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GLOBAL EXISTENCE'S SOLUTION OF A SYSTEM OF REACTION-DIFFUSION

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ABSTRACTS

Review of classical and exposed results, new results concerning the global existence's solution of a weakly coupled system of reaction-diffusion by order n

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INTRODUCTION

Recently, a class of systems of partials differentials equations of the parabolic type, called system of reaction diffusion, it received a large amount number of interest by the researchers, who are motivated by both the enrichment structure of the solution as well as it governs several chemical, ecological, biological, metallurgical phenomena and even in marketing ….

These systems spell in their simplest shape as follows:

$$\frac{\partial u}{\partial t} - D\Delta u = F(u), \quad \text{in } \Omega \times]0, +\infty[\tag{1a}$$

where, Ω is opened of R^n, $u{:}\Omega\times]0,+\infty[\rightarrow R^m$, i.e., $u(x, t) = (u_1(x, t), u_2(x, t), \ldots, u_m(x, t))$, F: $R^m \rightarrow R^m$,

$F(u(x, t)) = (F_1(u(x, t)),..., F_m(u(x, t)))$ is the term of the reaction (generally nonlinear). The terms of reaction are the result of any interaction between the constituents of the unknown u.

The objective of this work is contributed to the study of the global existence in times of the solution of (1a) with:

Newmann boundary condition:

$$\frac{\partial u}{\partial \eta} = 0 \text{ on } \partial\Omega \times]0, +\infty[\tag{1b}$$

which means that there is no immigration.

Initial data:

$$u(., 0) = u_0 \text{ in } \Omega \tag{1c}$$

such as $u_0 = (u_{0_1}, \ldots, u_{0_m})$ and $\forall i = \overline{1, m}$, u_{0_i} are a non-negative functions of $L^1(\Omega)$.

History of the problem: Most studies which are made about the system of reaction diffusion are essentially based on some particular cases of (1a), where the mathematical model:

$$\frac{\partial u}{\partial t} - \propto \Delta u = f(u, v) \tag{2a}$$

$$\frac{\partial v}{\partial t} - \beta \Delta v = g(u, v) \tag{2b}$$

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$$\frac{\partial u}{\partial \eta}=\frac{\partial v}{\partial \eta}=0 \tag{2c}$$

$$u\,(.,0) = u_0,\ v(.,0) = v_0 \tag{2d}$$

where, α and β are two positive constants, is the most approached by the researchers.

When f and g are « enough regular » and u_0, v_0 are bounded, the local existence in times of the solution (u, v) is classical, furthermore, it is not negative if u_0 and v_0 too. If the condition of the balance is satisfied i.e., $f + g = 0$, so by application of maximum principal, if for example g is not negative, we have the estimation in priori:

$$\|u(t)\|_\infty \le \|u_0\| \ \ \forall t \in [0, T_{max}[$$

where, T_{max} the maximal time of the existence is:

$$\|u\|_\infty = inf\{C > 0 : |u| < C \text{ p.p}\}$$

If it was possible to establish estimation in priori for v, the global existence would result from it, but such estimation is not evident only in the coarse case $\propto = \beta$ where:

$$\|(u+v)(t)\|_\infty \le \|u_0 + v_0\|_\infty$$

because:

$$\frac{\partial u}{\partial t}(u+v) - \propto \Delta(u+v) = 0$$

M. Medved (1998) consideres this problem and proved a global existence result. He also proved that

$$\lim_{t\to+\infty}\|u(t)\|_\propto = 0.$$

When $f(u,v) = -g(u,v) = -uv^\sigma$, Alikakos (1979) established the global existence of the solution for $1 < \sigma < \frac{n+2}{n}$ following method of « Bootstrap » , based on the injections of sobolev. The extension of this result for $\sigma > 1$ is obtained by Masuda (1983).Then Haraux and Youkana (1988) Generalized the result of Masuda Via the functional of Lyapunov by putting $f(u,v) = -g(u,v) = -u\varphi(v)$, where φ is a nonlinear function satisfying the condition:

$$\lim_{v\to\infty}\frac{\log(1+\varphi(v))}{v} = 0$$

Barabanova (1994) generalize the result of Haraux and youkana concerning the global existence of nonnegative solutions of a reaction-diffusion equation with exponential nonlinearity

The existence of a global solution is bounded by the problem (2a-d) was introduced by Hollis and al. (1987) with a method based on the theory of L^p-regularity for the operator of the heat and a principle of duality, Kouachi (2001) looked for the global existence of the solution by using the functional of Lyapunov.

Equally, the existence of a global solution of the problem (2a-d) was established by Pierre (1987) under weak conditions, by using a technique based on L^1-estimation. Bonafede and Schmitt (1998) were able to generalize the method of Pierre by studying the existence of the nonexistence of a global solution of the problem (2a-d). And M. Pierre, D. Schmitt (1997) have built examples of reaction-diffusion systems with L^∞ initial data, satisfying only

$$f(x,t,0,v) \ge 0, \quad g(x,t,u,0) \ge 0 \quad \text{for all } u, v \ge 0 \text{ and a.e. } x, t.$$

and $$\propto f(x,t,u,v) + g(x,t,u,v) \le k(u+v+1)$$

and for which blow-up in finite time of solutions occurs.

Global existence of the solution: We are going to show the global existence of the problem (1a-c) under the following hypotheses

H_1: F is a quasi-positive function

H_2: It exists positive constant β_i for $i = \overline{1,m}$ such as:

$$\sum_{i=1}^{m}\beta_i F_i\,(\xi) \le C\left(1 + \sum_{i=1}^{m}\xi_i\right)$$

for everything $\xi \in R_+^m$, where C is a constant independent of ξ.

H_3: It exists two nonnegative constants C_1 and C_2 such as:

$$|F_i(\xi)| \leq C_1 \left[1 + \sum_{i=1}^{m} \xi_i\right]^{C_2} \text{ for all } \xi \in \mathbb{R}_+^m \text{ and } i = \overline{1,m}$$

Theory 1: Suppose that the hypotheses (H_i), $i = \overline{1,3}$ are satisfied, so it exists $u = (u_1,\dots, u_m)$ solution of:

$$\begin{cases} u_1 \in C\big([0,+\infty[, L^1(\Omega)\big) \\ F_1 \epsilon L^1(Q) \text{ where } Q = \Omega \times [0,T[\\ u_1(x,t) - S_1(t)u_{0_i} + \int_0^t S_i\,(t-s)F_i\big(u_1(s),\dots u_m(s)\big)ds, \\ \qquad\qquad\qquad \forall t \in [0,T[\\ 1 \leq i \leq m \end{cases} \tag{3a}$$

where, $S_i(t)$ are the semi-groups in $L^1(\Omega)$ generatated by $-\alpha_i\Delta, i = \overline{1,m}$.

To show this theory, we need following reminders:

Local solution: Let A m-dissipatif operator of dense domain in the Banach space X and $S(t)$ a semi-group engendered by A, f a function locally Lipchitz, so $\forall u_0 \in X$ it exists $T(u_0) = T_{max}$ such as the problem:

$$\begin{cases} u \in C\big([0,\mathrm{T}], \mathrm{D(A)}\big) \cap \mathrm{C}^1([0,\mathrm{T}], \mathrm{X}) \\ \frac{\partial u}{\partial \mathrm{t}} - \mathrm{A}\Delta\mathrm{u} = f(u) \\ u(0) = u_0 \end{cases} \tag{3b}$$

admits a unique solution u verifying:

$$u(t) = S(t)u_0 + \int_0^t S(t-s)f\big(u(s)\big)ds, \forall t \in [0, T_{max}]$$

Study of a particular system: For all $n > 0$, we define the functions $u_{0_i}^n, i = \overline{1,m}$, by $u_{0_i}^n = \min\big(u_{0_i}, n\big)$, it is clear that:

$$u_{0_i}^n \in L^1(\Omega) \text{and } u_{0_i}^n \geq 0 \ \forall i = \overline{1,m}$$

Let us consider the following system:

$$\begin{cases} \frac{\partial u_n}{\partial t} - D\Delta u_n = F(u_n) \quad \text{in } \Omega \times [0,T] \\ \frac{\partial u_n}{\partial \eta} = 0 \qquad\qquad \text{on } \partial\Omega \times [0,T] \\ u_n(0,.) = u_{0_n} \end{cases} \tag{S_n}$$

Local existence of the system (S_n):

Let us put:

$$u_n = \begin{pmatrix} u_{1_n} \\ \vdots \\ u_{m_n} \end{pmatrix}, F = \begin{pmatrix} F_1 \\ \vdots \\ F_m \end{pmatrix}, A = \begin{pmatrix} \alpha_1 \Delta u_{1_n} \\ \vdots \\ \alpha_m \Delta u_{m_n} \end{pmatrix} \text{ and } u_{0_n} = \begin{pmatrix} u_{0_1}^n \\ \vdots \\ u_{0_m}^n \end{pmatrix}$$

so the system (S_n) can be returned to the shape of the system (3b), thus, if $\big(u_{1_n},\dots,u_{m_n}\big)$ is a solution of (S_n) so it veries the integral equations:

$$u_{i_n}(x,t) = S_i(t)u_{0\,i}^n + \int_0^t S_i(t-s)F_i\left(u_{1_n}(s),\dots,u_{m_n}(s)\right)ds, i = \overline{1,m} \tag{3c}$$

Theory 2: It exists $T_M > 0$ and $\big(u_{1_n},\dots,u_{m_n}\big)$a local solution of ($S_n$) for all $t \in [0, T_M]$, furthermore $u_{1_n}, i = \overline{1,m}$ are positive.

Proof: We know that $S_i(t), i = \overline{1,m}$ are semigroups of contraction and as F is locally Lipschitz $0 \leq u_{0_i}^n \leq n$,

$i=\overline{1,m}$ so we have $\exists T_M>0$ and $(u_{1_n},\dots,u_{m_n})$ is a local solution of (S_n) on $[0,T_M]$. and according to the hypothesis (H_1) and the positivity of $u^n_{0_i}$, the solutions u_{i_n} are positive, for $i=\overline{1,m}$.

Global existence of the solution of the system (S_n): To prove the global existence of the solution of the system (S_n) for all t positive it is enough to find an estimation of the solution for everything t ≥ 0, according to Haraux and Kirane (1983).

The lemma according to us shows the existence of an estimation of the solution of (S_n) in $L^1(\Omega)$.

Lemma 1: Let u_n the solution of the system (S_n) where $u_n=(u_{1_n},\dots,u_{m_n})$ so it exists $M(T)$ which depends only of *t*, such as for all $0\le t\le T_M$, we have:

$$\left\|\sum_{i=1}^{m} u_{i_n}\right\|_{L^1(\Omega)} \le M(t)$$

Proof: We can write the system (S_n) under the following shape:

$$\begin{cases} \dfrac{\partial u_{1_n}}{\partial t} - \alpha_1\,\Delta u_{1_n} = F_1(u_n) \text{ in } \Omega\times]0,+\infty[\\ \quad\vdots \\ \dfrac{\partial u_{m_n}}{\partial t} - \alpha_m\,\Delta u_{m_n} = F_m(u_n) \text{ in } \Omega\times]0,+\infty[\\ \dfrac{\partial u_{i_n}}{\partial \eta} = 0, i=\overline{1,m} \text{ on } \partial\Omega\times]0,+\infty[\\ u_{i_n}(0,.) = u^n_{o_i}(.)\ge 0, i=\overline{1,m} \end{cases}$$

Let us multiply every equation by $\beta_i, i=\overline{1,m}$ we obtain:

$$\begin{cases} \beta_1\dfrac{\partial u_{1_n}}{\partial t} - \beta_1\alpha_1\Delta u_{1_n} = \beta_1 F_1(u_n) \\ \beta_m\dfrac{\partial u_{m_n}}{\partial t} - \beta_m\alpha_m\Delta u_{m_n} = \beta_m F_m(u_n) \end{cases}$$

By taking into account of (H_2), we have:

$$\sum_{i=1}^{m}\beta_i\frac{\partial u_{i_n}}{\partial t} - \sum_{i=1}^{m}\beta_i\alpha_i\,\Delta u_{i_n} = \sum_{i=1}^{m}\beta_i F_i(u_n) \le C\left(1+\sum_{i=1}^{m}u_{i_n}\right)$$

Let us integrate on Ω and apply the formula of Green, we find:

$$\beta_{min}\int_\Omega \sum_{i=1}^{m}\frac{\partial u_{i_n}}{\partial t}dx \le C\int_\Omega \left(1+\sum_{i=1}^{m}u_{i_n}\right)dx$$

such as $\beta_{min}=\min_{1\le i\le m}\beta_i$ so:

$$\beta_{min} = \frac{\int_\Omega \sum_{i=1}^{m}\frac{\partial u_{i_n}}{\partial t}dx}{\int_\Omega\left(1+\sum_{i=1}^{m}u_{i_n}\right)dx} \le C$$

Integrate on $[0,t]$ we find:

$$\beta_{min}\log\frac{\int_\Omega\left(1+\sum_{i=1}^{m}u_{i_n}\right)dx}{\int_\Omega\left(1+\sum_{i=1}^{m}u^n_{0_i}\right)dx} \le Ct$$

Thus:

$$\int_\Omega\left(1+\sum_{i=1}^{m}u_{i_n}\right)dx \le exp(K_1t)\int_\Omega\left(1+\sum_{i=1}^{m}u^n_{0_i}\right)dx, \text{for } K_1=\frac{Ct}{\beta_{min}}$$

Let us put:

$$M(t) = K_1\exp(K_2t)\left\|1+\sum_{i=1}^{m}u^n_{0_i}\right\|_{L^1(\Omega)}$$

Thus:

$\left\|\sum_{i=1}^{m} u_{i_n}\right\|_{L^1(\Omega)} \leq M(t),\ \ 0 \leq t \leq T_M$

We can conclude from this estimation that the solution $\left(u_{1_n}, \dots, u_{m_n}\right)$ given by the theory 2 is a global solution.

Global existence of the solution of the system (1a-c):

Lemma 2: For quite solution $\left(u_{1_n}, \dots, u_{m_n}\right)$ of (S$_n$), there is a constant $K(t)$ which depends only of *t*, such as:

$$\left\|\sum_{i=1}^{m} u_{i_n}(t)\right\|_{L^1(\Omega)} \leq K(t)\left\|\sum_{i=1}^{m} u_{0_i}^{n}\right\|_{L^1(\Omega)} + 1$$

Proof: To prove this lemma, we use the results given in Bonafede and Schmitt (1998): $\forall \theta \in C_0^{\infty}(Q), \theta \geq 0$, there is a nonnegative function $\phi \in C^{2,1}(Q)$ with ϕ a solution of the problem:

$$\begin{cases} \dfrac{-\partial\phi}{\partial t} - \propto_1 \Delta\phi = \theta \ \ \text{in } Q \\ \dfrac{-\partial\phi}{\partial \eta} = 0 \ \ \text{on } \partial\Omega \times [0,T] \\ \phi = 0 \qquad \text{in } \Omega \end{cases}$$

furthermore ϕ verify:

$$\exists C' \geq 0, \text{ such as } \|\phi\|_{L^p(Q)} \leq C'\|\theta\|_{L^q(Q)}$$

We have according to Bonafede and Schmitt (1998):

$$\int_{\Omega} S_1(t) u_{0_i}^{n}(x)\left(\frac{-\partial\phi}{\partial t} - \propto_1 \Delta\phi\right) dxdt = \int_{\Omega} u_{0_i}^{n}(x)\phi(x,0)dx$$

and that:

$$\int_{Q}\left(\int_0^t S_1(t-s)F_1(u_n)\,ds\right).\left(\frac{-\partial\phi}{\partial t} - \propto_1 \Delta\phi\right) dxdt = \int_{Q} F_1(u_n)\,\phi(x,s)dxds$$

where from:

$$\int_{\Omega} S_1(t)\, u_{0_1}^{n}(x)\theta dxdt = \int_{\Omega} u_{0_1}^{n}(x)\Phi(x,0)dx \qquad (3d)$$

and

$$\int_{Q}\left(\int_0^t S_1(t-s)F_1(u_n)ds\right)\theta dxdt = \int_{Q} F_1(u_n)\phi\,(x,s)dxds \qquad (3e)$$

Let us multiply the Eq. (3c) for i = 1 by θ and let us integrate on Q by using (3d) and (3e), we obtain:

$$\begin{aligned} \int_{\Omega} u_{1_n}\theta dxdt &= \int_{Q} S_1(t)u_{0_1}^{n}\theta dxdt + \int_{Q}\left(\int_0^t S_1(t-s)F_1(u_n(s))\,ds\right)\theta\, dx \\ &= \int_{\Omega} u_{0_1}^{n}(x)\phi(x,0)dx + \int_{Q} F_1(u_n)\Phi(x,s)dxds \\ &\leq \int_{\Omega} u_{0_1}^{n}(x)\phi(x,0)dx + \int_{Q} \beta_1 F_1(u_n)\phi(x,s)dxds \ \ \text{for } \beta_1 \geq 0 \end{aligned}$$

also we find:

$$\begin{aligned} \int_{\Omega} u_{i_n}\theta dxdt &= \int_{\Omega} u_{0_i}^{n}(x)\phi(x,0)dx + \int_{Q} F_i(u_n)\phi(x,s)dxds \\ &\leq \int_{\Omega} u_{0_i}^{n}(x)\phi(x,0)dx + \int_{Q} \beta_i F_i(u_n)\phi(x,s)dxds \end{aligned}$$

for β_i positive constants and $i = \overline{2,m}$. thus:

$$\int_{\Omega}\left(\sum_{i=1}^{m} u_{i_n}\right)\theta dxdt \leq \int_{\Omega}\sum_{i=1}^{m} u_{0_i}^{n}\,\phi(x,0)dx + C\int_{Q}\left(1 + \sum_{i=1}^{m} u_i\right)\phi(x,s)dxds$$

We use the Holder's inequality:

$$\int_{\Omega}\left(\sum_{i=1}^{m} u_{i_n}\right)\theta dxdt \leq \left\|\sum_{i=1}^{m} u_{0_i}^{n}\right\|_{L^1(\Omega)} .\|\phi(.,0)\|_{L^\infty(\Omega)} + C\left\|\sum_{i=1}^{m} u_{i_n}+1\right\|_{L^1(\Omega)} .\|\phi\|_{L^\infty(Q)}$$
$$\leq K_1\left(\left\|\sum_{i=1}^{m} u_{0_i}^{n}\right\|_{L^1(\Omega)} + \left\|\sum_{i=1}^{m} u_{i_n}\right\|_{L^1(Q)}+1\right)\|\phi\|_{L^\infty(Q)}$$

as θ is arbitrary in $C_0^\infty(Q)$ we have:

$$\left\|\sum_{i=1}^{m} u_{i_n}\right\|_{L^1(Q)} \leq K_1\left(\left\|\sum_{i=1}^{m} u_{0_i}^{n}\right\|_{L^1(\Omega)} + \left\|\sum_{i=1}^{m} u_{i_n}\right\|_{L^1(Q)}+1\right)$$

We take $K(t)=\frac{1}{1-K_1(t)}$ we find:

$$\left\|\sum_{i=1}^{m} u_{i_n}\right\|_{L^1(Q)} \leq K(t)\left(\left\|\sum_{i=1}^{m} u_{0_i}^{n}\right\|_{L^1(\Omega)}+1\right)$$

see Hollis and Morgan (1992b)(1992b):

Proof of the theory 1: Let us define the application L by:

$$L:(w_0,h)\to S_\alpha(t)w_0+\int_0^t S_\alpha(t-s)h(s)ds$$

where $S_\alpha(t)$ the semigroup of contraction generated by $\propto\Delta$, according to the compactness of the application L of $L^1(Q)\times L^1(Q)$ in $L^1(Q)$ Baras and al. (1977); there is a subsequence $\left(u_{i_n}^{j}\right)_{1\leq i\leq m}$ of $\left(u_{i_n}\right)_{1\leq i\leq m}$ and

$(u_i)_{1\leq i\leq m}\in L^1(Q)\times L^1(Q)\times\ldots\times L^1(Q)$ such as $\left(u_{i_n}^{j}\right)_{1\leq i\leq m}$ converge towards $\left(u_{i_n}\right)_{1\leq i\leq m}$.

Let us show now that $(u_1,\ldots,u_m)$ is a solution of (3c).

We have:

$$\begin{cases} u_{i_n}^{j}(x,t)=S_i(t)u_{0_i}^{j}\int_0^t S_i(t-s)F_i\left(u_{1_n}^{j}(s),\ldots,u_{m_n}^{j}(s)\right)ds \\ \qquad 1\leq i\leq m \end{cases} \tag{S_j}$$

so it is enough to show that $(u_1,\ldots,u_m)$ verify (3a).

It is clear that $j\to+\infty$ we have the following limits:

$$F_i\left(u_{1_n}^{j},\ldots,u_{m_n}^{j}\right)\to F_i(u_1,\ldots,u_m)\text{p. p}, i=\overline{1,m} \tag{3f}$$

and

$$u_{0_i}^{n^j}\to u_{0_i}, i=\overline{1,m} \tag{3g}$$

and according to the lemma 2 and using the theory of convergence dominated by Lebesgue, we can conclude that $(u_{i_n}^{j},\ldots,u_{m_n}^{j})$ converge towards $(u_1,\ldots,u_m)$ in $L^1(Q)$:

Thus to show that $(u_1,\ldots,u_m)$ verify (3a) it remains to show that:

$$F_i\left(u_{1_n}^{j},\ldots,u_{m_n}^{j}\right)\to F_i(u_1,\ldots,u_m)\ i=\overline{1,m}\text{ in }L^1(Q)$$

We integrate the equations of (S_n) on Q by taking into account that:

$$-\alpha_i\int\Delta u_{i_n}dxdt=0, 1\leq i\leq m$$

we have:

$$\int_\Omega u_{i_n}dx - \int_\Omega u_{0_i}^j dx = \int_Q F_i\left(u_{1_n}^j, \dots, u_{m_n}^j\right) dxdt$$

where from:

$$-\int_Q F_i\left(u_{1_n}^j, \dots, u_{m_n}^j\right)dxdt \le \int_\Omega u_{0_i}dx \tag{3h}$$

Let us put:

$$N_{i_n} = C_1\left[\sum_{i=1}^m u_{i_n}^j + 1\right]^{C_2} - \left|F_i\left(u_{1_n}^j, \dots, u_{m_n}^j\right)\right|, i = \overline{1,m}$$

It is clear that N_{in} is positive according to (H_3) of (3h) we obtain:

$$\int_Q N_{i_n}dxdt \le C_1\int_Q\left[\sum_{i=1}^m u_{i_n}^j + 1\right]^{C_2} + \int_\Omega u_{0_i}dx$$

the lemma 2 gives us:

$$\int_Q N_{i_n}dxdt < +\infty$$

Which implies:

$$\int_Q \left|F_i\left(u_{1_n}^j, \dots, u_{m_n}^j\right)\right| dx \le C_1\int_Q\left[\sum_{i=1}^m u_{i_n}^j + 1\right]^{C_2} + \int_Q N_{i_n}dxdt < +\infty$$

Let $h_{n_i} = C_1\left[\sum_{i=1}^m u_{i_n}^j + 1\right]^{C_2} + N_{i_n}, i = \overline{1,m}$

h_{n_i} are in $L^1(Q)$ and positive and furthermore $\left|F_i\left(u_{1_n}^j, \dots, u_{m_n}^j\right)\right| \le h_{n_i}$ p.p, $i = \overline{1,m}$.

Let us combine this result with (3f) and we apply the theory of convergence dominated by Lebesgue.

We obtain:

$$F_i\left(u_{1_n}^j, \dots, u_{m_n}^j\right) \to F_i(u_1, \dots, u_m)\ i = \overline{1,m} \text{ in } L^1(Q)$$

by passage in the limit $j \to +\infty$ of (S_j) in $L^1(Q)$we find:

$$u_i(x,t) = S_i(t)u_{0_i} + \int_0^t S_i(t-s)F_i\big(u(s)\big)ds,\ 1 \le i \le m$$

Theory 3: The problem (1a-c) admits a global solution at time i.e.,$T_{max} = +\infty$.

Proof: Comes down from the theory 1 and the lemma 2.

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