

## Generalized soft $g\beta$ closed sets and soft $gs\beta$ closed sets in soft topological spaces

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### ABSTRACT

The focus of this paper is to introduce soft  $g\beta$  closed sets and soft  $gs\beta$  closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Further we obtain some properties in the light of these defined sets.

**Keywords:** Soft Topological Spaces, Soft Closed, Soft Generalized Closed, Soft  $g\beta$  Closed, Soft  $gs\beta$  Closed,  $T^*$ Space.

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### 1. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. This new set theory has found its applications in Game Theory, Operations Research, Theory of Probability, Riemann Integration, Perron Integration, Smoothness of functions, etc. The main objective behind application of such theory is to derive an effective solution from an uncertain and inadequate data. Molodtsov's [9] Soft Set Theory was originally proposed as general mathematical tool for dealing with uncertainty problems. He proposed Soft Set Theory, which contains sufficient parameters such that it is free from the corresponding difficulties, and a series of interesting applications of the theory instability and regularization, Game Theory, Operations Research, Probability and Statistics.

Maji *et al* [8] proposed several operations on soft sets and some basic properties. Shabir and Naz [10] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. K. Kannan [6] studied soft generalized closed sets in soft topological spaces along with its properties. The concept of generalized closed sets was introduced by N. Levine [7]. Many mathematicians [1][2][3] extended the results of generalized closed sets in many directions.

In this present study, we introduce some new concepts in soft topological spaces such as  $g\beta$  closed sets and soft  $gs\beta$  closed sets and some relation between these sets.

### 2. PRELIMINARIES

**Definition: 2.1** [9] Let  $U$  be an initial universe and  $E$  be a set of parameters. Let  $P(U)$  denote the power set of  $U$  and  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

**Definition: 2.2** The union of two soft sets  $(F, A)$  and  $(G, B)$  over the common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,  $H(e) = F(e)$  if  $e \in A - B$ ,  $H(e) = G(e)$  if  $e \in B - A$  and  $H(e) = F(e) \cup G(e)$  if  $e \in A \cap B$ . We write  $(F, A) \cup (G, B) = (H, C)$ .

**Definition: 2.3** The intersection  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , denoted  $(F, A) \cap (G, B)$ , is defined as  $C = A \cap B$ , and  $H(e) = F(e) \cap G(e)$  for all  $e \in C$ .

**Definition: 2.4** Let  $\tau$  be the collection of soft sets over  $X$ , then  $\tau$  is said to be a soft topology on  $X$  if (1)  $\Phi, X$  belong to  $\tau$ , (2) the union of any number of soft sets in  $\tau$  belongs to  $\tau$ , (3) the intersection of any two soft sets in  $\tau$  belongs to  $\tau$ . The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . Let  $(X, \tau, E)$  be a soft space over  $X$ , then the members of  $\tau$  are said to be soft open sets in  $X$ .

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**Definition: 2.5** Let  $(X, \tau, E)$  be a soft topological space over  $X$  and  $(F, E)$  be a soft set over  $X$ . Then, the soft closure of  $(F, E)$ , denoted by  $cl(F, E)$  is the intersection of all soft closed supersets of  $(F, E)$ .

Clearly  $(F, E)$  is the smallest soft closed set over  $X$  which contains  $(F, E)$ . The soft interior of  $(F, E)$ , denoted by  $int(F, E)$  is the union of all soft open subsets of  $(F, E)$ . Clearly  $(F, E)$  is the largest soft open set over  $X$  which is contained in  $(F, E)$ .

**Definition: 2.6** A soft set  $(A, E)$  is called soft generalized closed set ( soft g-closed) in a soft topological space  $(X, \tau, E)$  if  $cl(A, E) \widetilde{\subset} (U, E)$  whenever  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$

**Definition: 2.7** A soft topological space  $(X, \tau, E)$  is a soft  $T_{1/2}$  space if every soft g closed set is soft closed in  $X$

**Definition: 2.8** A subset  $A$  of a topological space  $(X, \tau, E)$  is called a soft pre open set if  $(A, E) \widetilde{\subset} int(cl(A, E))$

**Definition: 2.9** A subset  $A$  of a topological space  $(X, \tau, E)$  is called a soft semi open set if  $(A, E) \widetilde{\subset} cl(int(A, E))$

**Definition: 2.10** A subset  $A$  of a topological space  $(X, \tau)$  is called a pre open set if  $A \subseteq int(cl(A))$ .

**Definition: 2.11** A subset  $A$  of a topological space  $(X, \tau)$  is called a semi open set if  $A \subseteq cl(int(A))$

**Definition: 2.10** A subset  $A$  of a topological space  $X$  is called  $\beta$ -closed if  $int(cl(int(A))) \subset A$ .

**Definition: 2.11** A subset  $A$  of a topological space  $X$  is called regular open if  $A = int(cl(A))$

**Definition: 2.12** A subset  $A$  of a topological space  $X$  is called generalized closed (g-closed) if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ .

**Definition: 2.13** A subset  $A$  of a topological space  $X$  is called semi-generalized closed (sg-closed) if  $scl(A) \subset U$  whenever  $A \subset U$  and  $U$  is semi-open in  $X$ .

**Definition: 2.14** A subset  $A$  of a topological space  $X$  is called generalized-semi closed (gs-closed) if  $scl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$

**Definition: 2.15** A subset  $A$  of a topological space  $X$  is called regular generalized closed (rg-closed) if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular open in  $X$ .

**Definition 2.16** A subset  $A$  of a topological space  $X$  is called  $\alpha$  generalized-closed ( $\alpha$  g -closed) if  $\alpha cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ .

**Definition: 2.17** A subset  $A$  of a topological space  $X$  is called semi-closed if  $int(cl(A)) \subset A$ .

**Definition: 2.18** A subset  $A$  of a topological space  $X$  is said to be clopen if  $A$  is closed and open in  $X$ .

**Definition: 2.19** A subset  $A$  of a topological space  $X$  is called g $\beta$ -closed if  $\beta cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $(X, \tau)$ .

**Definition: 2.20** A subset of a topological space  $X$  is called gs $\beta$  -closed if  $\beta cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is semi-open in  $(X, \tau)$ .

**Definition: 2.21** A subset of a topological space  $X$  is called gs $\beta$  -closed if  $\beta cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is semi-open in  $(X, \tau)$ .

**Definition: 2.22[7]** A subset  $A$  in a topological space is defined to be a Q set iff  $int(cl(A)) = cl(int(A))$

Now we shall define the generalized  $\beta$  closed set using soft set in the following definition:

**3. SOFT GENERALIZED  $\beta$  CLOSED SETS** **Definition: 3.1** A subset  $(A, E)$  of a topological space  $X$  is called soft generalized  $\beta$  closed (soft g $\beta$  -closed) in a soft topological space  $(X, \tau, E)$ , if  $\beta cl(A, E) \widetilde{\subset} (U, E)$ , whenever  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ .

**Definition: 3.2** A subset  $(A, E)$  of a topological space  $X$  is called soft semi-generalized closed (soft sg-closed) if  $scl(A, E) \widetilde{\subset} (U, E)$  whenever  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft semi-open in  $X$ .

**Definition: 3.3** A subset  $(A, E)$  of a topological space  $X$  is called soft generalized-semi closed (soft gs-closed) if  $scl(A, E) \widetilde{\subset} (U, E)$  whenever  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ .

**Definition: 3.4** A subset  $(A, E)$  of a topological space  $X$  is called soft  $\beta$ -closed if  $\text{int}(\text{cl}(\text{int}(A, E))) \widetilde{\subset} (A, E)$

**Definition: 3.5** A subset  $(A, E)$  of a topological space  $X$  is called soft  $\alpha$  generalized-closed (soft  $\alpha$  g -closed) if  $\alpha \text{cl}(A, E) \widetilde{\subset} (U, E)$  whenever  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ .

**Definition: 3.6** A subset  $(A, E)$  of a topological space  $X$  is called regular generalized closed (rg-closed) if  $\text{cl}(A, E) \widetilde{\subset} (U, E)$  whenever  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft regular open in  $X$ .

**Theorem: 3.7**

- Every soft closed set is soft  $g\beta$  -closed.
- Every soft g-closed set is soft  $g\beta$  -closed.
- Every soft sg-closed set is soft  $g\beta$  -closed.
- Every soft gs-closed set is soft  $g\beta$  -closed.
- Every soft  $\beta$ -closed set is soft  $g\beta$  -closed
- Every soft  $\alpha$ g-closed set is soft  $g\beta$  -closed.
- Every soft  $\alpha$ -closed set is soft  $g\beta$  -closed
- Every soft rg-closed set is soft  $g\beta$  -closed

**Proof:**

a) Suppose  $(A, E)$  is a soft closed set,  $(U, E)$  be soft open in  $X$ , such that  $(A, E) \widetilde{\subset} (U, E)$ ,  $\text{cl}(A, E) = (A, E) \widetilde{\subset} (U, E)$ .

Thus  $\beta \text{cl}(A, E) \widetilde{\subset} \text{cl}(A, E) \widetilde{\subset} (U, E)$ . Hence  $(A, E)$  is soft  $g\beta$  - closed.

b) Suppose  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ . By assumption  $\text{cl}(A, E) \widetilde{\subset} (U, E)$

$\Rightarrow \beta \text{cl}(A, E) \widetilde{\subset} (U, E)$ . Hence  $(A, E)$  is soft  $g\beta$  -closed

c) The proof is obvious and straight forward.

d) Let  $(A, E)$  be soft gs-closed subset of  $X$ , Suppose  $(A, E) \widetilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ ,  $scl(A, E) \widetilde{\subset} (U, E)$

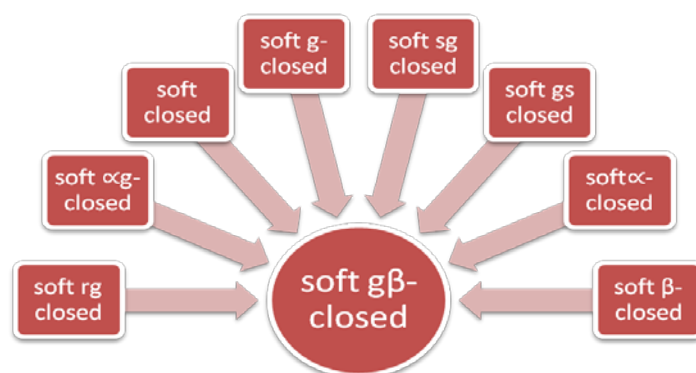
$\Rightarrow \beta \text{cl}(A, E) \widetilde{\subset} (U, E)$ , Hence  $(A, E)$  is soft  $g\beta$  -closed.

e) The proof is obvious and straight forward.

f) Let  $(A, E) \widetilde{\subset} (U, E)$  where  $(U, E)$  is soft open in  $X$ .  $\alpha \text{cl}(A, E) \widetilde{\subset} (U, E) \Rightarrow \beta \text{cl}(A, E) \widetilde{\subset} \alpha \text{cl}(A, E) \widetilde{\subset} (U, E)$ . Thus  $(A, E)$  is soft  $g\beta$  -closed.

The proof of (g) and (h) are obvious and straight forward

From the above results, the following implication is made:



**Theorem: 3.8** If  $(A, E)$  is soft open and soft  $g\beta$  - closed then  $(A, E)$  is soft  $\beta$  - closed.

**Proof:** Suppose  $(A, E)$  is soft open and soft  $g\beta$  - closed. Then, by definition  $\beta cl(A, E) \widetilde{\subseteq} (A, E)$ . But always  $(A, E) \widetilde{\subseteq} \beta cl(A, E)$ . Thus  $(A, E) = \beta cl(A, E) \Rightarrow (A, E)$  is soft  $\beta$  - closed.

**Theorem: 3.9** For a subset  $(A, E) \widetilde{\subseteq} X$ , the following conditions are equivalent

1.  $(A, E)$  is soft open and soft  $gs$ -closed.
2.  $(A, E)$  is soft regular open.

**Proof:**

**1  $\Rightarrow$  2:**  $scl(A, E) \widetilde{\subseteq} (A, E)$ , since  $(A, E)$  is soft open and soft  $gs$  - closed. Thus  $int(cl(int(A, E))) \widetilde{\subseteq} (A, E)$  since  $scl(A, E) = (A, E) \cup int(cl(A, E))$ . Since  $(A, E)$  is open, then  $(A, E)$  is clearly soft pre - open. Thus  $(A, E) \widetilde{\subseteq} int(cl(A, E)) \Rightarrow int(cl(A, E)) \widetilde{\subseteq} (A, E) \widetilde{\subseteq} int(cl(A, E)) \Rightarrow (A, E) = int(cl(A, E))$ .

**2  $\Rightarrow$  1:** Let  $(A, E)$  is soft regular open  $\Rightarrow (A, E) = int(cl(A, E))$ . From this we have  $int(cl(A, E)) \widetilde{\subseteq} (A, E)$ . Thus  $(A, E)$  is soft semi - closed  $\Rightarrow (A, E)$  is soft  $sg$  - closed.

But every soft  $sg$ -closed set is soft  $gs$ -closed. Hence  $(A, E)$  is soft  $gs$ -closed.

**Corollary: 3.10** If  $(A, E)$  is soft open and soft  $gs$ -closed set of  $(X, \tau, E)$  then  $(A, E)$  is soft semi - closed.

**Proof:** Let  $(A, E)$  be soft open and soft  $gs$ -closed  $\Rightarrow scl(A, E) \widetilde{\subseteq} (A, E)$  But  $(A, E) \widetilde{\subseteq} scl(A, E) \Rightarrow (A, E) = scl(A, E) \Rightarrow (A, E)$  is soft semi - closed.

**Theorem: 3.11** For a subset  $(A, E) \widetilde{\subseteq} X$ , the following conditions are equivalent

1.  $(A, E)$  is soft open and soft  $g\beta$  -closed.
2.  $(A, E)$  is soft regular open.

**Proof:**

**1  $\Rightarrow$  2:** By 1,  $\beta cl(A, E) \widetilde{\subseteq} (A, E)$ . Since  $(A, E)$  is soft open and soft  $g\beta$ -closed. Thus  $int(cl(A, E)) \widetilde{\subseteq} (A, E)$ , since  $\beta cl(A, E) = (A, E) \cup int(cl(A, E))$ .

Since  $(A, E)$  is soft open,  $(A, E)$  is clearly soft pre - open. Thus  $(A, E) \widetilde{\subseteq} int(cl(A, E)) \Rightarrow (A, E)$  is soft regular open.

**2  $\Rightarrow$  1:** Let  $(A, E)$  be soft regular open,  $\Rightarrow (A, E)$  is soft open. Since  $(A, E)$  is soft regular open,  $(A, E) = int(cl(A, E))$ . From this we have  $int(cl(A, E)) \widetilde{\subseteq} (A, E)$ . Thus  $(A, E)$  is soft semi - closed.  $\Rightarrow (A, E)$  is soft  $sg$ -closed. By theorem 3.7(c),  $(A, E)$  is soft  $g\beta$  - closed.

**Theorem: 3.12** For a subset  $(A, E) \widetilde{\subseteq} X$ , the following conditions are equivalent.

1.  $(A, E)$  is soft clopen.
2.  $(A, E)$  is soft open, a  $Q$  - set and soft  $g\beta$  - closed.

**Proof:**

**1  $\Rightarrow$  2:** Let  $(A, E)$  be soft clopen. Since  $(A, E)$  is soft closed,  $(A, E)$  is soft  $g\beta$  - closed. And we have  $int(cl(A, E)) = (A, E) = cl(int(A, E))$ . Thus  $(A, E)$  is a  $Q$  - set. Hence  $(A, E)$  is soft open, a  $Q$ -set and soft  $g\beta$  - closed.

**2  $\Rightarrow$  1:** Let  $(A, E)$  be soft open, a  $Q$  - set and soft  $g\beta$  - closed.

Since  $(A, E)$  is soft open and soft  $g\beta$  - closed, by theorem 3.11,  $(A, E)$  is soft regular open.

Since  $(A, E)$  is soft regular open,  $(A, E) = int(cl(A, E)) = cl(int(A, E)) = cl(A, E)$ . Thus  $(A, E)$  is soft closed. Equivalently,  $(A, E)$  is soft clopen.

**Theorem: 3.13** A set  $(A, E)$  is soft  $g\beta$  closed in  $X$  iff  $\beta cl(A, E) \setminus (A, E)$  contains only null soft closed set.

**Proof:** Suppose  $(A, E)$  is soft  $g\beta$  closed in  $X$ , Let  $(F, E)$  be soft closed  $\Rightarrow (F, E) \subseteq \beta cl(A, E) \setminus (A, E)$   
 $\Rightarrow (F, E) \subseteq \beta cl(A, E)$  and  $(F, E) \subseteq (A, E)$ . Hence  $(A, E) \subseteq (F, E)$   
 $\Rightarrow \beta cl(A, E) \subseteq (F, E)$   
 $\Rightarrow (F, E) \subseteq (\beta cl(A, E))'$ . Thus  $(F, E)$  is null set.

Conversely,  $\beta cl(A, E) \setminus (A, E) = \emptyset \Rightarrow \beta cl(A, E) = (A, E)$ .  $(A, E)$  is soft  $\beta$  closed, hence soft  $g\beta$  closed.

#### 4. SOFT $gs\beta$ CLOSED SETS

**Definition: 4.1** A subset  $(A, E)$  of a topological space  $X$  is called soft  $gs\beta$  closed if  $\beta cl(A, E) \subseteq (U, E)$  whenever  $(A, E) \subseteq (U, E)$  and  $(U, E)$  is soft semiopen in  $(X, \tau, E)$

**Theorem: 4.2**

- (i) Every soft closed set is soft  $gs\beta$  - closed.
- (ii) Every soft  $sg$ - closed set is soft  $gs\beta$  - closed.
- (iii) Every soft semi-closed set is soft  $gs\beta$  - closed.
- (iv) Every soft  $\beta$  - closed set is soft  $gs\beta$  - closed.
- (v) Every soft  $\alpha$  - closed set is soft  $gs\beta$  - closed.

**Proof:**

- (i) Let  $(A, E)$  be a soft closed set such that  $(A, E) \subseteq (U, E)$  where  $(U, E)$  is soft semi-open in  $X$ .

Since  $(A, E)$  is soft closed,  $(A, E) = cl((A, E))$ .

$$\Rightarrow \beta cl((A, E)) \subseteq cl((A, E)) \subseteq (U, E)$$

$$\Rightarrow \beta cl((A, E)) \subseteq (U, E)$$

Hence the proof.

- (ii) Proof is obvious and straight forward.

- (iii) Let  $(A, E)$  be a soft semi-closed set such that  $(A, E) \subseteq (U, E)$  where  $(U, E)$  is soft semi-closed.

Thus  $(A, E) = scl((A, E)) \subseteq (U, E)$ . Hence  $\beta cl((A, E)) \subseteq scl((A, E)) \subseteq (U, E)$   
 $\Rightarrow \beta cl((A, E)) \subseteq (U, E)$

- (iv) Proof is obvious

- (v) Assume  $(A, E)$  to be a soft  $\alpha$  - closed set.

Let  $(A, E) \subseteq (U, E)$  where  $(U, E)$  is soft semi-open in  $X$ .

$$\alpha cl((A, E)) \subseteq (U, E) \Rightarrow \beta cl((A, E)) \subseteq \alpha cl((A, E)) \subseteq (U, E) \Rightarrow \beta cl((A, E)) \subseteq (U, E)$$

Thus  $(A, E)$  is soft  $gs\beta$  - closed.

**Theorem: 4.3** Every soft  $gs\beta$  - closed set is soft  $g\beta$  - closed.

**Proof:** Let  $(A, E)$  be a soft  $gs\beta$  - closed set.

Assume  $(A, E) \subseteq (U, E)$  where  $(U, E)$  is soft open.

Since, every soft open set is soft semi-open and  $\beta cl((A, E)) \subseteq (U, E)$

Hence  $(A, E)$  is soft  $g\beta$  - closed.

## 5. SOFT $T^*$ SPACES

**Definition: 5.1** A space  $X$  is said to be soft  $T^*$  space if every soft  $g\beta$  – closed set is soft  $\beta$  - closed.

**Theorem: 5.2** For a space  $(X, \tau, E)$  the following conditions are equivalent:

1.  $X$  is a soft  $T^*$  space
2. Every singleton of  $X$  is soft closed or soft  $\beta$  - open.
3. Every singleton of  $X$  is soft closed or soft pre-open.

**Proof:**

(1)  $\Rightarrow$  (2): Assume that for some  $x \in X$ ,  $\{x\}$  is not soft closed.

Then  $X \setminus \{x\}$  is not soft open.

Thus the only soft open set containing  $X \setminus \{x\}$  is  $X$  itself and hence  $X \setminus \{x\}$  is trivially soft  $g\beta$  – closed

By (1)  $X \setminus \{x\}$  soft closed  $\Rightarrow X \setminus \{x\}$  is soft  $\beta$  – closed  $\Rightarrow \{x\}$  is soft  $\beta$ -open

(2)  $\Rightarrow$  (3): Assume that for some  $x \in X$ , the set  $\{x\}$  is not soft pre -open.

Thus  $\{x\}$  is nowhere dense, since it is well known that in every space a Singleton is either soft pre – open or nowhere dense

We have  $\text{cl}(\text{int}(\text{cl}(\{x\}))) = \emptyset$

Hence  $\{x\} \not\subset \text{cl}(\text{int}(\text{cl}(\{x\})))$

$\Rightarrow \{x\}$  is not soft  $\beta$  - open

By (2)  $\{x\}$  is soft closed.

(3)  $\Rightarrow$  (1) Let  $(A, E) \widetilde{\subset} X$  be soft  $g\beta$  – closed. to prove  $\beta\text{cl}(A, E) = (A, E)$ , let  $x \in \beta\text{cl}(A, E)$

By assumption  $\{x\}$  is either soft closed or soft pre – open.

**WE CONSIDER THESE TWO CASES:**

**Case (1):** Let  $\{x\}$  be soft closed.

$\beta\text{cl}(A, E) \setminus (A, E)$  does not contain non- empty soft closed set,

Thus  $\{x\} \not\subset \beta\text{cl}(A, E) \setminus (A, E)$ . Hence  $x \in (A, E) \Rightarrow \beta\text{cl}(A, E) \widetilde{\subset} (A, E)$

But  $(A, E) \widetilde{\subset} \beta\text{cl}((A, E)) \Rightarrow \beta\text{cl}((A, E)) = (A, E)$

**Case (2):** Let  $\{x\}$  be soft pre – open.  $\Rightarrow$  Clearly  $\{x\}$  is soft  $\beta$  - open

And  $x \in \beta\text{cl}((A, E))$ , then  $\{x\} \cap (A, E) \neq \emptyset$

Hence  $x \in (A, E) \Rightarrow \beta\text{cl}((A, E)) \widetilde{\subset} (A, E)$ , So  $(A, E) = \beta\text{cl}((A, E))$

$\Rightarrow (A, E)$  is soft  $\beta$  - closed. Hence  $X$  is a soft  $T^*$  -space.

**Theorem: 5.3** Every soft  $T_{1/2}$  - space is a soft  $T^*$  - space.

**Proof:** Suppose  $X$  is a soft  $T_{1/2}$  - space.

Every singleton of  $X$  is soft closed or soft open.

$\Rightarrow$  Every singleton of  $X$  is soft closed or soft pre – open.

Hence by theorem 5.2,  $X$  is a  $T^*$  -space.

## 6. CONCLUSION

In this paper, soft  $g\beta$  closed sets and soft  $gs\beta$  closed sets were introduced and studied with already existing sets in soft topological spaces. A new space soft  $T^*$  is also been introduced. The scope for further research can be focused on the applications of soft topological spaces.

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