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On $\alpha g\delta$-regular and $\alpha g\delta$- normal spaces

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ABSTRACT

We introduce a new types of separation axioms say $\alpha\delta T_i$- spaces, for i = 1/2, b and we study some of their properties. We define also the class of $\alpha g\delta$-regular and $\alpha g\delta$-normal spaces and show that $\alpha g\delta$-regularity and $\alpha g\delta$-normality are preserve under bijective continuous and pre-$\alpha g\delta$-open mappings. Several properties of these spaces are discussed.

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1. INTRODUCTION

In 1998, Devi and et. al [3] defined the notion of αT_b-spaces, while in 2005, Nasef and EL-Maghrabi [11] introduced the notion of δT_b-spaces. The aim of this paper is to introduce and study some separation axioms, say $\alpha\delta T_i$-spaces, for i = 1/2, b. Also, we construct the regularity and the normality of $\alpha g\delta$-closed sets.

2. PRELIMINARIES

Throughout the present paper, spaces mean topological spaces (X,τ)(or simply, X) on which no separation axioms are assumed unless explicitly stated. $f:(X,\tau)\to(Y,\sigma)$ (or simply, $f:X\to Y$) denotes a mapping from a space (X,τ) into a space (Y,σ). The closure (resp. the interior, the complement) of A for a space X are denoted by cl(A) (resp. int(A) , X-A). Some definitions and results which will be needed in this paper are recalled in the following stated.

Definition 2.1. A subset A of a space X is said to be:
(i) regular open [14] if A= int(cl(A)),
(ii) α-open [10] if A $\subseteq$ int(cl(int(A))),
(iii) δ-open [15] if it is the union of regular open sets.

The complement of a regular open (resp. δ-open, α-open) set is said to be regular closed (resp. δ-closed, α-closed). The intersection of all regular closed (resp. δ-closed, α-closed) sets containing A is called the regular closure [14] (resp. δ-closure [15], α-closure [2]) of A and is denoted by r-cl(A) (resp. $cl_\delta(A)$, α-cl(A)). The family of all regular open (resp. δ-open, α-open) sets in a space (X,τ) is denoted by $RO(X,\tau)$ (resp. τ^δ,τ^α). It is known that $\tau^\delta\subseteq\tau\subseteq\tau^\alpha$ and τ^α, τ^δ forms a topology on X [10, 15].

Definition 2.2. A subset A of a space (X,τ) is called:
(i) a generalized closed (briefly, g-closed) [1,5] set if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open,
(ii) a δ- dezilareneg-closed (briefly, δg-closed) [4] set if $cl_\delta(A)\subseteq U$ whenever $A\subseteq U$ and U is open,
(iii) an δ- generalized -closed (briefly, αg-closed) [3] set if α-$cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open,
(iv) a δ- generalized -closed (briefly, $g\alpha$-closed) [7] set if α-$cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open.

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Definition 2.3. A subset A of a space (X,τ) is called generalized open (resp. δ- dezilareneg open, α- generalized–open dezilareneg α- open) set if its complement X-A is g-closed (resp. δg-closed, αg-closed, $g\alpha$-closed) and denoted by g-open (resp. δg -open, αg-open, $g\alpha$-open)

Definition 2.4. A mapping f: $(X,\tau)\to(Y,\sigma)$ is called:

(i) δ-continuous [13] if $f^{-1}(V)$ is an δ-open set in (X,τ), for each open set V in (Y,σ),

(ii) α-continuous [8] if $f^{-1}(V)$ is an α-open set in (X,τ), for each open set V in (Y,σ),

(iii) α- irresolute [6] if $f^{-1}(V)$ is an α-open set in (X,τ), for each α-open set V in (Y,σ),

(iv) δ-open [9] if $f(V)$ is an δ-open set in (Y,σ), for each open set V in (X,τ),

(v) pre-α-closed [3] if $f(V)$ is an α-closed set in (Y,σ), for each α-closed set V in (X,τ),

Definition 2.5. A topological space (X,τ) is said to be:

(i) a $T_{1/2}$-space [5] if every g-closed set is closed,

(ii) $1/2\,T_\alpha$-space [3] if every αg-closed set is α-closed,

(iii) $\alpha T_{1/2}$-space [7] if every gα-closed set is α-closed,

(iv) αT_b-space [3] if every αg-closed set is closed,

(v) δT_b-space [11] if every δg-closed set is closed,

(vi) α-regular space [3] if for each closed set F of X and each point $x\in$ X-F, there exist disjoint α-open sets U and V such that $F\subseteq U$ and $x\in V$,

(vii) α-normal [12] if for any pair of disjoint closed sets F_1 and F_2, there exist disjoint α-open sets U and V such that $F_1\subseteq U$ and $F_2\subseteq V$.

3. MAIN RESULTS

Definition 3.1. A subset A of a space (X,τ) is said to be:

(i) an α-generalized δ- closed (briefly, $\alpha g\delta$-closed) set if α-cl(A)$\subseteq$U whenever A$\subseteq$U and U is δ-open,

(ii) an α-generalized δ-open (briefly, $\alpha g\delta$-open) set if its complement X-A is $\alpha g\delta$-closed.

$$\begin{tikzcd}
\delta\text{-closed} \arrow[r] & \text{closed} \arrow[r] \arrow[d] & \delta g\text{-closed} \arrow[r] \arrow[d, phantom, "\ddagger\ddagger"] & g\text{-closed} \arrow[d] & \\
& \alpha\text{-closed} \arrow[r] & g\alpha\text{-closed} \arrow[r] & \alpha g\text{-closed} \arrow[r] & \alpha g\delta\text{-closed}
\end{tikzcd}$$

Definition 3.2. A mapping f: $(X,\tau)\to(Y,\sigma)$ is called:

(i) $\alpha g\delta$-continuous if $f^{-1}(V)$ is $\alpha g\delta$-open in (X,τ), for each open set V in (Y,σ),

(ii) $\alpha g\delta$-irresolute if $f^{-1}(V)$ is $\alpha g\delta$-open, for each $\alpha g\delta$-open set V in (Y,σ),

(iii) $\alpha g\delta$-closed if f(V) is $\alpha g\delta$-closed in (Y,σ), for each closed set V in (X,τ),

(iv) pre-$\alpha g\delta$-closed if f(V) is $\alpha g\delta$-closed in (Y,σ), for each $\alpha g\delta$-closed set V in (X,τ),

(v) pre-$\alpha g\delta$-open if f(V) is $\alpha g\delta$-open in (Y,σ), for each $\alpha g\delta$-open set V in (X,τ),

(vi) α-generalized δ-C-homeomorphism if f is bijective, $\alpha g\delta$-irresolute and pre-$\alpha g\delta$-open.

Theorem 3.1. If A is an $\alpha g\delta$-closed set, then α-cl(A)-A contains no non empty δ-closed set.

Proof. Let F be a δ-closed subset of α-cl(A)-A. Then F$\subseteq\alpha$-cl(A) (1)

Let A$\subseteq$X-F, where X-F is δ-open. Since, A is $\alpha g\delta$-closed, then α-cl(A) $\subseteq$X-F (2)

Hence, from (1), (2) we have F$\subseteq\alpha$-cl(A) $\cap$(X- α-cl(A))= φ and so, F is empty.

Theorem 3.2. If A is $\alpha g\delta$ -closed in X and if f: X $\to$ Y is pre-α-closed and δ-continuous mapping, then f (A) is $\alpha g\delta$ -closed in Y.

Proof. Let A be an $\alpha g\delta$ -closed set in X and G be a δ-open set of Y such that $f(A)\subseteq G$. Then $A\subseteq f^{-1}(G)$. Hence, $\alpha\text{-}cl(A)\subseteq f^{-1}(G)$. Then $f(\alpha\text{-}cl(A))\subseteq G$ and therefore $f(\alpha\text{-}cl(A))$ is $\alpha g\delta$ -closed set in Y which implies that $\alpha\text{-}cl(f(A))\subseteq\alpha\text{-}cl(f(\alpha\text{-}cl(A)))\subseteq G$. Hence, $f(A)$ is $\alpha g\delta$ -closed set in Y.

Theorem 3.3. The inverse image of each $\alpha g\delta$ -closed set is $\alpha g\delta$ -closed under bijective $\alpha g\delta$ -irresolute and δ-open mappings.

Proof. Let B be an $\alpha g\delta$ -closed set in Y and $f^{-1}(B)\subseteq U$, where U is a δ-open set in X. Then f (U) is δ-open and hence $B\subseteq f(U)$. Since B is $\alpha g\delta$ -closed, hence α-cl(B)$\subseteq$f(U). Then $f^{-1}(\alpha\text{-cl}(B))\subseteq U$. But, $\alpha\text{-}cl(f^{-1}(B))\subseteq\alpha\text{-}cl(f^{-1}(\alpha\text{-}cl(B)))=f^{-1}(\alpha\text{-}cl(B))\subseteq U$. Therefore, f^{-1}(B) is $\alpha g\delta$ -closed.

4. $\alpha\delta T_{1/2}$ -spaces

Definition 4.1. A space (X,τ) is called $\alpha\delta T_{1/2}$ -space if every $\alpha g\delta$ -closed is α-closed.

Remark 4.1. By Definition 4.1, we have the following diagram.

$$\boxed{\alpha\delta T_{1/2}\text{-space}}\to 1/2T_{\alpha}\text{-space}\to\alpha T_{1/2}\text{-space}.$$

However, the converses of the above implications are not true in general as is shown by [3] and the following example.

Example 4.1. If X= {a, b, c} with the topology $\tau=\{X,\varphi,\{a\},\{b\},\{a,b\},\{b,c\}\}$, then (X,τ) is $1/2T_{\alpha}$ but it is not $\alpha\delta T_{1/2}$.

Theorem 4.1. For a space (X,τ), the following are equivalent:

(i) (X,τ) is $\alpha\delta T_{1/2}$,

(ii) for each $x\in$ X, then {x} is δ-closed or α-open.

Proof. (i) ⇒ (ii). Suppose that {x} is not δ-closed, for some $x\in$ X. Since, X is the only δ-open set containing X-{x}. Then X-{x} is $\alpha g\delta$ -closed. But (X,τ) is $\alpha\delta T_{1/2}$, then X-{x} is α-closed. Hence, {x} is α-open.

(ii) ⇒ (i). Let A be $\alpha g\delta$ -closed with $x\in\alpha-cl(A)$. We consider the following two cases:

Case: 1. Let{x} be α-open. Since, $x\in\alpha-cl(A)$, then $\{x\}\cap A\neq\varphi$. This shows that $x\in A$.

Case: 2. Let{x} be δ-closed. If suppose that $x\notin A$. Then we would have $x\in\alpha-cl(A)-A$ which cannot happen according to Theorem 3.1. Hence, $x\in A$. So, in both cases we have $\alpha-cl(A)\subseteq A$. Hence, $\alpha-cl(A)=A$. Then A is α -closed and so, (X,τ) is $\alpha\delta T_{1/2}$.

Theorem 4.2. A space (X,τ) is $\alpha\delta T_{1/2}$ if and only if every subset of X is the intersection of all α-open sets and all δ-closed sets containing it.

Proof. Firstly. Let X be $\alpha\delta T_{1/2}$ with $B\subseteq X$ arbitrary. Then $B=\{X-\{x\}:x\notin B\}$ is the intersection of α-open and δ-closed by Theorem 4.1.

Secondly. For each $x\in X$, then X-{x} is the intersection of all α-open sets and δ-closed sets containing it. Thus X-{x} is either α-open or δ-closed and X is $\alpha\delta T_{1/2}$.

Proposition 4.1. For a mapping $f:(X,\tau)\to(Y,\sigma)$, then the following statement are equivalent
(i) If X is an $\alpha\delta T_{1/2}$ -space, then the concept of α -continuous and $\alpha g\delta$ -continuous are coincident,
(ii) If X, Y are $\alpha\delta T_{1/2}$ -spaces, then the concept of α -irresolute and $\alpha g\delta$ -irresolute are coincident.

Proof. (i) Let B any closed set in Y. Since, f is $\alpha g\delta$ -continuous, then $f^{-1}(B)$ is $\alpha g\delta$ -closed set in X. But X is $\alpha\delta T_{1/2}$ - space, then $f^{-1}(B)$ is α -closed which implies that f is α -continuous.
(ii) Similar to (i).

Theorem 4.3. A space (X,τ) is $\alpha\delta T_{1/2}$ if and only if $\alpha O(X,\tau)=\alpha G\delta O(X,\tau)$.

Proof. Firstly. Let (X,τ) be $\alpha\delta T_{1/2}$ and $A\in\alpha G\delta O(X,\tau)$. Then X-A is $\alpha g\delta$ -closed. By hypothesis X-A is α -closed and thus $A\in\alpha O(X,\tau)$. Then $\alpha G\delta O(X,\tau)\subseteq\alpha O(X,\tau)$. Hence, $\alpha O(X,\tau)=\alpha G\delta O(X,\tau)$.

Secondly. Let $\alpha O(X,\tau)=\alpha G\delta O(X,\tau)$ and let A be $\alpha g\delta$ -closed. Then X-A is $\alpha g\delta$ -open. Hence, X-A $\in\alpha O(X,\tau)$. Thus A is α -closed which implies that (X,τ) is $\alpha\delta T_{1/2}$ - space.

Theorem 4.4. If a space (Y,σ) is $\alpha\delta T_{1/2}$ and $f:(X,\tau)\to(Y,\sigma)$ is bijective pre α -closed, α - irresolute and δ - continuous mappings, then (X,τ) is $\alpha\delta T_{1/2}$.

Proof. Let A be an $\alpha g\delta$ -closed set of (X,τ). Hence by Theorem 3.2. We have f(A) is $\alpha g\delta$ -closed. And by the assumption f(A) is α -closed and hence, A is α -closed in X. Therefore, (X,τ) is $\alpha\delta T_{1/2}$.

Theorem 4.5. If (X,τ) is an $\alpha\delta T_{1/2}$ -space and f: $(X,\tau)\to(Y,\sigma)$ is bijective $\alpha g\delta$ - irresolute, pre- α -closed and δ -open mappings, then (Y,σ) is $\alpha\delta T_{1/2}$ -space.

Proof. Let A be an $\alpha g\delta$ -closed set of (Y,σ). Then by Theorem 3.3, f^{-1}(A) is $\alpha g\delta$ - closed in (X,τ). Since, (X,τ) is $\alpha\delta T_{1/2}$ -space, hence f^{-1}(A) is α -closed and therefore A is α -closed. Hence, (Y,σ) is $\alpha\delta T_{1/2}$ - space.

5. $\alpha\delta T_b$ -spaces

Definition 5.1. A space (X,τ) is called $\alpha\delta T_b$ -space if every $\alpha g\delta$ -closed set is closed.

Lemma 5.1. For a space (X,τ), every $\alpha\delta T_b$ -space is αT_b -space.

The converses of above lemma need not be true as is shown by the following example.

Example 5.1. Let X= {a, b, c} with the topologies τ ={X, φ ,{a}, {b}, {a, b} ,{b, c}}. Then a space X is αT_b , but not $\alpha\delta T_b$, since {b} is $\alpha g\delta$ -closed but not closed

Theorem 5.1. If $f:(X,\tau)\to(Y,\sigma)$ is surjective closed and $\alpha g\delta$ -irresolute mappings, then (Y,σ) is $\alpha\delta T_b$ -space, if (X,τ) is $\alpha\delta T_b$ -space.
Proof. Let B be an $\alpha g\delta$ -closed subset of (Y,σ). Then $f^{-1}(B)$ is $\alpha g\delta$ -closed in (X,τ). Since, (X,τ) is $\alpha\delta T_b$ -space, then $f^{-1}(B)$ is closed in (X,τ). Hence, B is closed in (Y,σ) and so, (Y,σ) is $\alpha\delta T_b$ -space.

Corollary 5.1. A space (X,τ) is $\alpha\delta T_b$ if and only if $\tau=\alpha G\delta O(X,\tau)$.

Proposition 5.1. Let $f:(X,\tau)\to(Y,\sigma)$ be a $\alpha g\delta$-closed mapping and (Y,σ) be $\alpha\delta T_b$ -space, Then $f:(X,\tau)\to(Y,\sigma)$ is closed.

Theorem 5.2. Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping and (X,τ) be an $\alpha\delta T_b$-space. Then f is continuous if one of the following conditions are hold:
(i) f is $\alpha g\delta$ -continuous,
(ii) f is $\alpha g\delta$ -irresolute.

Proof. (i) Let F be a closed set in (Y,σ). Then $f^{-1}(F)$ is $\alpha g\delta$ -closed in (X,τ). But (X,τ) is an $\alpha\delta T_b$-space, then $f^{-1}(F)$ is closed. Hence, f is continuous.
(ii) Obvious.

Theorem 5.3. Let $f:(X,\tau_X)\to(Y,\tau_Y)$ and $h:(Y,\tau_Y)\to(Z,\tau_Z)$ be two mappings and (Y,τ_Y) be an $\alpha\delta T_b$-space. Then:
(i) $h\circ f$ is $\alpha g\delta$ -continuous if f and h are $\alpha g\delta$ -continuous,
(ii) $h\circ f$ is $\alpha g\delta$ -closed if f and h are $\alpha g\delta$ -closed.

Proof. (i) Let V be a closed set of (Z,τ_Z). Then $h^{-1}(V)$ is $\alpha g\delta$ -closed in (Y,τ_Y). But, (Y,τ_Y) is $\alpha\delta T_b$-space, then $h^{-1}(V)$ is closed in (Y,τ_Y). Since f is $\alpha g\delta$ - continuous, then $(h\circ f)^{-1}(V)$ is $\alpha g\delta$ -closed in (X,τ_X). $h\circ f$ is $\alpha g\delta$ - continuous.
(ii) Obvious.

Corollary 5.2. For a mapping $f:(X,\tau)\to(Y,\sigma)$ we have:
(i) f is pre-$\alpha g\delta$ -closed, if f is $\alpha g\delta$ -closed and (X,τ) is $\alpha\delta T_b$-space,
(ii) f is $\alpha g\delta$ -irresolute, if f is $\alpha g\delta$ -continuous and (Y,σ) is $\alpha\delta T_b$-space.

Theorem 5.4. A space (X,τ) is $\alpha\delta T_b$ if and only if , for each $x\in X$,{x} is δ -closed or open .

Proof. Necessity. Suppose that for some $x\in X$, {x} is not δ -closed. Since X is the only δ -open containing X-{x}. Then X-{x} is $\alpha g\delta$ -closed. Hence, {x} is open.

Sufficiency. Let A be $\alpha g\delta$ -closed with $x\in\delta-cl(A)$. If {x} is open, $\{x\}\cap A\neq\varphi$. Otherwise{x} is δ -closed and $\varphi\neq\delta-cl(\{x\})\cap A=\{x\}\cap A$. In either case $x\in A$. Then $\delta\text{-}cl(A)\subseteq A$. Hence, $\delta\text{-}cl(A)=A$. Then A is δ -closed and so, X is $\alpha\delta T_b$.

Theorem 5.5. A space $(X,\tau)$is $\alpha\delta T_b$ if and only if, every subset of X is the intersection of all open sets and all δ -closed sets containing it.

6. $\alpha g\delta$ - regular spaces

Definition 6.1. A space (X,τ) is said to be $\alpha g\delta$ -regular if for each closed set F of X and each point $X\in X$ -F , there exist disjoint $\alpha g\delta$ -open sets U and V such that $F\subseteq U$ and $X\in V$.

Lemma 6.1. For a space (X,τ) every α -regular space is $\alpha g\delta$ -regular space.

The converse of the above lemma is not true as is shown by the following example.

Example 6.1. In Example 4.1, a space X is $\alpha g\delta$ -regular but it is not α -regular.

Remark 6.1. An $\alpha g\delta$ -regular and $\alpha\delta T_{1/2}$ -spaces is α -regular space.

Theorem 6.1. Let X be a space. Then the following are equivalent:
(i) X is $\alpha g\delta$ -regular,
(ii) For each $F\subseteq X$ and $p\in X$-F, there exists an $\alpha g\delta$ -open set U such that $p\in U\subseteq\alpha g\delta$ -cl(U)$\subseteq$X-F.

Proof. (i)→ (ii) Let X be an $\alpha g\delta$ -regular space, $F \subseteq X$ and p $\in$ X-F. Then there exist disjoint $\alpha g\delta$ -open sets U and V such that $p \in U$ and $F \subseteq V = X - \alpha g\delta$ -cl(U). This implies that $\alpha g\delta$ -cl(U) $\subseteq$ X-F and hence, $p \in U \subseteq \alpha g\delta$ -cl(U) $\subseteq$ X-F.

(ii)→(i). Let p $\in$ X and $F \subseteq$ X -{p} be a closed set. Then there exists an $\alpha g\delta$ -open set such that $p \in U \subseteq \alpha g\delta$ -cl(U) $\subseteq$ X-F. Then $F \subseteq X - \alpha g\delta$ -cl(U) which is an $\alpha g\delta$ -open set and $U \cap (X - \alpha g\delta\text{-cl}(U)) = \varphi$. Hence, (X, τ) is an $\alpha g\delta$ -regular space.

Theorem 6.2. For an $\alpha g\delta$ -regular space, for any two points x, y of X, then either $\alpha g\delta$ -cl({x}) = $\alpha g\delta$ -cl({y}) or $\alpha g\delta$ -cl({x}) $\cap$ $\alpha g\delta$ -cl({y}) = φ.

Proof. Suppose that $\alpha g\delta$ -cl({x}) $\neq$ $\alpha g\delta$ -cl({y}), then either x $\notin$ $\alpha g\delta$ -cl({y}) or y $\notin$ $\alpha g\delta$ -cl({x}). Suppose that y $\notin$ $\alpha g\delta$ -cl({x}). Since, X is $\alpha g\delta$ -regular, then there exist disjoint an $\alpha g\delta$ -open sets G and H such that $\alpha g\delta$ -cl({x}) $\subseteq$ G and $y \in H \subseteq$ X-G , where X-G is $\alpha g\delta$ -closed this implies that $\alpha g\delta$ -cl({y}) $\subseteq$ X-G. Therefore, $\alpha g\delta$ -cl({x}) $\cap$ $\alpha g\delta$ -cl({y}) $\subseteq G \cap (X - G) = \varphi$.

Theorem 6.3. Let f be $\alpha g\delta$ -irresolute, closed mappings and Y be an $\alpha g\delta$ -regular. Then X is $\alpha g\delta$ -regular space.

Proof. Let V be any closed set of X and $x \in$ X-V. Then f(V) is closed in Y and f(x) $\in$ Y-f(V). Then there exist disjoint $\alpha g\delta$ -open sets G and H such that $f(x) \in G$ and $f(V) \subseteq H$. Since, f is $\alpha g\delta$ -irresolute, then $f^{-1}(G)$ and $f^{-1}(H)$ is $\alpha g\delta$ -open in X. Then $x \in f^{-1}(G)$, $V \subseteq f^{-1}(H)$ and $f^{-1}(G) \cap f^{-1}(H) = \varphi$. Hence, X is $\alpha g\delta$ -regular.

The following theorems are shown that $\alpha g\delta$ -regular space is preserved under bijective continuous and pre- $\alpha g\delta$ -open.

Theorem 6.4. If f is a bijective continuous and pre- $\alpha g\delta$ -open map, then Y is $\alpha g\delta$ - regular, if X is $\alpha g\delta$ -regular space.

Proof. Let F be any closed set of Y and $y \in$ Y-F. Then $f^{-1}(F)$ is closed in X and $x \notin f^{-1}(F)$. Since, X is $\alpha g\delta$ -regular, then there exist disjoint $\alpha g\delta$ -open sets G and H such that $x \in G$ and $f^{-1}(F) \subseteq H$. Then $y \in$ f(G) and $F \subseteq$ f(H) and f(G) $\cap$ f(H)= φ. Hence, Y is $\alpha g\delta$ -regular.

Theorem 6.5. The property of being $\alpha g\delta$ -regular space is a topological property.

Proof. Let a mapping $f : X \to Y$ be $\alpha g\delta$ C-homeomorphism from an $\alpha g\delta$ -regular space X into a space Y. Then f is bijective $\alpha g\delta$ -irresolute and pre - $\alpha g\delta$ -open. and hence f is bijective continuous and pre- $\alpha g\delta$ -open. then for Theorem 6.4, $\alpha g\delta$ -regular is a topological property.

7. $\alpha g\delta$ - normal spaces

Definition 7.1. A space (X, τ) is said to be $\alpha g\delta$ -normal if for any pair of disjoint closed sets F_1 and F_2, there exist disjoint $\alpha g\delta$ -open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Lemma 7.1. For a space (X, τ), the following are hold

(i) Every α -normal space is $\alpha g\delta$ -normal,

(ii) Every $\alpha g\delta$ -regular space is $\alpha g\delta$ -normal.

Proof. (i) Let (X, τ) be an α -normal space. Then for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint α -open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$. Then by definition 3.1, there exist disjoint $\alpha g\delta$ -open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$ and therefore (X, τ) is $\alpha g\delta$ -normal space.

(ii) Let F_1, F_2 be two disjoint closed sets of X. Then for each $x \in F_1$ implies $x \notin F_2$. Since, X is $\alpha g\delta$ -regular, then there exist two disjoint $\alpha g\delta$ -open sets U and V_X such that $F_2 \subseteq U$ and $x \in V_X$. But, $x \in F_1$ and $x \in V_X$, then, $F_1 \subseteq V$ and $F_2 \subseteq U$ and $U \cap V = \varphi$. Therefore, (X, τ) is an $\alpha g\delta$ - normal space.

The converse of the above lemma is not true. as is shown by the following examples.

Example 7.1. Let X = {a, b, c} with topologies τ_1 = {X, φ, {a}, {a, b}, {a, c}} and τ_2 = {X, φ, {a}, {b},{a, b}}. Then

(i) a space (X, τ_1) is $\alpha g\delta$ -normal but not α -normal. Since, {b}, {c} are disjoint closed sets while there are not exist disjoint α -open sets U and V such that $\{b\} \subseteq U$, $\{c\} \subseteq V$.

(ii) a space (X, τ_2) is $\alpha g\delta$ -normal but not $\alpha g\delta$ -regular. Since, {c} is a closed set and $a \in X$-{c} there are not exist disjoint $\alpha g\delta$ -open sets U and V such that $\{c\} \subseteq U$, $a \in V$.

Proposition 7.1. An $\alpha g\delta$ -normal and $\alpha\delta T_{1/2}$ spaces is α -normal space.

Theorem 7.1. For a space (X, τ), then the following are equivalent:

(i) X is $\alpha g\delta$ -normal,

(ii) For any pair of disjoint closed sets F_1, F_2 of X , there exists an $\alpha g\delta$ -open set H such that $F_1 \subseteq H$ and $\alpha g\delta$ -cl(H) disjoint of F_2.

(iii) For any closed set F of X and any open set U containing F, there exists an $\alpha g\delta$ - open set H such that $F \subseteq H \subseteq \alpha g\delta$ -cl(H) $\subseteq U$.

Proof. (i)→ (ii). Let F_1 and F_2 be any non-empty disjoint closed sets of an $\alpha g\delta$ -normal space X. Then there exist two $\alpha g\delta$ -open sets H, W of X such that $F_1 \subseteq H$, $F_2 \subseteq W$ and $W \cap H = \varphi$ .Then $\alpha g\delta$ -cl(X-W) $\subseteq$ X- F_2 and $\alpha g\delta$ -cl(H) $\subseteq$ X- F_2. Hence, $F_1 \subseteq H$ and $\alpha g\delta$ -cl(H) $\cap$ $F_2 = \varphi$.

(ii)→ (iii). Let F be any closed set and U be an open set containing F. Then by hypothesis, there exists an $\alpha g\delta$ -open set H such that $F \subseteq H$ and $\alpha g\delta$ -cl(H) $\cap$ (X-U) = φ this implies that $F \subseteq H \subseteq \alpha g\delta$ -cl(H) $\subseteq U$.

(iii)→(i). Let F_1 and F_2 be any disjoint closed sets of X. Then X- F_2 is an open set containing F_1. Hence by hypothesis, there exists an $\alpha g\delta$ -open set H such that $F_1 \subseteq H \subseteq \alpha g\delta$ -cl(H) $\subseteq$ X- F_2. If we put V = X- $\alpha g\delta$ -cl(H), then H and V are disjoint $\alpha g\delta$ -open sets such that $F_1 \subseteq H$ and $F_2 \subseteq V$. Hence, X is $\alpha g\delta$ -normal.

Theorem 7.2. If f: $(X, \tau) \to (Y, \sigma)$ is $\alpha g\delta$ -irresolute and closed mappings, then X is $\alpha g\delta$ -normal, if Y is $\alpha g\delta$ - normal.

Proof. Let F_1 and F_2 be any two disjoint closed sets of X. Then f(F_1), f(F_2) are disjoint closed sets of Y. By $\alpha g\delta$ -normality, there exists disjoint $\alpha g\delta$ -open sets G, H such that f(F_1) $\subseteq$ G and f(F_2) $\subseteq$ H. Then $F_1 \subseteq f^{-1}$(G), $F_2 \subseteq f^{-1}$(H) and f^{-1}(G) $\cap$ f^{-1}(H) = φ. Hence, X is $\alpha g\delta$ -normal.

Theorem 7.3. If f: $(X, \tau) \to (Y, \sigma)$ is a bijective continuous and pre- $\alpha g\delta$ -open mappings, then Y is $\alpha g\delta$ - normal, if X is $\alpha g\delta$ - normal space.

Proof. Let A and B be any two disjoint closed sets of Y. Then f^{-1}(A), f^{-1}(B) are disjoint closed sets of X. By $\alpha g\delta$ -normality, there exists disjoint $\alpha g\delta$ -open set G, H such that f^{-1}(A) $\subseteq$ G and f^{-1}(B) $\subseteq$ H. and by using a bijective pre- $\alpha g\delta$ -open mapping, we obtain A $\subseteq$ f(G), B $\subseteq$ f(H) and f(G) $\cap$ f(H) = φ . Hence, Y is $\alpha g\delta$ -normal.

Theorem 7.4. The property of being $\alpha g\delta$ -normal space is a topological property.

Proof. Let a mapping $f : X \rightarrow Y$ be a $\alpha g\delta$ C-homeomorphism from an $\alpha g\delta$ -normal space X into a space Y. Then f is bijective $\alpha g\delta$ -irresolute and pre- $\alpha g\delta$ -open mapping. Then f is bijective continuous and pre- $\alpha g\delta$ -open. Therefore by using Theorem 7.3. $\alpha g\delta$ -normal space is a topological property.

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