International Journal of Mathematical Archive-4(2), 2013, 17-23
Available online through www.ijma.info ISSN 2229 – 5046

Generalized soft gβ closed sets and soft gsβ closed sets in soft topological spaces

***I. Arockiarani**
Department of Mathematics, Nirmala College for Women, Coimbatore, India

****A. Arokia Lancy**
Department of Mathematics, KTVR Knowledge Park for Engineering and Technology, Coimbatore, India

(Received on: 05-11-12; Revised & Accepted on: 12-12-12)

ABSTRACT

The focus of this paper is to introduce soft gβ closed sets and soft gsβ closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Further we obtain some properties in the light of these defined sets.

Keywords: *Soft Topological Spaces, Soft Closed, Soft Generalized Closed, Soft gβ Closed, Soft gsβ Closed, T^* Space.*

2010 AMS Classification: *06D72, 54A40.*

1. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. This new set theory has found its applications in Game Theory, Operations Research, Theory of Probability, Riemann Integration, Perron Integration, Smoothness of functions, etc. The main objective behind application of such theory is to derive an effective solution from an uncertain and inadequate data. Molodtsov's **[9]** Soft Set Theory was originally proposed as general mathematical tool for dealing with uncertainty problems. He proposed Soft Set Theory, which contains sufficient parameters such that it is free from the corresponding difficulties, and a series of interesting applications of the theory instability and regularization, Game Theory, Operations Research, Probability and Statistics.

Maji *et al* **[8]** proposed several operations on soft sets and some basic properties. Shabir and Naz **[10]** introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. K. Kannan **[6]** studied soft generalized closed sets in soft topological spaces along with its properties. The concept of generalized closed sets was introduced by N. Levine **[7].** Many mathematicians **[1][2][3]** extended the results of generalized closed sets in many directions.

In this present study, we introduce some new concepts in soft topological spaces such as gβ closed sets and soft gsβ closed sets and some relation between these sets.

2. PRELIMINARIES

Definition: 2.1 [9] Let U be an initial universe and E be a set of parameters. Let P (U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: $A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A).

Definition: 2.2 The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, H(e) = F(e) if $e \in A – B$, H(e)=G(e) if $e \in B – A$ and $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$. We write $(F, A) \cup (G, B) = (H, C)$.

Definition: 2.3 The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) $\cap$ (G, B), is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition: 2.4 Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if (1) Φ, X belong to τ, (2) the union of any number of soft sets in τ belongs to τ, (3) the intersection of any two soft sets in τ belongs to τ. The triplet (X, τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Corresponding author: ***A. Arokia Lancy*
Department of Mathematics, KTVR Knowledge Park for Engineering and Technology, Coimbatore, India

Definition: 2.5 Let (X, τ , E) be a soft topological space over X and (F, E) be a soft set over X. Then, the soft closure of (F, E), denoted by cl (F, E) is the intersection of all soft closed supersets of (F, E).

Clearly (F, E) is the smallest soft closed set over X which contains (F, E). The soft interior of (F, E), denoted by int (F, E) is the union of all soft open subsets of (F, E). Clearly (F, E) is the largest soft open set over X which is contained in (F, E).

Definition: 2.6 A soft set (A,E) is called soft generalized closed set (soft g-closed) in a soft topological space (X, τ , E) if cl(A,E) $\tilde{\subset}$ (U,E) whenever (A, E) $\tilde{\subset}$ (U, E) and (U, E) is soft open in X

Definition: 2.7 A soft topological space (X, τ , E) is a soft $T_{1/2}$ space if every soft g closed set is soft closed in X

Definition: 2.8 A subset A of a topological space (X, τ , E) is called a soft pre open set if (A,E) $\tilde{\subset}$ int(cl(A,E))

Definition: 2.9 A subset A of a topological space (X, τ , E) is called a soft semi open set if (A,E) $\tilde{\subset}$cl(int(A,E))

Definition: 2.10 A subset A of a topological space (X, τ) is called a pre open set if A $\subseteq$ int(cl(A)) .

Definition: 2.11 A subset A of a topological space (X, τ) is called a semi open set if A $\subseteq$ cl(int(A))

Definition: 2.10 A subset A of a topological space X is called β-closed if int (cl (int(A))) $\subset$ A.

Definition: 2.11 A subset A of a topological space X is called regular open if A=int (cl(A))

Definition: 2.12 A subset A of a topological space X is called generalized closed (g-closed) if cl (A) $\subset$ U whenever A $\subset$ U and U is open in X.

Definition: 2.13 A subset A of a topological space X is called semi-generalized closed (sg-closed) if scl (A) $\subset$ U whenever A $\subset$ U and U is semi-open in X.

Definition: 2.14 A subset A of a topological space X is called generalized-semi closed (gs-closed) if scl (A) $\subset$ U whenever A $\subset$ U and U is open in X

Definition: 2.15 A subset A of a topological space X is called regular generalized closed (rg-closed) if cl (A) $\subset$ U whenever A $\subset$ U and U is regular open in X.

Definition 2.16 A subset A of a topological space X is called α generalized-closed **(** α g -closed**)** if α cl (A) $\subset$ U whenever A $\subset$ U and U is open in X.

Definition: 2.17 A subset A of a topological space X is called semi-closed if int (cl (A)) $\subset$ A.

Definition: 2.18 A subset A of a topological space X is said to be clopen if A is closed and open in X.

Definition: 2.19 A subset A of a topological space X is called gβ-closed if βcl (A) $\subset$ U whenever A $\subset$ U and U is open in (X, τ).

Definition: 2.20 A subset of a topological space X is called gsβ –closed if βcl (A) $\subset$ U whenever A $\subset$ U and U is semi-open in(X, τ).

Definition: 2.21 A subset of a topological space X is called gsβ –closed if βcl (A) $\subset$ U whenever A $\subset$ U and U is semi-open in(X, τ).

Definition: 2.22[7] A subset A in a topological space is defined to be a Q set iff int (cl(A))=cl(int(A))

Now we shall define the generalized β closed set using soft set in the following definition:

3. SOFT GENERALIZED β CLOSED SETS Definition: 3.1 A subset (A, E) of a topological space X is called soft generalized β closed (soft gβ –closed) in a soft topological space (X, τ, E), if βcl(A,E) $\tilde{\subset}$ (U,E), whenever (A,E) $\tilde{\subset}$ (U,E) and (U,E) is soft open in X.

Definition: 3.2 A subset (A, E) of a topological space X is called soft semi-generalized closed (soft sg-closed) if scl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft semi-open in X.

Definition: 3.3 A subset (A, E) of a topological space X is called soft generalized-semi closed (soft gs-closed) if scl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

Definition: 3.4 A subset (A, E) of a topological space X is called soft β-closed if int(cl (int(A,E))) $\tilde{\subseteq}$ (A,E)

Definition: 3.5 A subset (A,E) of a topological space X is called soft α generalized-closed **(**soft αg -closed**)** if αcl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

Definition: 3.6 A subset (A, E) of a topological space X is called regular generalized closed (rg-closed) if cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft regular open in X.

Theorem: 3.7

a) Every soft closed set is soft gβ –closed.
b) Every soft g-closed set is soft gβ –closed.
c) Every soft sg-closed set is soft gβ –closed.
d) Every soft gs-closed set is soft gβ –closed.
e) Every soft β-closed set is soft gβ –closed
f) Every soft αg-closed set is soft gβ –closed.
g) Every soft α-closed set is soft gβ –closed
h) Every soft rg-closed set is soft gβ –closed

Proof:

a) Suppose (A, E) is a soft closed set, (U,E) be soft open in X ,such that (A,E) $\tilde{\subseteq}$ (U,E), cl(A,E)= (A,E) $\tilde{\subseteq}$ (U,E). Thus β cl(A,E) $\tilde{\subseteq}$ cl(A,E) $\tilde{\subseteq}$ (U,E). Hence (A, E) is soft gβ - closed.

b) Suppose (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X. By assumption cl(A,E) $\tilde{\subseteq}$ (U,E) $\Rightarrow$ β cl(A,E) $\tilde{\subseteq}$ (U,E).Hence (A,E) is soft gβ –closed

c) The proof is obvious and straight forward.

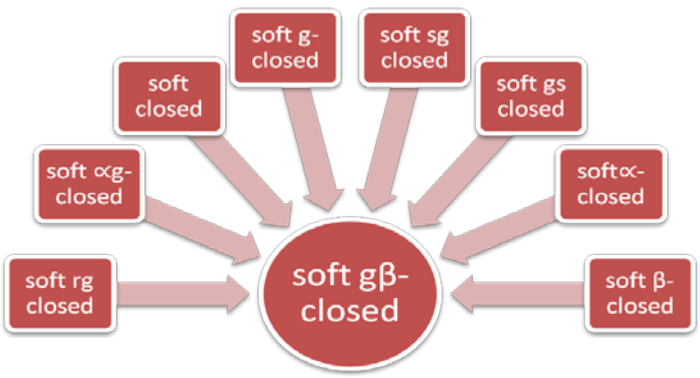
d) Let (A, E) be soft gs-closed subset of X, Suppose (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X, scl(A,E) $\tilde{\subseteq}$ (U,E) $\Rightarrow$ β cl(A,E) $\tilde{\subseteq}$ (U,E) , Hence (A,E) is soft gβ –closed.

e) The proof is obvious and straight forward.

f) Let (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft open in X. αcl(A,E) $\tilde{\subseteq}$ (U,E) $\Rightarrow$ βcl(A,E) $\tilde{\subseteq}$ αcl(A,E) $\tilde{\subseteq}$ (U,E). Thus (A, E) is soft gβ –closed.

The proof of (g) and (h) are obvious and straight forward

From the above results, the following implication is made:



Theorem: 3.8 If (A, E) is soft open and soft gβ - closed then (A, E) is soft β - closed.

Proof: Suppose (A, E) is soft open and soft gβ - closed. Then, by definition βcl (A, E) $\tilde{\subset}$ (A, E). But always (A, E) $\tilde{\subset}$ βcl(A,E). Thus (A,E) = βcl(A,E) $\Rightarrow$ (A,E) is soft β - closed.

Theorem: 3.9 For a subset (A, E) $\tilde{\subset}$ X, the following conditions are equivalent
1. (A, E) is soft open and soft gs-closed. 2. (A, E) is soft regular open.

Proof:
1 $\Rightarrow$ 2: scl (A,E) $\tilde{\subset}$ (A,E), since (A,E) A is soft open and soft gs – closed. Thus int (cl(int(A,E))) $\tilde{\subset}$ (A,E) since scl (A,E)=(A,E) $\cup$ int(cl(A,E)).Since(A, E) is open, then (A,E) is clearly soft pre - open. Thus (A,E) $\tilde{\subset}$ int(cl(A,E)) $\Rightarrow$ int(cl(A,E)) $\tilde{\subset}$ (A,E) $\tilde{\subset}$ int(cl(A,E)) $\Rightarrow$ ((A,E) =int(cl((A,E))).

2 $\Rightarrow$ 1: Let (A, E) is soft regular open $\Rightarrow$ (A, E) = int (cl((A,E))). From this we have int (cl((A,E))) $\tilde{\subset}$ (A,E). Thus (A, E) is soft semi - closed $\Rightarrow$ (A,E) is soft sg – closed.

But every soft sg-closed set is soft gs-closed. Hence (A, E) is soft gs-closed.

Corollary: 3.10 If (A, E) is soft open and soft gs-closed set of (X, τ, E) then (A,E) is soft semi – closed.

Proof: Let (A, E) be soft open and soft gs-closed $\Rightarrow$ scl(A,E) $\tilde{\subset}$ (A,E) But (A,E) $\tilde{\subset}$ scl((A,E))
$\Rightarrow$ (A, E)= scl((A,E)) $\Rightarrow$ (A,E) is soft semi – closed.

Theorem: 3.11 For a subset (A, E) $\tilde{\subset}$ X, the following conditions are equivalent
1. (A, E) is soft open and soft g β -closed.
2. (A, E) is soft regular open.

Proof:
1 $\Rightarrow$ 2: By 1, βcl(A) $\tilde{\subset}$ A, Since A is soft open and soft gβ–closed. Thus int (cl(A,E)) $\tilde{\subset}$ (A,E), since βcl(A,E)= (A,E) $\cup$ int(cl(A,E)).

Since (A, E) is soft open, (A, E) is clearly soft pre - open. Thus (A, E) $\tilde{\subset}$ int(cl((A,E))). $\Rightarrow$ (A, E) is soft regular open.

2 $\Rightarrow$ 1: Let (A, E) be soft regular open, $\Rightarrow$ (A, E) is soft open. Since (A, E) is soft regular open, (A, E) = int(cl((A,E))). From this we have int(cl((A,E))) $\tilde{\subset}$ (A,E). Thus (A, E) is soft semi – closed. $\Rightarrow$ (A, E) is soft sg-closed. By theorem 3.7(c), (A, E) is soft gβ - closed.

Theorem: 3.12 For a subset (A, E) $\tilde{\subset}$ X, the following conditions are equivalent.
1. (A, E) is soft clopen.
2. (A, E) is soft open, a Q – set and soft gβ - closed.

Proof:
1 $\Rightarrow$ 2: Let (A, E) be soft clopen. Since (A, E) is soft closed, (A, E) is soft gβ - closed. And we have int (cl((A,E))) = (A,E) = cl(int((A,E))). Thus (A,E) is a Q – set. Hence (A, E) is soft open, a Q-set and soft gβ - closed.

2 $\Rightarrow$ 1: Let (A, E) be soft open, a Q – set and soft gβ – closed.

Since (A, E) is soft open and soft gβ - closed, by theorem 3.11, (A, E) is soft regular open.

Since (A,E) is soft regular open,(A,E)=int(cl((A,E))) = cl(int((A,E))) = cl((A,E)). Thus (A, E) is soft closed.
Equivalently, (A, E) is soft clopen.

Theorem: 3.13 A set (A, E) is soft gβ closed in X iff βcl (A, E) \ (A, E) contains only null soft closed set.

Proof: Suppose (A,E) is soft gβ closed in X, Let (F,E) be soft closed $\Rightarrow$ (F,E) $\tilde{\subset}$ βcl(A,E)\(A,E)

$\Rightarrow$ (F, E) $\tilde{\subset}$ βcl(A,E) and (F,E) $\tilde{\subset}$ (A,E)'. Hence (A, E) $\tilde{\subset}$ (F, E)'

$\Rightarrow$ βcl(A,E) $\tilde{\subset}$ (F,E)'

$\Rightarrow$ (F, E) $\tilde{\subset}$ (βcl(A,E))'. Thus (F, E) is null set.

Conversely, βcl(A,E)\(A,E)= ϕ $\Rightarrow$ βcl(A, E) = (A, E). (A, E) is soft β closed, hence soft gβ closed.

4. SOFT gs β CLOSED SETS

Definition: 4.1 A subset (A, E) of a topological space X is called soft gsβ closed if β *cl (A, E)* $\tilde{\subset}$ *(U, E)* whenever (A,E) $\tilde{\subset}$ (U,E) and (U,E) is soft semiopen in (X, τ ,E)

Theorem: 4.2
(i) Every soft closed set is soft gsβ – closed.
(ii) Every soft sg- closed set is soft gsβ – closed.
(iii) Every soft semi-closed set is soft gsβ - closed.
(iv) Every soft β - closed set is soft gsβ – closed.
(v) Every soft α - closed set is soft gsβ – closed.

Proof:
(i) Let (A, E) be a soft closed set such that (A, E) $\tilde{\subset}$ (U, E) where (U, E) is soft semi-open in X.

Since (A, E) is soft closed, (A, E) = cl((A, E)).
$\Rightarrow$ βcl((A,E)) $\tilde{\subset}$ cl((A,E)) $\tilde{\subset}$ (U,E)
$\Rightarrow$ βcl((A,E)) $\tilde{\subset}$ (U,E)
Hence the proof.

(ii) Proof is obvious and straight forward.

(iii) Let (A, E) be a soft semi-closed set such that (A,E) $\tilde{\subset}$ (U,E) where (U,E) is soft semi-closed.
Thus (A, E) = scl((A,E)) $\tilde{\subset}$ (U,E). Hence βcl((A,E)) $\tilde{\subset}$ scl((A,E)) $\tilde{\subset}$ (U,E)
$\Rightarrow$ βcl((A,E)) $\tilde{\subset}$ (U,E)

(iv) Proof is obvious

(v) Assume (A, E) to be a soft α - closed set.

Let (A, E) $\tilde{\subset}$ (U,E) where (U,E) is soft semi-open in X.
α cl((A,E)) $\tilde{\subset}$ (U,E) $\Rightarrow$ βcl ((A,E)) $\tilde{\subset}$ α cl((A,E)) $\tilde{\subset}$ (U,E) $\Rightarrow$ βcl((A,E)) $\tilde{\subset}$ (U,E)

Thus (A, E) is soft gsβ - closed.

Theorem: 4.3 Every soft gsβ - closed set is soft gβ – closed.

Proof: Let (A, E) be a soft gsβ – closed set.

Assume (A, E) $\tilde{\subset}$ (U,E) where (U,E) is soft open.
Since, every soft open set is soft semi-open and βcl((A,E)) $\tilde{\subset}$ (U,E)
Hence (A, E) is soft gβ - closed.

5. SOFT T^* SPACES

Definition: 5.1 A space X is said to be soft T^* space if every soft gsβ – closed set is soft β - closed.

Theorem: 5.2 For a space (X, τ , E) the following conditions are equivalent:

1. X is a soft T^* space
2. Every singleton of X is soft closed or soft β - open.
3. Every singleton of X is soft closed or soft pre-open.

Proof:
(1) $\Rightarrow$ **(2):** Assume that for some x $\in$ X, {x}is not soft closed.

Then X\ {x} is not soft open.

Thus the only soft open set containing X\{x} is X itself and hence X\{x}is trivially soft gsβ – closed

By (1) X\{x} soft closed $\Rightarrow$ X\{x} is soft β – closed $\Rightarrow$ {x} is soft β-open

(2) $\Rightarrow$ **(3):** Assume that for some x $\in$ X, the set {x} is not soft pre -open.

Thus {x} is nowhere dense, since it is well known that in every space a Singleton is either soft pre – open or nowhere dense

We have cl(int(cl({ x }))) = ϕ

Hence {x} $\not\subset$ cl(int(cl { x }))

$\Rightarrow$ {x} is not soft β - open

By (2) {x} is soft closed.

(3) $\Rightarrow$ (1) Let (A, E) $\tilde{\subset}$ X be soft gsβ – closed. to prove βcl(A,E) = (A,E), let x $\in$ βcl (A,E)

By assumption {x} is either soft closed or soft pre – open.

WE CONSIDER THESE TWO CASES:

Case (1): Let {x} be soft closed.
βcl(A,E)\ (A,E) does not contain non- empty soft closed set,

Thus {x} $\not\subset$ βcl (A, E) \(A,E).Hence x $\in$ (A,E) $\Rightarrow$ βcl (A,E) $\tilde{\subset}$ (A,E)

But (A, E) $\tilde{\subset}$ βcl((A,E)) $\Rightarrow$ βcl((A,E))= (A,E)

Case (2): Let {x} be soft pre – open. $\Rightarrow$ Clearly {x} is soft β - open

And x $\in$ βcl ((A,E)),then {x} $\cap$ (A,E) $\neq \phi$

Hence x $\in$ (A,E) $\Rightarrow$ βcl ((A,E)) $\tilde{\subset}$ (A,E),So (A,E) = βcl((A,E))

$\Rightarrow$ (A,E) is soft β - closed. Hence X is a soft T^* -space.

Theorem: 5.3 Every soft $T_{1/2}$ - space is a soft T^* - space.

Proof: Suppose X is a soft $T_{1/2}$ - space.

Every singleton of X is soft closed or soft open.
$\Rightarrow$ Every singleton of X is soft closed or soft pre – open.

Hence by theorem 5.2, X is a T^* -space.

6. CONCLUSION

In this paper, soft gβ closed sets and soft gsβ closed sets were introduced and studied with already existing sets in soft topological spaces. A new space soft T* is also been introduced. The scope for further research can be focused on the applications of soft topological spaces.

REFERENCES

[1] P. Bhattacharya and B. K. Lahiri, *"On Semi- generalized closed sets in topology"*, Indian J Math. 29 (1987) no.3, 375-382.

[2] R. Devi, H. Maki and K. Balachandran, *"On Semi- generalized closed maps and generalized closed maps"*, Mem. Fac. Sci. Kochi-Univ. Ser. A. Math., 14(1993), 41-54.

[3] J. Dontchev, *"On generalized semi-pre open sets"*, Mem. Fac. Sci. Kochi-Univ. Ser. A. Math., 16, (1995).

[4] W. Dunham, *"On $T_{1/2}$– Spaces"*, Kyongpook Math. J., 17(1977) 161- 169.

[5] Ganster and D. Andrijevic, "*On some questions concerning semi- pre open sets*" J. Inst. Mathe & Comp. Sci (Math. ser), (1988), 65-72.

[6] K. Kannan, "*Soft Generalized closed sets in soft Topological Spaces"*, Journal of Theoretical and Applied Information Technology, Vol.37 (2012), 17-20.

[7] N. Levine, "*On Generalized Closed Sets in Topology*", Rend. Circ. Math. Palermo, 19(2) (1970) 89-96.

[8] P. K. Maji, R. Biswas and A. R. Roy, "*Soft Set Theory*" Computers and Mathematics with Applications (2003), 555-562.

[9] D. Molodtsov, "*Soft Set Theory-First results*", computers and Mathematics with Applications, (1999), 19-31.

[10] M. Shabir and M. Naz, "*On Soft Topological Spaces*", Computers and Mathematics with Applications, (2011), Vol.61, Issue 7, 1786-1799.

[11] P. Sundaram, H. Maki and K. Balachandran, "*On Semi-generalized continuous maps and semi $T_{1/2}$ –Spaces*", Bull. Fukuoka Univ.Ed Part III 40(1991), 33-40.

Source of support: Nil, Conflict of interest: None Declared