

WEEKLY GENERALIZED SEMI-PRE CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce intuitionistic fuzzy weakly generalized semi-pre continuous mappings. We investigate some of its properties. Also we provide the relation between intuitionistic fuzzy weakly generalized semi-pre continuous mappings and other intuitionistic fuzzy continuous mappings.

Key words and phrases: *Intuitionistic fuzzy topology, intuitionistic fuzzy weakly generalized semi-pre continuous mappings.*

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [10], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. Intuitionistic fuzzy semi-pre continuous mappings in intuitionistic fuzzy topological spaces are introduced by Young Bae Jun and Seok-Zun Song [9]. R. Santhi and D. Jayanthi [6] introduced the notion of intuitionistic fuzzy generalized semi-pre continuous mappings and intuitionistic fuzzy generalized semi-pre irresolute mappings. In this paper we introduce intuitionistic fuzzy weakly generalized semi-pre continuous mappings. We investigate some of its properties.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$.

The intuitionistic fuzzy sets $0_\cdot = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\cdot = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

- (i) $0_\cdot, 1_\cdot \in \tau$
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

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In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X . The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{G / G \text{ is an intuitionistic fuzzy open set in } X \text{ and } G \subseteq A\}$$

$$\text{cl}(A) = \cap \{K / K \text{ is an intuitionistic fuzzy closed set in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$ [12].

Definition 2.5:[4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- (i) *intuitionistic fuzzy semi closed set* (IFSCS for short) if $\text{int}(\text{cl}(A)) \subseteq A$
- (ii) *intuitionistic fuzzy pre closed set* (IFPCS for short) if $\text{cl}(\text{int}(A)) \subseteq A$
- (iii) *intuitionistic fuzzy α closed set* (IF α CS for short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFSCSs, IFPCSs, and IF α CSs (respectively intuitionistic fuzzy semi open sets(IFSOs), intuitionistic fuzzy pre open sets(IFPOs) and intuitionistic fuzzy α open sets(IF α OSs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IFPC(X) and IF α C(X) (respectively IFSO(X), IFPO(X) and IF α O(X)).

Definition 2.6: [8] An IFS A is an intuitionistic fuzzy regular open set if $A = \text{int}(\text{cl}(A))$.

Definition 2.7: [9] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy semi-pre open set (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set B such that $B \subseteq A \subseteq \text{cl}(B)$.

Note that an IFS A is an IFSPOS if and only if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ [5].

Definition 2.8: [8] An IFS A is an intuitionistic fuzzy generalized closed set (IFGCS for short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.

Definition 2.9: [5] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) . The complement A^c of an IFGSPCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized open set (IFGSPOS for short) in X .

Definition 2.10: Let A be an IFS in an IFTS (X, τ) . Then the generalized semi-pre interior and the generalized semi-pre closure of A are defined by

$$\text{gspint}(A) = \cup \{G / G \text{ is an IFGSPOS in } X \text{ and } G \subseteq A\}$$

$$\text{gspcl}(A) = \cap \{K / K \text{ is an IFGSPCS in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ) , we have $\text{gspcl}(A^c) = [\text{gspint}(A)]^c$ and $\text{gspint}(A^c) = [\text{gspcl}(A)]^c$.

Remark 2.11: If an IFS A in an IFTS (X, τ) is an IFGSPCS in X , then $\text{gspcl}(A) = A$. But the converse may not be true in general, since intersection does not exist in IFGSPCSs [5]

Remark 2.12: If an IFS A in an IFTS (X, τ) is an IFGSPOS in X , then $\text{gspint}(A) = A$. But the converse may not be true in general, since union does not exist in IFGSPOSs [5]

Definition 2.13: [4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy continuous mapping (IFCts.M for short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$

- Definition 2.14:** [4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an
- (i) *intuitionistic fuzzy semi continuous mapping* (IFSCts. for short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$
 - (ii) *intuitionistic fuzzy α - continuous mapping* (IF α Cts. for short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$
 - (iii) *intuitionistic fuzzy pre continuous mapping* (IFPCts. for short) if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$

Definition 2.15: [8] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized continuous mapping (IFGcts. for short) if $f^{-1}(B) \in \text{IFGC}(X)$ for every IFCS B in Y .

Definition 2.16: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy semi-pre continuous mapping (IFSPCts. for short) if $f^{-1}(B) \in \text{IFSPC}(X)$ for every $B \in \sigma$

Definition 2.17:[6] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi-pre continuous mapping (IFGSPCts. for short) if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ) .

Definition 2.18:[7] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy completely generalized semi-pre continuous mapping (IFcomGSPCts. for short) if $f^{-1}(V)$ is an IFRCS in (X, τ) for every IFGSPCS V of (Y, σ) .

3. INTUITIONISTIC FUZZY WEEKLY GENERALIZED SEMI-PRE CONTINUOUS MAPPINGS

In this section we introduce intuitionistic fuzzy weekly generalized semi-pre continuous mappings and study some of its properties

Definition 3.1: A mapping $f: X \rightarrow Y$ is said to be an intuitionistic fuzzy weekly generalized semi-pre continuous mapping (IFWGSPCts.M for short) if $f^{-1}(V) \subseteq \text{gspint}(f^{-1}(\text{cl}(V)))$ for each IFOS V in Y .

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.2_a, 0_b) \rangle$, $G_2 = \langle x, (0.4_a, 0.1_b), (0.2_a, 0.1_b) \rangle$, $G_3 = \langle y, (0.2_u, 0.2_v), (0.4_u, 0.4_v) \rangle$ and $G_4 = \langle y, (0.2_u, 0_v), (0.5_u, 0.4_v) \rangle$. Then $\tau = \{0_., G_1, G_2, 1_.\}$ and $\sigma = \{0_., G_3, G_4, 1_.\}$ are IFTS on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFWGSPCts.M.

Theorem 3.3: Every IFGSPCts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFGSPCts.M. Let V be any IFOS in Y . Then by hypothesis, $f^{-1}(V)$ is an IFGSPCS in X . Therefore $\text{gspint}(f^{-1}(V)) = f^{-1}(V)$. Now $f^{-1}(V) = \text{gspint}(f^{-1}(V)) \subseteq \text{gspint}(f^{-1}(\text{cl}(V)))$. Hence f is an IFWGSPCts.M.

Example 3.4: In Example 4.2, f is an IFWGSPCts.M but not an IFGSPCts.M, since $G_4^c = \langle y, (0.5_u, 0.4_v), (0.2_u, 0_v) \rangle$ is an IFCS in Y but $f^{-1}(G_4^c) = \langle x, (0.5_a, 0.4_b), (0.2_a, 0_b) \rangle$ is not an IFGSPCS, since $f^{-1}(G_4^c) \subseteq G_1$ but $\text{spcl}(f^{-1}(G_4^c)) = 1_. \subseteq G_1$.

Theorem 3.5: Every IFcomGSPCts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFcomGSPCts.M. Let V be any IFOS in Y . Since every IFOS is an IFGSPCS, V is an IFGSPCS. Then by hypothesis, $f^{-1}(V)$ is an IFRCS in X .

Therefore $\text{gspint}(f^{-1}(V)) = f^{-1}(V)$. Now $f^{-1}(V) = \text{gspint}(f^{-1}(V)) \subseteq \text{gspint}(f^{-1}(\text{cl}(V)))$. Hence f is an IFWGSPCts.M.

Example 3.6: In Example 4.2, f is an IFWGSPCts.M but not an IFcomGSPCts.M, since $G_4 = \langle y, (0.2_u, 0_v), (0.5_u, 0.4_v) \rangle$ is an IFGSPCS in Y but not an IFRCS in X , since $\text{cl}(\text{int}(G_4)) = 0_. \neq G_4$.

Theorem 3.7: Every IFCTs.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFCTs.M. Since every IFCTs.M is an IFGSPCts.M, by Theorem 4.3, f is an IFWGSPCts.M.

Example 3.8: In Example 4.2, f is an IFWGSPCts.M but not an IFCTs.M, since $G_4^c = \langle y, (0.5_u, 0.4_v), (0.2_u, 0_v) \rangle$ is an IFCS in Y but $f^{-1}(G_4^c) = \langle x, (0.5_a, 0.4_b), (0.2_a, 0_b) \rangle$ is not an IFCS in X , since $\text{cl}(f^{-1}(G_4^c)) = 1_. \neq f^{-1}(G_4^c)$.

Theorem 3.9: Every IFSCts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFSCts.M. Since every IFSCts.M is an IFGSPCts.M, by Theorem 4.3, f is an IFWGSPCts.M.

Example 3.10: In Example 4.2, f is an IFWGSPCts.M but not an IFSCts.M, since $G_4^c = \langle y, (0.5_u, 0.4_v), (0.2_u, 0_v) \rangle$ is an IFCS in Y but $f^{-1}(G_4^c) = \langle x, (0.5_a, 0.4_b), (0.2_a, 0_b) \rangle$ is not an IFSCS in X , since $\text{int}(\text{cl}(f^{-1}(G_4^c))) = 1_. \subseteq f^{-1}(G_4^c)$.

Theorem 3.11: Every IFPCts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFPCts.M. Since every IFPCts.M is an IFGSPCts.M, by Theorem 4.3, f is an IFWGSPCts.M.

Example 3.12: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.4_a, 0.1_b), (0.6_a, 0.7_b) \rangle$, $G_3 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.2_v) \rangle$. Then $\tau = \{0_., G_1, G_2, 1_.\}$ and $\sigma = \{0_., G_3, 1_.\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFWGSPCts.M but not an IFPCts.M. Since G_3 is an IFOS in Y but $f^{-1}(G_3) = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.2_b) \rangle \subseteq \text{int}(\text{cl}(f^{-1}(G_3))) = G_1$ is not an IFPOS.

Theorem 3.13: Every IFSPCts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFSPCts. M. Since every IFSPCts.M is an IFGSPCts. M, by Theorem 4.3, f is an IFWGSPCts.M.

Example 3.14: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.6_a, 0.7_b), (0.2_a, 0.3_b) \rangle$, $G_2 = \langle x, (0.2_a, 0.1_b), (0.7_a, 0.8_b) \rangle$, $G_3 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, and let $G_4 = \langle y, (0.5_u, 0.3_v), (0.5_u, 0.7_v) \rangle$. Then $\tau = \{0_., G_1, G_2, G_3, 1_.\}$ and $\sigma = \{0_., G_4, 1_.\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFWGSPCts.M but not an IFSPCts.M, since G_4 is an IFOS in Y but $f^{-1}(G_4) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) \rangle \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(G_4)))) = G_1^c$ is not an IFSPS.

Theorem 3.15: Every IF α Cts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IF α Cts.M. Since every IF α Cts.M is an IFGSPCts.M, by Theorem 4.3, f is an IFWGSPCts.M.

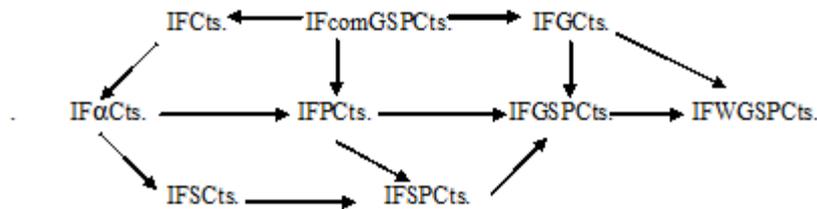
Example 3.16: In Example 4.2, f is an IFWGSPCts.M but not an IF α Cts.M, since $G_4^c = \langle y, (0.5_u, 0.4_v), (0.2_u, 0_v) \rangle$ is an IFCS in Y but $f^{-1}(G_4^c) = \langle x, (0.5_a, 0.4_b), (0.2_a, 0_b) \rangle$ is not an IF α CS in X , since $\text{cl}(\text{int}(\text{cl}(f^{-1}(G_4^c)))) = 1_. \subseteq f^{-1}(G_4^c)$.

Theorem 3.17: Every IFGCts.M is an IFWGSPCts.M but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFGCts.M. Since every IFGCts.M is an IFGSPCts.M, by Theorem 4.3, f is an IFWGSPCts.M.

Example 3.18: In Example 4.2, f is an IFWGSPCts.M but not an IFGCts.M, since $G_4^c = \langle y, (0.5_u, 0.4_v), (0.2_u, 0_v) \rangle$ is an IFCS in Y but $f^{-1}(G_4^c) = \langle x, (0.5_a, 0.4_b), (0.2_a, 0_b) \rangle$ is not an IFGCS, since $f^{-1}(G_4^c) \subseteq G_1$ but $\text{cl}(f^{-1}(G_4^c)) = 1_. \subseteq G_1$.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram.



The reverse implications are not true in general in the above diagram

Theorem 3.19: For a mapping $f: X \rightarrow Y$, the following are equivalent

- (i) f is an IFWGSPCts.M.
- (ii) $f^{-1}(\text{int}(A)) \subseteq \text{gspint}(f^{-1}(A))$ for each IFCS A in Y
- (iii) $f^{-1}(\text{int}(\text{cl}(A))) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$ for each IFOS A in Y
- (iv) $f^{-1}(A) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$ for each IFPOS A in Y
- (v) $f^{-1}(A) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$ for each IF α OS A in Y

Proof: (i) \Rightarrow (ii) Let $A \subseteq Y$ be an IFCS. Then $\text{int}(A) = \text{int}(\text{cl}(A))$, which is an IFROS. Hence $\text{int}(A)$ is an IFOS in Y . Therefore by (i) $f^{-1}(\text{int}(A)) \subseteq \text{gspint}(f^{-1}(\text{cl}(A))) = \text{gspint}(f^{-1}(A))$, since $\text{cl}(A) = A$. Hence $f^{-1}(\text{int}(A)) \subseteq \text{gspint}(f^{-1}(A))$.

(ii) \Rightarrow (iii) Let $A \subseteq Y$ be an IFOS. Then $\text{int}(A) = A$. Now $\text{cl}(A) = \text{cl}(\text{int}(A))$, which is an IFRCS. Therefore $\text{cl}(A)$ is an IFCS in Y . By (ii) $f^{-1}(\text{int}(\text{cl}(A))) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$.

(iii) \Rightarrow (iv) Let A be an IFPOS in Y . Then $A \subseteq \text{int}(\text{cl}(A))$. Now $f^{-1}(A) \subseteq f^{-1}(\text{int}(\text{cl}(A)))$. Since $\text{int}(\text{cl}(A))$ is an IFROS, it is an IFOS. Hence (iii) implies $f^{-1}(\text{int}(\text{cl}(\text{int}(\text{cl}(A)))) \subseteq \text{gspint}(f^{-1}(\text{cl}(\text{int}(\text{cl}(A)))) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$. That is $f^{-1}(\text{int}(\text{cl}(A))) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$.

Thus $f^{-1}(A) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$.

(iv) \Rightarrow (v) is obvious, since every IF α OS is an IFPOS.

(v) \Rightarrow (i) Let $A \subseteq Y$ be an IFOS. Since every IFOS is an IF α OS, A is an IF α OS. Hence by (v), $f^{-1}(A) \subseteq \text{gspint}(f^{-1}(\text{cl}(A)))$. Therefore f is an IFWGSPCts.M.

Theorem 3.20: Let $f: X \rightarrow Y$ be a bijective mapping. Then the following are equivalent.

- (i) f is an IFWGSPCts.M.
- (ii) $\text{gspcl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$ for each IFOS A in Y
- (iii) $\text{gspcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(A)$ for each IFCS A in Y

Proof: (i) \Rightarrow (iii) Let $A \subseteq Y$ be an IFCS. Then A^c is an IFOS in Y . By (i) $f^{-1}(A^c) \subseteq \text{gspint}(f^{-1}(\text{cl}(A^c)))$. This implies $\text{gspcl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$.

(iii) \Rightarrow (i) is obvious.

(ii) \Rightarrow (iii) Let $A \subseteq Y$ be an IFCS. Then $\text{int}(A)$ is an IFOS in Y . By (ii), $\text{gspcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(\text{cl}(\text{int}(A))) \subseteq f^{-1}(\text{cl}(A)) = f^{-1}(A)$, since $\text{cl}(A) = A$. Hence $\text{gspcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(A)$.

(iii) \Rightarrow (ii) Let $A \subseteq Y$ be an IFOS, then $\text{int}(A) = A$ and $\text{cl}(A)$ is an IFCS. By (iii), $\text{gspcl}(f^{-1}(\text{int}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$. Now $\text{gspcl}(f^{-1}(A)) = \text{gspcl}(f^{-1}(\text{int}(A))) \subseteq \text{gspcl}(f^{-1}(\text{int}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$. Hence $\text{gspcl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$.

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