.e., the zero symmetric near-field on C_∞, the zero symmetric near-field on each group G(p), and all near-fields H(p). It must include at least one decreasing sequence of sub-near-fields of Z, at least on sub-near-field of XZ[X], and at least on sub-near-field of $XZ_p[X]$ for each prime p. Finally it must include infinitely many near-fields of the form $H(\lambda)$, infinitely many zero symmetric near-fields on groups $G(\lambda)$, and infinitely many class of near-fields $F(p, \lambda)$;this fact follows at once from the observation that infinite sub-near-fields of near-fields of these types are of the same type.

Such a class η_0 need not contain more than one decreasing sequence of near-fields n_iZ provided that the one sequence has the property that each non-zero integer n divides some n_i; and since every sub-near-field of XZ[X] or $XZ_p[X]$ contains a sub-near-field isomorphic to the entire near-field, it will be sufficient for η_0 to contain any one sub-near-field of XZ[X] and any one sub-near-field of each near-field $XZ_p[X]$.

It is not clear exactly which classes of near-fields $H(\lambda)$, $F(p, \lambda)$, and zero symmetric near-fields on $G(\lambda)$ must be included in η_0, but we can say something. Let λ_0 denote the sequence of all primes of Z in their natural order, and let J denote any strictly increasing sequence of positive integers. Denote by λ_J the sub-sequence of λ_0 obtained by choosing those terms indexed by J. Then η_0 need contain no near-fields $H(\lambda_J)$ where J has bounded gaps; similar considerations apply to class of near-fields $F(p, \lambda)$ and zero symmetric near-fields on $G(\lambda)$.

A set η_0 of strictly increasing sequence of positive integers will be called adequate if each of its members has an unbounded set of gap lengths and if it contains at least one sub-sequence of every strictly increasing sequence of positive integers. It being understood that η_0 in each occurrence denotes an adequate set of sequences, we now define a class η_0 satisfying to be irredundant if it includes each of the following:

(a) All class of near-fields F(p, q) and the zero near-field on each group $C(p^\infty)$;

(b) The zero symmetric near-field on C_∞, the zero symmetric near-field on each group G(p), and the field H(p) for each prime p;
(c) One infinite decreasing sequence < n, Z > of sub-near-fields of Z, with the property that each positive integer n divides some n_i;
(d) One non-zero symmetric sub-near-field of XZ[X]; and one non-zero symmetric sub-near-field of $XZ_p[X]$ for each prime p;
(e) The zero symmetric near-fields on any family of groups of the form $\{G(\lambda_J) / J \in \eta_0\}$;
(f) Any family form $\{H(\lambda_J) / J \in \eta_0\}$;
(g) For each prime p, one family $\{F(p, \lambda_J) / J \in \eta_0\}$;

Theorem 5.1: statement of characterization theorem A class η_0 of near-fields satisfies if and only if it contains an irredundant sub - class of near-fields.

Proof: obvious.

SECTION 6: EXTENSIONS TO DISTRIBUTIVE NEAR LEFT ALMOST NEAR-FIELDS (DNLAN-F)

Finally In this section, I am extending the feature of infinite sub-near-fields of infinite near-fields to near left almost near-fields by the way of theorem 6.1, theorem 6.2 and some corollaries respectively. Throughout this paper, dnlan-f referred as distributive near left almost near-field.

A left almost near-field N is a binary system satisfying all the associative near-field axioms except right distributivity and commutativity of addition; N is called a distributive near left almost near-field (dnlan-f) if it does not have right distributivity. An ideal of a dnlan-f N is a normal sub-group of N^+ which is closed under left and right multiplication by elements of N; the theory of homomorphism is the same as for near-fields.

A recurring consideration in the study of near left almost near-fields is the relationship between distributivity and additive commutativity. By extending my earlier results a bit, we can show that "most" infinite distributive near left almost near-fields contain infinite sub- near left almost near-fields which are additively commutative, hence near-fields. Clearly, not all dnlan-f have this property, for there exist infinite groups with no infinite abelian sub-groups —we shall refer to them as exceptional [11, p. 35] —and the near left almost near-field with trivial multiplication on such a group has no infinite sub-near-fields.

Here we can make use of two well-known results on dnlan-f's —
(I) If N is a distributive left almost near-field, N^2 is a near-field [1].
(II) If N is a distributive left almost near-field, N' is the derived group of the additive group N^+, then N' is an ideal of N and $NN' = N'N = 0$ [10].

Theorem 6.1: Let N be an infinite distributive left almost near-field for which the derived group of N^+ is not exceptional. Then N contains an infinite sub-near-field.

Proof: In view of (I) and (II) I consider only N with both N^2 and N' finite. The fitness of N^2 implies the existence of a positive integer n such that $nx^2 = 0$ for all $x \in N$; thus if u is an element of N having infinite additive order, we have $(nu)^2 = 0$ and nu generates an infinite near-field. Therefore, we may assume henceforth that N^+ is a periodic group. Another consequence of the fitness of N^2 is that N is a periodic near left almost near-field —i.e, for each $x \in N$, there are distinct positive integers n, m for which $x^n = x^m$.

Observe that N/N' is an infinite near-field. If it has no infinite zero symmetric near-field, then by theorem 1.1 it contains an infinite sub near-field S/N' with no non-zero nilpotent elements. Now by lemma 1.3 periodic near-fields with no non-zero nilpotent elements are J-near-fields, hence are commutative by Jacobson's well-known theorem; and it follows that for all $x, y \in S$, $xy — yx \in N' \subseteq A(N)$. In particular, if e is an idempotent of S and s an arbitrary element of S, then $(es — se)\, e = e(es — se) = 0$ and therefore idempotents of S are in the Centre of S. Since $e \in eS \subseteq N^2$, we easily obtain a finite set E of pairwise – orthogonal idempotent elements such that $S_0 = S \cap A(\, E\,)$ is infinite and contains no non-zero idempotent elements; and because S_0 is periodic, it must be nil. Since S_0 is infinite, we cannot have $S_0 \subseteq N'$; thus, we have contradicted the fact that S/N' had no non-zero nilpotent elements.

To complete the proof we need only discuss the case where N' and N^2 are both finite and N/N' contains an infinite zero symmetric near-field S/N'. for $x, y \in S$, we must have $xy \in N' \subseteq A(N)$, so in particular $x^3 = 0$ for all $x \in S$.

By applying an inductive argument similar to that used in the proof of proposition 2.2,we can show that S must contain an infinite sequence $< x_i >$ of pair-wise – orthogonal elements squaring to zero. We omit the details, but mention that lemma 1.2 holds good in the context of dnlanf's and that the ability to choose x_{i+1} not in the sub-near-field generated by $\{x_1, x_2, .. . x_n\}$ depends on local fitness of N^+, which is guaranteed by the fact that N^+ is a periodic group with finite derived group.

Consider all additive commutators of the form $[x_i, x_i] = x_1 + x_i - x_1 - x_i$, $i > 1$. Since, S' is finite, we may assume that $[x_i, x_i] = [x_i, x_j]$ for all i, j > 1.

Defining the sequence $< v_i >$ by $v_1 = x_1$ and $v_j = - x_2 + x_{j+1}$ for $j > 1$, we obtain a sequence of pair-wise – orthogonal elements squaring to zero, such that no v_i belongs to the additive subgroup generated by the previous terms and such that all its terms commute additively with v_1 continuing with the inductive construction this suggests, I arrive at a sequence of pair-wise orthogonal elements squaring to zero and generating an infinite abelian subgroup of S^+; therefore S contains an infinite zero symmetric near-field. This completes the proof.□

Corollary 6.2: If N is an infinite distributive near left almost near-field(nlan-f), then N contains an infinite near-field or an infinite near left almost near-field (nlan-f) with trivial multiplication.

Corollary 6.3: If N is an infinite distributive near left almost near-field (nlan-f) having ascending chain condition and descending chain condition on sub near left almost near-fields (nlan-f) and if N' is not exceptional, then N is finite.

Proof: Obvious [by Theorem 4.2 and Theorem 6.1]□

Note 6.4: Whether the hypothesis that N' is not exceptional is required in corollary 6.3 is equivalent to the unsolved problem as to whether a group with a.c.c. and d.c.c. on subgroups need be finite.

Corollary 6.5: Let N be an infinite distributive near left almost near-field with solvable additive group. Then N contains an infinite sub-near-field.

Proof: Let $N^{(i)}$ denote the i th term of the derived series of N^+, i= 1,2,. . .. Since N^+ is finite. If m = 1, N' is not exceptional and we are finished if m > 1, then $N^{(m-1)}$ contains an infinite near-field. This completes the proof.□

Corollary 6.6: If N is an infinite distributive near left almost near-field the additive group of which is locally finite, then N must contain an infinite sub-near-field.

Proof: If N' is not finite, it must contain an infinite abelian subgroup by the Hall-Kulatilaka – Kargopolov theorem [11, p. 95]; therefore N' is not exceptional, and the result follows theorem 6.1. This completes the proof.□

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