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# ON $g^{\mu}b$ –TOTALLY CONTINUOUS FUNCTIONS IN SUPRA TOPOLOGICAL SPACES

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***ABSTRACT***

*In this paper, we introduce new generalization of strong continuous function called $g^{\mu}b$-totally continuous function, which is stronger than $totally^{\mu}$ continuous. We characterize and obtain the properties of $g^{\mu}b$-totally continuous function. Further $g^{\mu}b$-totally open function is also discussed.*

***Keywords:*** *Supra b – closed set, generalized b - closed sets.*

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## 1. INTRODUCTION

The generalization of closed sets in topological space and a class of topological spaces called $T_{1/2}$ spaces were introduced by Levine [7] in 1970. Extensive research on generalizing closedness was done in recent years by many Mathematicians [3, 4, 7, 8]. In 1980, R. C. Jain [6] defined totally continuous functions. Andrijevic [1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [5] under the name of $\gamma$ - open sets.

In 1983, A. S. Mashhour *et al* [9] investigated the notion of supra topological spaces and studied S-S continuous functions and $S^*$- continuous functions. In 2010, O. R. Sayed and Takashi Noiri [10] introduced supra b - open sets and supra b - continuity. The purpose of this paper is to develop a new generalization of strong continuity called $g^{\mu}b$-totally continuity, which is stronger than $totally^{\mu}$ continuity. Furthermore, the properties and preservation theorems of $g^{\mu}b$-totally continuous functions are investigated. Also $g^{\mu}b$-totally open functions in supra topological spaces are studied.

## 2. PRELIMINARIES

**Definition: 2.1 [9]** A subfamily μ of X is said to be a supra topology on X if

i) $X,\phi \in \mu$

ii) if $A_i \in \mu$ for all i ∈ J, then $\cup A_i \in \mu$ .($X$,μ) is called a supra topological space. The elements of μ are called supra open sets in ($X$,μ) and complement of supra open set is called supra closed set and it is denoted by $\mu^c$.

**Definition: 2.2 [9]** The supra closure of a set A is defined as $Cl^{\mu}(A) = \cap \{B: B \text{ is supra closed and } A \subseteq B\}$ The supra interior of a set A is defined as $Int^{\mu}(A) = \cup \{B: B \text{ is supra open and } A \supseteq B\}$

**Definition: 2.3 [12]** A function $f:(X,\tau) \to (Y,\sigma)$ is said to be $g^{\mu}$ b –continuous if $f^{-1}(V)$ is $g^{\mu}$ b - closed in $(X,\tau)$ for every supra closed set V of $(Y,\sigma)$.

**Definition: 2.4 [12]** A function $f:(X,\tau) \to (Y,\sigma)$ is said to be $g^{\mu}$ b –irresolute if $f^{-1}(V)$ is $g^{\mu}$ b - closed in $(X,\tau)$ for every $g^{\mu}$ b - closed set V of $(Y,\sigma)$.

**Definition: 2.5 [2]** Let ($X$,μ) be a supra topological space. A set A of X is called supra generalized b-closed set (simply $g^{\mu}$ b - closed) if $bcl^{\mu}(A) \subseteq$ U whenever A⊆U and U is supra open. The complement of supra generalized b - closed set is supra generalized b - open set.

## 3. CHARACTERIZATIONS OF $g^{\mu}b$-TOTALLY CONTINUOUS FUNCTIONS

**Definition: 3.1** A function $f: X \to Y$ is said to be supra -totally continuous function if the inverse image of every supra open subset of Y is $Cl^{\mu} open^{\mu}$ in X.

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**Definition: 3.2** A function $f: X \to Y$ is said to be $g^\mu b$-totally continuous function if the inverse image of every $g^\mu b$-open subset of Y is $Cl^\mu open^\mu$ in X.

**Theorem: 3.3** A bijective function $f: X \to Y$ is $g^\mu b$-totally continuous if and only if the inverse image of every $g^\mu b$-closed subset of Y is $Cl^\mu open^\mu$ in X.

**Proof:** Let F be any $g^\mu b$- closed set in Y. Then Y-F is $g^\mu b$-open set in Y.By definition $f^{-1}(Y-F)$ is $Cl^\mu open^\mu$ in X. That is X$-f^{-1}(F)$ is $Cl^\mu open^\mu$ in X. This implies $f^{-1}(F)$ is $Cl^\mu open^\mu$ in X.

Conversely, if V is $g^\mu b$-open in Y, then Y−V is $g^\mu b$-closed in Y. By assumption, $f^{-1}(Y-V) = X - f^{-1}(V)$ is $Cl^\mu open^\mu$ in X, which implies $f^{-1}(V)$ is $Cl^\mu open^\mu$ in X. Therefore $f$ is $g^\mu b$-totally continuous function.

**Theorem: 3.4**

(i) Every $g^\mu b$-totally continuous function is $totally^\mu$ continuous function.
(ii) Every $g^\mu b$-totally continuous function is $g^\mu b$-continuous.

**Proof:**

(i) Suppose $f: X \to Y$ is $g^\mu b$-totally continuous and U is any supra open subset of Y.Since every supra open set is $g^\mu b$-open, U is $g^\mu b$-open in Y and $f: X \to Y$ is $g^\mu b$ –totally continuous, it follows $f^{-1}(U)$ is $Cl^\mu open^\mu$ in X.
(ii) It is similar to (i).

**Remark: 3.5** The converse of the above theorem is not true as shown by the following example.

**Example: 3.6** Let X= {a, b, c};$\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{\phi, X, \{a\}\}$. Let $f: (X, \tau) \to (X, \sigma)$ be an identity function. Here$f$ is $totally^\mu$ continuous but $f^{-1}\{b\} = \{b\}$ is not supra closed in $(X, \tau)$. Therefore $f$ is not $g^\mu b$-totally continuous function.

**Example: 3.7** Let X = {a, b, c, d};$\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{\phi, X, \{a\}\}$. Let $f: (X, \tau) \to (X, \sigma)$ be defined by $f(a) = b; f(b) = c; f(c) = d$ and $f(d) = a$. Here $f$ is $g^\mu b$- continuous function but not $g^\mu b$-totally continuous.

**Theorem: 3.8** Let $f: X \to Y$ be a function, where X and Y are supra topological spaces.Then following are equivalent.

(i) $f$ is $g^\mu b$-totally continuous.
(ii) For each $x \in X$ and each $g^\mu b$-open set V in Y with $f(x) \in V$, there is a $Cl^\mu open^\mu$ set U in X such that $x \in U$ and $f(U) \subset V$.

**Proof:** (i)⇒(ii) Suppose $f$ is $g^\mu b$-totally continuous and V be any $g^\mu b$-open set in Y containing $f(x)$ so that $x \in f^{-1}(V)$. Since $f$ is $g^\mu b$-totally continuous, $f^{-1}(V)$ is $Cl^\mu open^\mu$ in X. Let $U = f^{-1}(V)$ then U is $Cl^\mu open^\mu$ set in X and $x \in U$. Also $f(U) = f\big(f^{-1}(V)\big) \subset V$. This implies $f(U) \subset V$.

(ii) ⇒(i) Let V be $g^\mu b$-open in Y. Let $x \in f^{-1}(V)$ be any arbitrary point. This implies $f(x) \in V$.Therefore by (ii) there is a $Cl^\mu open^\mu$ set $f(G_x) \subset X$ containing $x$ such that $f(G_x) \subset V$, which implies $G_x \subset f^{-1}(V)$ is $Cl^\mu open^\mu$ neighbourhood of $x$. Since $x$ is arbitrary, it implies $f^{-1}(V)$ is $Cl^\mu open^\mu$ neighborhood of each of its points. Hence it is $Cl^\mu open^\mu$ set in X.Therefore $f$ is $g^\mu b$-totally continuous.

**Definition: 3.9** A supra topological space $(X, \tau)$ is said to be $g^\mu b$ - space if every $g^\mu b$-closed set of X is supra closed in X.

**Definition: 3.10** A supra topological space X is said to be $locally^\mu$indiscrete if every supra open set of X is supra closed in X.

**Theorem: 3.11** For a function $f: X \to Y$ the following properties hold

(i) If f is supra continuous and X is $locally^\mu$indiscrete then $f$ is $totally^\mu$continuous.
(ii) If $f$ is $totally^\mu$continuous and Y is $g^\mu b$ –space then $f$ is $g^\mu b$-totally continuous.

**Proof:** (i) Let V be supra open in Y. Since $f$ is supra continuous and X is $locally^\mu$indiscrete, $f^{-1}(V)$ is supra open and supra closed in X. Hence $f^{-1}(V)$ is $Cl^\mu open^\mu$ in X. Therefore $f$ is $totally^\mu$continuous.

(ii)Let V be $g^\mu b$-open in Y. Then Y−V is $g^\mu b$-closed in Y. Since Y is $g^\mu b$ –space, Y−V is supra closed in Y, which implies V is supra open in Y. Since $f$ is $totally^\mu$continuous $f^{-1}(V)$ is $Cl^\mu open^\mu$ in X. Therefore $f$ is $g^\mu b$-totally continuous.

**Theorem: 3.12** The composition of two $g^{\mu}b$-totally continuous function is $g^{\mu}b$-totally continuous.

**Proof:** It is obvious.

**Theorem: 3.13**
(i) If $f: X \to Y$ is $g^{\mu}b$-totally continuous and $g: Y \to Z$ is $g^{\mu}b$ irresolute, then $g \circ f: X \to Z$ is $g^{\mu}b$-totally continuous.
(ii) If $f: X \to Y$ is $g^{\mu}b$-totally continuous and $g: Y \to Z$ is $g^{\mu}b$ -continuous, then $g \circ f: X \to Z$ is $totally^{\mu}$ continuous.

**Proof:** It is obvious.

**Theorem: 3.14** Let $f: X \to Y$ be $g^{\mu}b$-closed map and $g: Y \to Z$ be any function. If $g \circ f: X \to Z$ is $g^{\mu}b$-totally continuous then $g$ is $g^{\mu}b$-irresolute.

**Proof:** Let $g \circ f: X \to Z$ be $g^{\mu}b$-totally continuous. Let V be $g^{\mu}b$-open set in Z. Since $g \circ f$ is $g^{\mu}b$-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $Cl^{\mu}\, open^{\mu}$ in X. Since $f$ is $g^{\mu}b$-totally continuous,
$f\left(f^{-1}\left(g^{-1}(V)\right)\right)$ is $Cl^{\mu}\, open^{\mu}$ in Y. Then $g^{-1}(V)$ is $g^{\mu}b$-open in Y. Hence $g$ is $g^{\mu}b$-irresolute.

## 4. APPLICATIONS

**Definition: 4.1** A supra topological space X is said to be
(i) $g^{\mu}b$-$T_0$ if for each pair of distinct points in $X$, there exists a $g^{\mu}b$-open set containing one point but not the other.
(ii) $g^{\mu}b$-$T_1$(resp.$Cl^{\mu}\, open^{\mu}\, T_1$) if for each pair of distinct points $x$ and $y$ of $X$, there exist $g^{\mu}b$-open(resp. $Cl^{\mu}\, open^{\mu}$) sets $U$ and $V$containing $x$ and $y$ respectively such that $x \in U, y \notin U\ and\ x \notin V, y \in V$.
(iii) $g^{\mu}b$-$T_2$(resp.$ultra^{\mu}$ Hausdorff or $UT_2^{\mu}$) if every two distinct points of X can be separated by disjoint $g^{\mu}b$-open (resp. $Cl^{\mu}\, open^{\mu}$) sets.
(iv) $S^{\mu}$-normal (resp.$ultra^{\mu}$ normal) if each pair of non-empty disjoint supra closed sets can be separated by disjoint $g^{\mu}b$-open(resp. $Cl^{\mu}\, open^{\mu}$)sets.
(v) $S^{\mu}$-regular(resp. $ultra^{\mu}$-regular) if for each supra closed set F of X and each $x \notin F$, there exist disjoint $g^{\mu}b$-open(resp. $Cl^{\mu}\, open^{\mu}$) sets U and V such that $F \subset U$ and $x \in V$.
(vi) $g^{\mu}b$-normal(resp. $Cl^{\mu}\, open^{\mu}$ normal) if for each pair of disjoint $g^{\mu}b$-closed(resp. $Cl^{\mu}\, open^{\mu}$) sets U and V of X, there exist two disjoint $g^{\mu}b$-open(resp. supra open) sets G and H such that $U \subset G$ and $V \subset H$.
(vii) $g^{\mu}b$-regular($Cl^{\mu}\, open^{\mu}$ regular) if for each $g^{\mu}b$-closed(resp. $Cl^{\mu}\, open^{\mu}$) set F of X and each $x \notin F$, there exist disjoint $g^{\mu}b$-open(resp.supra open) sets U and V such that $F \subset U$ and $x \in V$.
(viii) $g^{\mu}b$-connected if X is not the union of two non-empty disjoint $g^{\mu}b$-open subsets of X.

**Theorem: 4.2** Every $g^{\mu}b$-regular is $S^{\mu}$-regular space.
(i) Every $g^{\mu}b$-normal is $S^{\mu}$-normal space.

**Proof:** It is obvious.

**Theorem: 4.3** If $f: X \to Y$ is $g^{\mu}b$-totally continuous injection and Y is $g^{\mu}b$-$T_1$, then X is $Cl^{\mu}\, open^{\mu}$-$T_1$

**Proof:** Let $x$ and $y$ be any two distinct points in X. Since $f$ is injective we have $f(x)$ and $f(y) \in Y$ such that $f(x) \neq f(y)$. Since Y is $g^{\mu}b$-$T_1$, there exist $g^{\mu}b$-open sets U and V in Y such that $f(x) \in U, f(y) \notin U$, $f(y) \in V$ and $f(x) \notin V$. Therefore we have $x \in f^{-1}(U)$, $y \notin f^{-1}(U), y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are $Cl^{\mu}\, open^{\mu}$ subsets of X because $f$ is $g^{\mu}b$-totally continuous. This shows that X is $Cl^{\mu}\, open^{\mu}$- $T_1$.

**Theorem: 4.4** If $f: X \to Y$ is $g^{\mu}b$-totally continuous injection and Y is $g^{\mu}b$- $T_0$, then X is $ultra^{\mu}$ Hausdorff.

**Proof:** Let $a$ and $b$ be any pair of distinct points of X and $f$ be injective. Then $f(a) \neq f(b)$ in Y. Since Y is $g^{\mu}b$- $T_0$, there exists a $g^{\mu}b$-open set containing say $f(a)$ but not $f(b)$. Then we have $a \in f^{-1}(U)$ and $b \notin f^{-1}(U)$. Since $f$ is $g^{\mu}b$-totally continuous $f^{-1}(U)$ is $Cl^{\mu}\, open^{\mu}$ in X. Also $a \in f^{-1}(U)$ and $b \in X - f^{-1}(U)$. This implies every pair of distinct points of X can be separated by disjoint $Cl^{\mu}\, open^{\mu}$ sets in X. Therefore X is $ultra^{\mu}$ Hausdorff.

**Theorem: 4.5** If $f: X \to Y$ is $g^{\mu}b$-totally continuous injection and Y is $g^{\mu}b$-$T_2$, then X is $ultra^{\mu}$ Hausdorff.

**Proof:** Let $x_1$, $x_2 \in X$ and $x_1 \neq x_2$. Then, since $f$ is injective, $f(x_1) \neq f(x_2)$ in Y. Further, since Y is $g^{\mu}b$- $T_2$, there exist $V_1$ and $V_2 \in g^{\mu}bo(Y)$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$, and $V_1 \cap V_2 = \phi$. This implies $x_1 \in f^{-1}(V_1)$ and $x_2 \in f^{-1}(V_2)$. Since $f$ is $g^{\mu}b$-totally continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $Cl^{\mu}\, open^{\mu}$ sets in X. Also $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2)=\phi$ . Thus every two distinct points of X can be separated by disjoint $Cl^{\mu}\, open^{\mu}$ sets. Therefore X is $ultra^{\mu}$ Hausdorff.

 

**Theorem: 4.6** If $f: X \to Y$ is $g^\mu b$-totally continuous, supra closed injection and Y is $g^\mu b$-normal, then X is $ultra^\mu$ normal.

**Proof:** Let $F_1$ and $F_2$ be disjoint supra closed subsets of X. Since $f$ is supra closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint supra closed subsets of Y. Since Y is $g^\mu b$-normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint $g^\mu b$-open sets $V_1$ and $V_2$ separately. Therefore we obtain,$F_1 \subset f^{-1}(V_1)$ and $F_2 \subset f^{-1}(V_2)$. Since $f$ is $g^\mu b$-totally continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $Cl^\mu open^\mu$ sets in X. Also $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2)$=$\phi$. Thus each pair of non-empty disjoint supra closed sets in X can be separated by disjoint $Cl^\mu open^\mu$ sets in X. Therefore X is $ultra^\mu$ normal.

**Theorem: 4.7** If $f: X \to Y$ is $g^\mu b$-totally continuous, surjection and X is supra connected then Y is $g^\mu b$-connected.

**Proof:** Suppose Y is not $g^\mu b$-connected. Let A and B form disconnection of Y. Then A and B are $g^\mu b$-open sets in Y and $Y = A \cup B$ where $A \cap B = \phi$. Also $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty $Cl^\mu open^\mu$ sets in X, because $f$ is $g^\mu b$-totally continuous. Further $f^{-1}(A) \cap f^{-1}(B)$= $f^{-1}(A \cap$ $B$=$\phi$.This implies X is not supra connected, which is a contradiction. Hence Y is $g\mu b$-connected.

**Theorem: 4.8** If $f: X \to Y$ is $totally^\mu$continuous injective $g^\mu b$-open function from a $Cl^\mu open^\mu\ regular^\mu$ space X onto a space Y, then Y is $s^\mu$-regular.

**Proof:** Let F be supra closed set in Y and $y \notin F$.Take $y$=$f(x)$.Since $f$ is $totally^\mu$ continuous, $f^{-1}(F)$ is $Cl^\mu open^\mu$ in X. Let $G = f^{-1}(F)$.Then we have $x \notin G$. Since X is $Cl^\mu open^\mu$regular, there exist disjoint supra open sets U and V such that $G \subset U$ and $x \in V$.This implies $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$. Further, since $f$ is injective and $g^\mu b$-open, we have $f(U \cap V) = f(\phi)$= $\phi$ and $f(U)$ and $f(V)$ are $g^\mu b$-open sets in Y. Thus, for each supra closed set F in Y and each $y \notin F$,there exist disjoint $g^\mu b$-open sets $f(U)$ and $f(V)$ in Y such that $F \subset f(U)$ and $y \in f(V)$. Therefore Y is $g^\mu b$-regular.

**Theorem: 4.9** If $f: X \to Y$ is $g^\mu b$-totally continuous injective $g^\mu b$-open function from a $Cl^\mu open^\mu$ regular space X onto a space Y, then Y is $g^\mu b$-regular.

**Proof:** It is similar to theorem 4.8

**Theorem: 4.10** Let $f: X \to Y$ be $g^\mu b$-totally continuous, supra closed injection. If Y is $g^\mu b$-regular then X is $ultra^\mu$ -regular.

**Proof:** Let F be supra closed set not containing $x$. Since $f$ is supra closed, we have $f(F)$ is supra closed in Y not containing $f(x)$. Since Y is $g^\mu b$- regular, there exists disjoint $g^\mu b$-open sets A and B such that $f(x) \in A$ and $f(F) \subset B$, which implies $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$ , where $f^{-1}(A)$ and $f^{-1}(B)$ are $Cl^\mu open^\mu$ sets in X because $f$ is $totally^\mu$continuous. Moreover, since $f$ is injective, we have$f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$. Thus, for a pair of point and supra closed set not containing the point, they can be separated by disjoint $Cl^\mu open^\mu$ sets. Therefore X is $ultra^\mu$ regular.

**Theorem: 4.11** Let $f: X \to Y$ be a $g^\mu b$-totally continuous and $g^\mu b$-closed injection. If Y is $g^\mu b$-regular, then X is $ultra^\mu$ -regular.

**Proof:** It is similar to theorem 4.10

**Theorem: 4.12** If $f: X \to Y$ is $totally^\mu$continuous, injective and $g^\mu b$-open function from a $Cl^\mu open^\mu$ normal space X onto a space Y then Y is $s^\mu$-normal.

**Proof:** Let $F_1$ and $F_2$ be any two distinct supra closed sets in Y. Since $f$ is $totally^\mu$continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are $Cl^\mu open^\mu$ subsets of X. Take $U = f^{-1}(F_1)$ and V=$f^{-1}(F_2)$. Since $f$ is injective, we have $U \cap V = f^{-1}(F_1) \cap f^{-1}(F_2) = f^{-1}(F_1 \cap F_2) = f^{-1}(\phi) = \phi$. Since X is $Cl^\mu open^\mu$ normal there exist disjoint supra open sets A and B such that $U \subset A$ and $V \subset B$. This implies $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$. Further, since $f$ is injective $g^\mu b$-open, $f(A)$ and $f(B)$ are disjoint $g^\mu b$-open sets. Thus, each pair of disjoint supra closed sets in Y can be separated by disjoint $g^\mu b$-open sets. Therefore Y is $g^\mu b$-normal.

**Theorem: 4.13** If $f: X \to Y$ is $g^\mu b$-totally continuous injective $g^\mu b$-open function from a$Cl^\mu open^\mu$ normal space X onto a space Y, then Y is $g^\mu b$-normal.

**Proof:** It is similar to theorem 4.12

**Definition: 4.14** A function $f: X \to Y$ is said to be $g^\mu b$-totally open if the image of every $g^\mu b$-open set in X is $Cl^\mu open^\mu$ in Y.

**Theorem: 4.15** If a bijective function $f: X \to Y$ is $g^\mu b$- totally open, then the image of every $g^\mu b$-open set in X is $Cl^\mu open^\mu$ in Y.

**Proof:** Let F be $g^\mu b$-closed set in X. Then X-F is $g^\mu b$-open in X. Since $f$ is $g^\mu b$-totally open, $f(X-F) = Y - f(F)$ is $Cl^\mu open^\mu$ in Y. This implies $f(F)$ is $Cl^\mu open^\mu$ in Y.

**Theorem: 4.16** A surjective function $f: X \to Y$ is $g^\mu b$-totally open iff for each subset B of Y and for each $g^\mu b$-closed set U containing $f^{-1}(B)$ there is a $Cl^\mu open^\mu$ set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

**Proof:** Suppose $f: X \to Y$ is surjective $g^\mu b$-totally open function and $B \subset Y$.Let U be $g^\mu b$-closed set of X such that $f^{-1}(B) \subset U$. Then $V = Y - f(X-U)$ is $Cl^\mu open^\mu$ subset of Y containing B such that $f^{-1}(V) \subset U$.

Conversely, suppose F is $g^\mu b$-closed set of X. Then $f^{-1}\big(Y - f(F)\big) \subset X - F$ is $g^\mu b$-open. By hypothesis, there exists a $Cl^\mu open^\mu$ set V of Y such that Y-f(F)$\subset V$, which implies $f^{-1}(V) \subset X - F$. Therefore F$\subset X - f^{-1}(V)$. Hence Y-V$\subset f(F) \subset f\big(X - f^{-1}(V)\big) \subset Y - V$. This implies,$f(F) = Y - V$, which is $Cl^\mu open^\mu$ in Y. Thus, the image of a $g^\mu b$-open set in X is$Cl^\mu open^\mu$ in Y. Therefore $f$ is $g^\mu b$-totally open function.

**Theorem: 4.17** For any bijective function $f: X \to Y$ the following statements are equivalent
(i) Inverse of $f$ is $g^\mu b$-totally continuous.
(ii) $f$ is $g^\mu b$-totally open.

**Proof:**
(i)⇒(ii) Let U be $g^\mu b$-open set of X. By assumption $(f^{-1})^{-1}(U) = f(U)$ is $Cl^\mu open^\mu$ in Y, So $f$ is $g^\mu b$-totally open.

(ii)⇒(i) Let F be $g^\mu b$-open in X. Then $f(V)$ is $Cl^\mu open^\mu$ in Y. That is $(f^{-1})^{-1}(V)$ is $Cl^\mu open^\mu$ in Y. Therefore $f^{-1}$ is $g^\mu b$-totally continuous.

**Theorem: 4.18** The composition of two $g^\mu b$-totally open function is again $g^\mu b$-totally open.

**Proof:** It is obvious.

**Theorem: 4.19** If $f: X \to Y$ is $g^\mu b$ irresolute and $g: Y \to Z$ is $g^\mu b$-totally continuous, then $g \circ f: X \to Z$ is $g^\mu b$-totally open.

**Proof:** It is obvious.

**Theorem: 4.20** Let $f: X \to Y$ and $g: Y \to Z$ be two functions such that $g \circ f: X \to Z$ is $g^\mu b$-totally open function. Then (i) If $f$ is supra irresolute and surjective, then $g$ is $g^\mu b$-totally open. (ii) If $g$ is$totally^\mu continuous$ and injective, then $f$ is $g^\mu b$-totally open.

**Proof:** It is obvious.

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