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On Weaker Forms of πgb-Continuous Functions

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ABSTRACT

We introduce slightly -πgb- continuous function and faintly πgb- continuous function which are weaker forms of πgb-continuous functions and its basic properties are obtained. Moreover their relationship with other spaces such as πgb-connected spaces, strongly πgb-normal spaces and πgb-compact spaces are investigated. Further πgb-closed graphs are discussed.

Keywords: *slightly πgb-continuous, faintly πgb-continuous, strongly πgbc-regular spaces, strongly πgbc-normal spaces, strongly πgb-co-closed and θ-πgb-graphs.*

1. INTRODUCTION

Functions and continuity plays a vital role in the area of research work in Mathematics. Different forms of continuity and its weaker forms have been discussed by many researchers. Few of them are α-continuity[14,15], α-irresoluteness[7], pre-continuity[2,13], semi-continuity[10], γ-continuity[4], slightly continuity[8,18], faintly continuity[12], almost continuity, somewhat continuity[9]. Levine [17] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{½}$ spaces. Andrijevic [1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [5] under the name of γ-open sets. Zaitsev [24] defined the concept of π-closed sets and a class of topological spaces called quasi-normal spaces. Dontchev and Noiri introduced the notion of πg-closed sets and characterized the properties and preservation theorems for quasi normal spaces. A point $x \in X$ is called a θ-cluster point of A if $Cl(V) \cap A \neq \phi$ for every open set V of X containing x. The set of all θ -cluster points of A is called the θ -closure of A and is denoted by $Cl_\theta(A)$. If $A = Cl_\theta(A)$, then A is said to be θ-closed. The complement of θ-closed set is said to be θ-open.

In this paper, slightly -πgb-continuity, faintly πgb-continuity, faintly πgb-closed graphs are introduced and its various characterizations are studied. Henceforth, basic properties and preservation theorems of these functions are obtained.

2. PRELIMINARIES

Definition 2.1: A subset A of (X, τ) is called πgb -closed [**21**] if $bcl(A) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ). By πGBC(τ) we mean the family of all πgb-closed subsets of the space (X, τ).

Definition 2.2: A function f: $(X, \tau) \to (Y, \sigma)$ is called
1) π- irresolute [2] if $f^{-1}(V)$ is π- closed in (X, τ) for every π-closed of (Y, σ);
2) b-irresolute: [5] if for each b-open set V in Y, $f^{-1}(V)$ is b-open in X;
3) b-continuous: [5] if for each open set V in Y, $f^{-1}(V)$ is b-open in X.
4) πgb- continuous [21] if every $f^{-1}(V)$ is πgb- closed in (X, τ) for every closed set V of (Y, σ).
5) πgb- irresolute [21] if $f^{-1}(V)$ is πgb- closed in(X, τ) for every πgb- closed set V in (Y, σ).
6) almost πgb-continuous if $f^{-1}(V)$ is πgb-open in (X, τ) for every regular open set V of (Y, σ).
7) faintly continuous [12] if $f^{-1}(V)$ is open in (X, τ) for every θ-open set V of (Y, σ).
8) slightly πg-continuous [16] if for each $x \in X$ and each clopen set V of Y containing f(x), there exists $U \in \pi GO(X, x)$ such that $f(U) \subset V$.

Definition 2.3: A topological space X is a πgb- space [21] if every πgb- closed set is closed.

Definition 2.4: A topological space (X, τ) is said to be πgb-$T_{1/2}$-space if every πgb-closed set is closed.

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Definition 2.5: A collection $\{A_i;\ i \in \Lambda\}$ of πgb-open sets in a topological space X is called πgb-open cover [22] of a subset B of X if $B \subset \cup \{A_i;\ i \in \Lambda\}$ holds.

Definition 2.6: A space (X, τ) is said to be. θ-compact [19] if each θ-open cover of X has a finite subcover.

Definition 2.7: A topological space X is πGBO-compact [22] if every πgb-open cover of X has a finite sub cover.

Definition 2.8: A subset B of a topological space X is said to be πGBO-compact[22] relative to X if, for every collection $\{A_i;\ i \in \Lambda\}$ of πgb-open subsets of X such that $B \subset \cup\{A_i;\ i \in \Lambda\}$, there exists a finite subset Λ_o of Λ such that $B \subset \cup\{A_i;\ i \in \Lambda_o\}$

Definition 2.9: A subset B of a topological space X is said to be πGBO-compact [22] if B is πGBO-compact as a subspace of X

Definition 2.10: A topological space X is said to be πGB-connected [22] if X cannot be expressed as a disjoint union of two non empty πgb-open sets.

A subset of X is πGB connected if it is πGB-connected as a subspace.

Definition 2.11: A map f: (X, τ)→(Y, σ) is said to be a M - πgb-closed map if the image f(A) is πgb-closed in Y for every πgb-closed set A in X.

Definition 2.12: A space X is said to be
(i) Mildly countably compact[18] if every clopen countably cover of X has a finite subcover.
(ii) Mildly Lindelof [18] if every open cover of X by clopen sets has a countable subcover.
(iii) πgb-Lindelof if every πgb- open cover of X has a countable subcover.
(iv) Countably πgb-compact if every πgb-open countably cover of X has a finite subcover.

Definition2.13: A space X is said to be mildly compact [18] (respectively πgb-compact) if every clopen cover (resp. πgb-open cover) of X has a finite subcover.

Definition 2.14: A topological space X is said to be

[1] πgb- regular[23] if for every closed set F and each point $x \notin F$, there exist disjoint πgb-open sets M and N such that $F \subseteq M$, $x \in N$, $M \cap N = \Phi$.

[2] πgbc-regular[23] if for every πgb-closed set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subseteq U$, $x \in V$ and $U \cap V = \Phi$

[3] πgbc-completely regular [23] if for every point $x \in X$ and each πgb-closed set F in X such that $x \notin F$, there exists a continuous map $f : X \to [0, 1]$ such that $f(x) = 0$ and $f(F) = \{1\}$.

[4] πgbc-normal [23] if for any pair of disjoint πgb-closed sets A and B, there exists disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

[5] clopen-regular[5] if for each clopen F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.

[6] clopen normal[5] if for each pair of disjoint clopen subsets A and B, there exists open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.15: A space X is said to be clopen T_1 [18] if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

Definition 2.16: A topological space (X, τ) is said to be
(i) πgb-T_1 (resp. θ-T_1 [19]) if for each pair of distinct points x and y, there exists πgb-open(resp. θ-open)sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.
(ii) πgb-T_2 (resp. θ-T_2 [19]) if for each pair of distinct points x and y, there exists disjoint πgb-open(resp. θ-open)sets U and V containing x and y respectively.

Definition 2.17: A space (X, τ) is strongly θ-regular [17] if for each θ-closed set F and each point $x \notin F$, there exists disjoint θ-open sets U and V such that $F_1 \subset U$ and $x \in V$.

3. Slightly -πgb- Continuous Functions

Definition 3.1: A function f: X → Y is called slightly -πgb- continuous if every $f^{-1}(V)$ is πgb- open in (X, τ) for every clopen set V of (Y, σ).

Theorem 3.2: Suppose πGBO(X) is closed under arbitrary unions. Let f: X → Y be a function. Then f is slightly πgb -continuous iff for each point x ∈ X and each clopen set V in Y containing f(x), there exists a πgb -open subset U in X containing x such that f(U) ⊂V .

Proof: Let x∈X and V be clopen set with f(x) ∈V. Since f is slightly πgb-continuous, $f^{-1}(V)$ is πgb-open in X. If we put U=$f^{-1}(V)$, then x∈U and f(U) ⊂V. Conversely, Let V be clopen subset of Y and let x∈$f^{-1}(V)$. Since f(x) ∈V, there exists a πgb-open set U_x in X containing x such that U_x⊂$f^{-1}(V)$. Hence $f^{-1}(V)$ =∪ {U_x: x ∈$f^{-1}(V)$}.This implies $f^{-1}(V)$ is πgb-open.

Theorem 3.3: Let (X, τ) and (Y, σ) be topological spaces. Suppose πGBO(X) is closed under arbitrary unions. The following statements are equivalent for a function f: X → Y:
(1) f is slightly πgb -continuous;
(2) for every clopen set V ⊂ Y , $f^{-1}(V)$ is πgb -open;
(3) for every clopen set V ⊂ Y , $f^{-1}(V)$ is πgb -closed;
(4) for every clopen set V ⊂ Y , $f^{-1}(V)$ is πgb -clopen.

Proof: Straight Forward.

Theorem 3.4:
a) If f is slightly continuous, then it is slightly πgb-continuous.
b) If f is continuous, then it is slightly πgb-continuous.

Remark 3.5: Converse of the above theorem need not be true

Example 3.6: Let X={a, b, c}, τ={Φ, X, {a}}, Y={p, q}, σ={ϕ, Y, {p},{q}}. Let f: (X, τ) → (Y, σ) be defined by f(a)=f(c)=q, f(b)=p. Then f is slightly πgb-continuous but not slightly continuous.

Example 3.7: Let X={a, b, c}, τ={Φ, X, {a},{b},{a, b}},Y={p,q}, σ={ϕ, Y, {p}}. Let f: (X, τ) → (Y, σ) be defined by f(a)=q=f(b), f(c)=p. Then f is slightly πgb-continuous but not πgb-continuous.

Remark 3.8: The following implication holds good for a topological space X.

T_1⇒ πgb-T_1.Converse need not be true.

Example 3.9: X= {a, b, c}; τ= {Φ, X, {a}, {b, c}}. (X, τ) is πgb-T_1 but not T_1.

Remark 3.10 [21]: Let A⊂Y⊂X.

(i) If Y is open in X, then A ∈πGBC(X) implies A∈ πGBC(Y).

(ii) If Y is regular open and πgb-closed in X, then A ∈πGBC(Y) implies A∈πGBC(X).

Theorem 3.11: Let f: (X, τ) → (Y, σ) be function.
(i) If f: (X, τ) → (Y, σ) is slightly πgb-continuous and Y is locally indiscrete, then f is πgb-continuous.
(ii) If f: (X, τ) → (Y, σ) is slightly πgb-continuous and X is πgb-space, then f is slightly-continuous.
(iii) Suppose πGBO(X) is closed under arbitrary unions. If f: (X, τ) → (Y, σ) is slightly πgb-continuous and U is open in X, then f/U: U→Y is slightly πgb-continuous.

Proof: (i) Let V be open in Y. Since Y is locally indiscrete, every open set is closed. Since f is slightly πgb-continuous, $f^{-1}(V)$ is πgb-open. Hence f is πgb-continuous.

(ii) Let V be clopen in Y. Since Y is slightly πgb-continuous, $f^{-1}(V)$ is πgb-open in X. Since X is πgb-space, every πgb-open in X. This implies $f^{-1}(V)$ is open in X. Hence f is slightly continuous.

(iii) Let V be clopen in Y. Since f is slightly πgb-continuous, $f^{-1}(V)$ is πgb-open in X.$(f/U)^{-1}(V)$=$f^{-1}(V)$∩U is πgb-open in X. Hence f/U is slightly continuous.

Theorem 3.12: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (z, η) be functions.
(i) If f is πgb-irresolute and g is slightly πgb-continuous, then g ∘ f is slightly πgb-continuous.

(ii) If f: (X, τ) → (Y, σ) is slightly πgb-continuous and g is continuous, then g ∘ f is slightly πgb-continuous.

(iii) If f: (X, τ) → (Y, σ) is πgb-irresolute and g is πgb-continuous, then g ∘ f is slightly πgb-continuous.

(iv) If f: (X, τ) → (Y, σ) is πgb-irresolute and g is slightly-πgb-continuous, then g ∘ f is slightly πgb-continuous.

Proof: Let V be clopen set in Z. Then g is slightly πgb-continuous implies $g^{-1}(V)$ is πgb-open in Y. Since f is πgb-irresolute,$f^{-1}(g^{-1}(V)$ is πgb-open in X. Hence g ∘ f is slightly πgb-continuous. Proof of (ii), (iii), (iv) is analogous to that of (i)

Theorem 3.13: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (z, η) be functions. If f is M- πgb-open surjective and g ∘ f is slightly πgb-continuous, then g is slightly πgb-continuous.

Proof: Let V be clopen set in Z. is g ∘ f is slightly πgb-continuous, $f^{-1}(g^{-1}(V)$ is πgb-open in(X, τ) Since f is M-πgb-open, $f(f^{-1}(g^{-1}(V))=g^{-1}(V)$ is πgb-open in (Y, σ). Hence g is slightly πgb-continuous.

Theorem 3.14: If f: (X, τ) → (Y, σ) be surjective, πgb-irresolute, M- πgb- open and and g: (Y, σ)→(z,η) be a function. Then g ∘ f is slightly πgb-continuous if and only if, then g is slightly πgb-continuous.

Proof: Previous two theorems 3.12 and 3.13.

Theorem 3.15: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous and K is πGBO-compact relative to X, then f(K) is mildly compact in Y.

Proof: Let $\{H_\alpha: \alpha\in I\}$ be any cover of f(K) by clopen sets of the subspace f(K).For α∈I, there exists a clopen set K_α of Y such that $H_\alpha= K_\alpha\cap f(K)$.For each x∈K, there exists $\alpha_k \in I$ such that $f(x) \in K_{\alpha x}$ and there exists $U_x \in \pi GBO(X, x)$ such that $f(U_x)\subset K_{\alpha x}$. Since the family is a cover of K by πgb-open sets of K, there exists a finite subset K_0 of K such that $K\subset\cup\{U_x:x\in K_0\}$. Therefore we obtain $f(K) \subset\cup\{U_x:x\in K_0\} \subset\cup\{K_{\alpha x}:x\in K_0\}$. Therefore we obtain $f(K)= \cup\{K_{\alpha x}:x\in K_0\}$.Hence f(K) is mildly compact.

Corollary 3.16: If f: (X, τ) → (Y, σ) is slightly πgb-continuous surjective and X is πGBO-compact, then Y is mildly compact.

Theorem 3.17: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous surjection, then the following statements hold:
(i) If X is πgb-Lindelof, then Y is mildly Lindelof.
(ii) If X is countably πgb-compact,then Y is mildly countably compact.

Proof: (i) Let $\{V_\alpha: \alpha\in I\}$ be any clopen cover of Y. Since f is slightly πgb- continuous, then $\{f^{-1}(V_\alpha: \alpha\in I\}$ is a πgb-open cover of X. Since X is πgb-L indelof, there exists a countable subset I_0 of I such that $X=\cup\{f^{-1}(V_\alpha): \alpha\in I_0\}$.Thus we have $Y=\cup\{V_\alpha: \alpha\in I_0\}$ and Y is mildly Lindelof.
(ii) Proof is similar to that of (i).

Theorem 3.18: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous surjection and X is πgb-connected space, then Y is connected space.

Proof: Suppose Y is not connected, then there exists non empty disjoint open sets U and V such that Y=U∪V. Therefore U and V are clopen sets in Y. Since f is slightly πgb-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint πgb-closed and πgb-open sets in (X, τ). Also $X= f^{-1}(U) \cup f^{-1}(V)$.This shows that X is not πgb-connected. Hence Y is connected.

Theorem 3.19: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous injection and Y is clopen T_1, then X is πgb-T_1.

Proof: Suppose Y is clopen T_1.For any two distinct points x and y in X, there exists clopen sets V and W in Y such that $f(x) \in V, f(y) \in W$, $f(x) \notin W$, $f(y) \notin V$. Since f is slightly πgb-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are πgb-open subsets of X such that $x\in f^{-1}(V)$, $y \notin f^{-1}(V)$,$x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. Hence X is πgb-T_1.

Theorem 3.20: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous injection and Y is clopen T_2, then X is πgb-T_2.

Proof: Suppose Y is clopen T_1.For any two distinct points x and y in X, there exists disjoint clopen sets V and W in Y such that $f(x) \in V$, $f(y) \in W$, $f(x) \notin W$, $f(y) \notin V$. Since f is slightly πgb-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are πgb-open subsets of X containing x and y respectively. Therefore $f^{-1}(U) \cap f^{-1}(V) = \Phi$ because $U \cap V = \Phi$. Hence X is πgb-T_2.

Theorem 3.21: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous injective and open from a πgbc-normal space X onto a space Y,then Y is clopen-normal.

Proof: Let A and B be two disjoint clopen subsets of Y. Since f is slightly πgb-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are πgb-closed sets in X. Take $U= f^{-1}(A)$ and $V= f^{-1}(B)$ .That is $U\cap V= \Phi$. Since X is πgbc-normal, there exists disjoint open sets F_1 and F_2 such that $U\subset F_1$ and $V\subset F_2$.Hence $f(U) \subset f(F_1)$ and $f(U) \subset f(F_2)$.This implies $A= f(U) \subset f(F_1)$ and $B =f(U) \subset f(F_2)$. Therefore $f(F_1)$ and $f(F_2)$ are disjoint open sets. Hence Y is clopen normal.

Theorem 3.22: If a function f: (X, τ) → (Y, σ) is slightly πgb-continuous injective and open from a πgbc-regular space X onto a space Y, then Y is clopen-regular.

Proof: Let F be a clopen set in Y and let $y \notin F$. Take y=f(x).Since f is slightly πgb-continuous, $f^{-1}(F)$ is πgb-closed. Let $G=f^{-1}(F)$. We have $x \notin f^{-1}(F)$.That is $x \notin G$. Since X is πgbc-regular, there exists disjoint open sets U and V such that $G\subset U$ and $x \in V$. Hence $f(G) \subset f(U)$ and $f(x) \in f(V)$. This implies $F=f(G) \subset f(U)$ and $f(x) \in f(V)$ such that f(U) and f(V) are disjoint open sets. Hence Y is clopen-regular.

Theorem 3.23: If f, g : (X, τ)→(Y, σ) is slightly πgb-continuous and Y is clopen Hausdorff, then $E=\{x\in X:f(x)=g(x)\}$ is πgb-closed.

Proof: Let $x\notin E$. Then $f(x)\neq g(x)$.Since Y is clopen Hausdorff, there exists disjoint clopen sets V_1 and V_2 in Y such that $f(x)\in V_1$ and $g(x)\in V_2$.Since f and g are slightly πgb-continuous, $f^{-1}(V_1)$ and $g^{-1}(V_2)$ are πgb-open sets with $x\in f^{-1}(V_1)\cap g^{-1}(V_2)$.Let $U=f^{-1}(V_1) \cap g^{-1}(V_2)$.Therefore $f(U)\cap g(U)= \phi$. Then U is a πgb-open set and $U\cap E= \phi$ implies $x\notin$πgb-cl(E).Hence E is πgb-closed.

Definition 3.24: A graph G(f) of a function f: (X, τ)→(Y, σ) is said to be strongly πgb-co-closed if for each $(x, y) \in (X \times Y)$-G(f), there exist $U \in$πgbCO(X) containing x and $V \in$ CO(Y) containing y such that $(U\times V) \cap G(f) = \phi$.

Lemma 3.25: A graph G(f) of a function f: (X, τ)→(Y, σ) is strongly πgb-co-closed in $X \in Y$ if and only if for each $(x, y) \in (X \times Y)$-G(f), there exist $U \in$ πgbCO(X)containing x and $V \in$ CO(Y) containing y such that $f(U) \cap V = \phi$.

Theorem 3.26: If f: (X, τ) →(Y, σ) is slightly πgb-continuous and Y is clopen T_1, then G(f) is strongly πgb-co-closed in X × Y.

Proof: Let $(x, y) \in (X \times Y)$-G (f), then $f(x) \neq y$ and there exists a clopen set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is slightly πgb-continuous, then $f^{-1}(V) \in$ πgbCO(X) containing x. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Therefore, we obtain $f(U) \cap (Y -V) = \phi$ and $Y -V \in$CO(Y) containing y. This shows that G(f) is strongly πgb-co-closed in X × Y.

Corollary 3.27: If f: (X, τ) →(Y, σ) is slightly πgb-continuous and Y is clopen T_2, then G(f) is strongly πgb-co-closed in X × Y .

Theorem 3.28: Let f: (X, τ) →(Y, σ) has a strongly πgb-co-closed graph G(f). If f is injective, then X is πgb-T_1.

Proof: Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y)$-G (f). By Lemma, there exist a πgb-clopen set U of X and $V \in$ CO(Y) such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \phi$. Hence $U \cap f^{-1}(V) = \phi$ and $y \notin U$. This implies that X is πgb-T_1.

Theorem 3.29: Let f: (X, τ) →(Y, σ) has a strongly πgb-co-closed graph G(f). If f is surjective M-πgb-open function, then Y is πgb-T_2.

Proof: Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y2) \in (X \times Y)$-G (f). By hypothesis, there exist a πgb-clopen set U of X and $V \in$ CO(Y) such that $(x, y_2) \in U \times V$ and $(U \times V)$ -G (f) = ϕ. Then, we have $f(U) \cap V = \phi$. Since f is M-πgb- open, then f(U) is πgb-open such that $f(x) = y_1 \in f(U)$. This implies that Y is πgb- T_2.

4. Faintly πgb-Continuous Functions

Definition 4.1: A function f: (X, τ) → (Y, σ) is faintly πgb-continuous if $f^{-1}(V)$ is πgb-open in (X, τ) for every θ-open set V of (Y, σ).

Theorem 4.2: Let (X, τ) be a πgb- $T_{1/2}$-space.Then a function f: (X, τ) → (Y, σ) is faintly πgb-continuous if and only if it is faintly continuous.

Proof: Let V be θ-open in Y. Let f be faintly πgb-continuous, then $f^{-1}(V)$ is πgb-open in X. By hypothesis of πgb-$T_{1/2}$ space,$f^{-1}(V)$ is open in X. Hence f is faintly continuous.
Converse: Let f be faintly continuous. Let V be θ-open in Y. Then $f^{-1}(V)$ is open in (X, τ).This implies $f^{-1}(V)$ is πgb-open in (X, τ).Hence f is faintly πgb-continuous.

Theorem 4.3: If a function f: (X, τ) → (Y, σ) is faintly πgb-continuous and (Y, σ) is a regular space, then f is πgb-continuous.

Proof: Let V be any open set in Y. Since Y is regular, V is θ-open in Y. Since f is faintly πgb-continuous, $f^{-1}(V)$ is πgb-open in X. This implies f is πgb-continuous.

Remark 4.4: Every πgb-continuous function is almost πgb-continuous.

Theorem 4.5: If a function f: (X, τ) →(Y, σ) is faintly πgb-continuous and (Y, σ) is regular, then f is almost πgb-continuous.

Proof: Previous remark 4.4 and theorem 4.3.

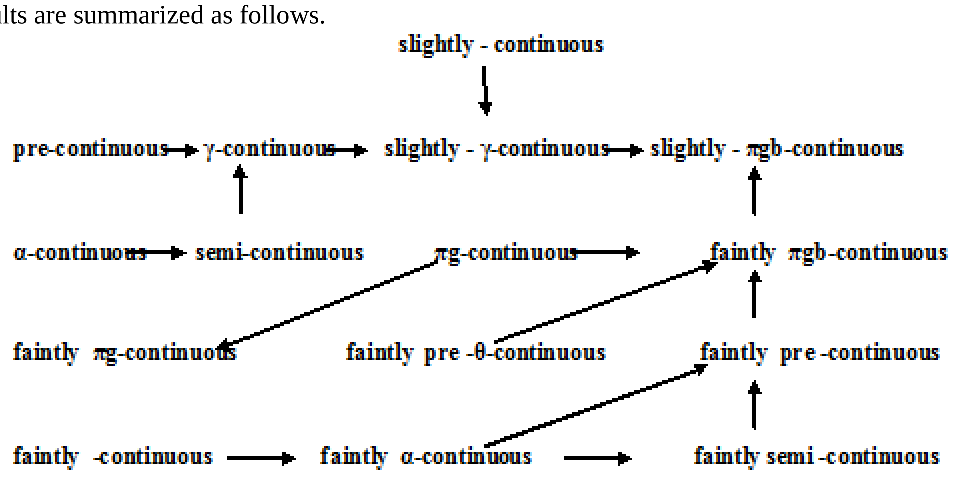
Theorem 4.6: If a function f: (X, τ) →(Y, σ) is faintly πgb-continuous, then it is slightly πgb-continuous.

Proof: Let x∈X and V be any clopen subset of Y containing f(x).Then V is θ-open in Y. Since f is faintly πgb-continuous; $f^{-1}(V)$ is πgb-open in X. This implies f is slightly πgb-continuous.

Theorem 4.7: Let f: (X, τ) →(Y, σ) be a function and g: (X, τ) → (X×Y, τ ×σ) be the graph function of f defined by g(x) =(x, f(x)) for every x∈X. If g is πgb-continuous, then f is faintly πgb-continuous.

Proof: Let U be θ-open set of (Y, σ).Then X×U is a θ-open set in X×Y. It follows that $f^{-1}(U) = g^{-1}(X\times U)$ ∈πGBO(X).This implies f is faintly πgb-continuous.

The above results are summarized as follows.



Theorem 4.8: Let f: (X, τ) →(Y, σ) be a faintly πgb-continuous surjective function. Then the following statements holds good.
(i) If X is πgb-Lindelof, then Y is θ-Lindelof.
(ii) If X is countably πgb-compact, then Y is countably θ-compact.

Proof (i) Let f: (X, τ) →(Y, σ) be a faintly πgb-continuous from X onto Y. Let $\{G_\alpha:\alpha\in I\}$ is a θ -open cover of Y. Since f is faintly πgb-continuous,$\{f^{-1}(G_\alpha):\alpha \in I\}$ is a πgb-open cover of X. Since X is πgb-Lindelof, every πgb-open cover of X has a countable subcover. This implies $\{G_i:i=1,2,\ldots n\}$ is a countable subcover which cover Y.Hence Y is θ-Lindelof.
(ii)Proof similar to that of (i)

Theorem 4.9: If f: (X, τ) →(Y, σ) is faintly πgb-continuous injection and Y is a θ-T_1space,then X is πgb-T_1-space.

Proof: Suppose Y is θ-T_1.For any two distinct points x and y in X, there exists V, W are θ-open sets in Y such that f(x) ∈V, f(y) ∉V and f(x) ∉ W, f(y) ∈ W. Since f is faintly πgb-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are πgb-open in (X, τ) such that x∈$f^{-1}(V)$,y∉$f^{-1}(V)$ and x∉$f^{-1}(W)$,y∈$f^{-1}(W)$.This implies X is πgb-T_1.

Theorem 4.10: If f: (X, τ) →(Y, σ) is faintly πgb-continuous injection and Y is a θ-T_2space, then X is πgb-T_2-space.

Proof: Suppose Y is θ-T_2.For any two distinct points x and y in X, there exists V, W are θ-open sets in Y such that f(x) ∈V, f(y) ∉V and f(x) ∉ W, f(y) ∈ W. Since f is faintly πgb-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are πgb-open in (X, τ) such that x∈$f^{-1}(V)$ and y∈$f^{-1}(W)$.Hence $f^{-1}(V)$ ∩$f^{-1}(W)$ =Φ .This implies X is πgb-T_2.

Definition 4.11: A space (X, τ) is strongly πgbc-regular if for each πgb-closed set F and each point x∉F, there exists disjoint πgb-open sets U and V such that F_1⊂U and x ∈V.

Definition 4.12: A space (X, τ) is strongly πgbc-normal if for any pair of disjoint πgb-closed subsets F_1 and F_2 of X, there exists disjoint πgb-open sets U and V such that F_1⊂U and F_2⊂V.

Definition 4.13: A space (X, τ) is strongly θ-normal[17] if for any pair of disjoint θ-closed subsets F_1 and F_2 of X, there exists disjoint θ-open sets U and V such that F_1⊂U and F_2⊂V.

Definition 4.14:A function f : (X, τ)→(Y, σ) is called
(i) πgb-θ-open if f(V) is θ-open in Y for each V ∈ πGBO(X).
(ii) πgb-θ-closed if f(V) is θ-closed for each V ∈ πGBC(X).

Theorem 4.15: If f : (X, τ)→(Y, σ) is faintly πgb -continuous, πgb-θ-open injective function from a strongly πgbc-regular space (X, τ) onto (Y, σ),then (Y, σ) is strongly θ-regular.

Proof: Let F be θ-closed subset of Y and y∉F. Take y=f(x).Since f is faintly πgb-continuous,$f^{-1}(F)$ is πgb-closed in X such that $f^{-1}(y)$=x∉$f^{-1}(F)$.Let G=$f^{-1}(F)$.This implies x∉G. Since X is strongly πgbc-regular space, then there exists disjoint πgb-open sets U and V in X such that G⊂U and x∈V. Hence F=f(G)⊂f(U) and y∈f(x)∈f(V) such that f(U) and f(V) are disjoint θ-open sets. This implies Y is strongly θ-regular.

Theorem 4.16: If f: (X, τ) →(Y, σ) is faintly πgb-continuous, πgb-θ-open injective function from a strongly πgbc-normal space (X, τ) onto (Y, σ), then (Y, σ) is strongly θ-normal.

Proof: Let F_1 andF_2 be disjoint θ-closed subsets of Y. Since f is faintly πgb-continuous,$f^{-1}(F_1)$ and $f^{-1}(F_2)$ are πgb-closed in X. Let U=$f^{-1}(F_1)$ and V=$f^{-1}(F_2)$ such that U∩V=Φ. Since X is strongly πgbc-normal space, then there exists disjoint πgb-open sets A and B in X such that U⊂A and V⊂B. We get F_1=f(U)⊂f(A) and F_2=f(V) ⊂f(B) such that f(A) and f(B) are disjoint θ-open sets. Hence Y is strongly θ-normal.

Theorem 4.17: If f: (X, τ) →(Y, σ) is a faintly πgb-continuous function and (X, τ) is a πgb-connected space, then Y is connected space.

Proof: Assume that (Y, σ) is not connected. Then there exist non empty open sets V_1 and V_2 such that V_1∩V_2 = ϕ and V_1∪V_2 =Y Hence we have $f^{-1}(V_1)$ ∩$f^{-1}(V_2)$= ϕ and $f^{-1}(V_1)$ ∪$f^{-1}(V_2)$=X. Since f is surjective,$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non empty subsets of X. Since V_i is open and closed, V_i is θ-open for each i=1,2.Since f is faintly πgbcontinuous, $f^{-1}(V_i)$∈πGBO(X).Therefore,(X, τ) is not πgb-connected. This is a contradiction and hence (Y, σ) is connected.

Theorem 4.18: The surjective faintly πgb-continuous function from a πgb-compact space is θ-compact.

Proof: Let f: (X, τ) → (Y, σ) be a faintly πgb-continuous function from a πgb-compact space onto a space Y. Let {G_i:α∈I} be any θ-open cover of Y. Since f is faintly πgb-continuous,{$f^{-1}(G_i)$:i=1,2,….n} of X. Then it follows that {G_i:i=1,2,…n} is a finite subfamily which cover Y. Hence Y is θ-compact.

Definition 4.19: A graph G(f) of a function f: (X, τ) → (Y, σ) is said to be θ -πgb-closed if for each (x, y) ∈ (X × Y)-G(f), there exist U ∈ πGBO(X, x) and V is θ-open in Y containing y such that (UxV)∩G(f)= φ.

Lemma 4.20: A graph G(f) of a function f : (X, τ) → (Y, σ) is θ -πgb-closed in X ×Y if and only if for each (x, y) ∈ (X ×Y) -G(f), there exist U∈πGBO(X, x) and V is θ-open in Y containing y such that f(U) ∩ V = φ.

Proof: It is an immediate consequence of Definition.

Theorem 4.21: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is faintly πgb-continuous function and (Y, σ) is ϴ-T_2, then G(f) is ϴ -πgb-closed.

Proof: Let $(x, y) \varepsilon (X \times Y)$-G (f), then $f(x) \neq y$. Since Y is ϴ -T_2, there exist ϴ -open sets V and W in Y such that $f(x) \in V$, $y \in W$ and $V \cap W = \varphi$. Since f is faintly πgb-continuous, $f^{-1}(V) \in \pi GBO(X, x)$. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Therefore, we obtain $f(U) \cap W = \varphi$. This shows that G(f) is ϴ-πgb closed.

Theorem 4.22: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ has ϴ-πgb-closed graph G(f). If f is a faintly πgb-continuous injection, then (X, τ) is πgb-T_2.

Proof: Let x and y be any two distinct points of X. Since f is injective, we have $f(x) \neq f(y)$. Then, we have $(x, f(y)) \in (X \times Y)$-G (f). By Lemma 4.20 , $U \in \pi GBO(X)$ and V is ϴ-open in (Y, σ) such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \varphi$.

Hence $U \cap f^{-1}(V) = \varphi$ and $y \notin U$. Since f is faintly πgb-continuous, there exists $W \in \pi GBO(X, y)$ such that $f(W) \subset V$.

Therefore, we have $f(U) \cap f(W) = \varphi$. Since f is injective, we obtain $U \cap W = \varphi$. This implies that (X, τ) is πgb-T_2.

Theorem 4.23: If f: $(X, \tau) \rightarrow (Y, \sigma)$ has the ϴ -πgb-closed graph, then f(K) is ϴ-closed in (Y, σ) for each subset K which is πgb-compact relative to X.

Proof: Suppose that $y \notin f(K)$. Then $(x, y) \notin G(f)$ for each $x \in K$. Since G(f) is ϴ -πgb-closed, there exist $U_x \in \pi GBO(X, x)$ and a ϴ-open set V_x of Y containing y such that $f(U_x) \cap V_x = \varphi$ by lemma 4.20 **.** The family $\{U_x : x \in K\}$ is a cover of K by πg-open sets. Since K is πg-compact relative to (X, τ), there exists a finite subset K_0 of K such that $K \subset \cup\{U_x : x \in K_0\}$. Set $V = \cap\{V_x : x \in K_0\}$. Then V is a ϴ -open set in Y containing y. Therefore, we have $f(K) \cap V \subset [\cup_{x \in K0} f(U_x)] \cap V \subset \cup_{x \in K0} [f(Ux) \cap V] = \varphi$. It follows that $y \notin Cl_ϴ(f(K))$. Therefore, f(K) is ϴ -closed in (Y, σ).

Corollary 4.24: If f : $(X, \tau) \rightarrow (Y, \sigma)$ is faintly πgb-continuous and (Y, σ) is ϴ-T_2, then f(K) is ϴ-closed in (Y, σ) for each subset K which is πgb-compact relative to (X, τ).

Proof: The proof follows from theorems 4.21 and 4.23.

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