

MASS TRANSFER AND THERMAL RADIATION EFFECTS OVER A STRETCHING SHEET IN A MICROPOLAR FLUID WITH HEAT GENERATION

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ABSTRACT

An analysis is carried out to study the effects of mass transfer and thermal radiation on a steady boundary layer flow induced by a linearly stretching sheet immersed in an incompressible micropolar fluid with constant surface temperature in presence of heat generation. Similarity transformation is employed to transform the governing partial differential equations into ordinary ones, which are then solved numerically using the Runge-Kutta fourth order along Shooting method. Results for the velocity, temperature, concentration, heat transfer rate and mass transfer rate profiles are presented for different values of the governing parameters. Comparisons with previously published work are performed and results are found to be in very good agreement.

Key-words: Micropolar fluid, Stretching sheet, mass transfer, thermal radiation and heat generation.

INTRODUCTION

During the past few decades, considerable progress has been made in the investigations of non-Newtonian fluids due to their applications in engineering and industry. The rheological characteristics of such fluids are very useful in describing the salient features associated with several fluids in nature like shampoo, ketchup, mud, paints, cosmetic products, etc. classical Navier-Stokes equations are unable to describe the properties, such as micro-rotation, spin-inertia, couple stress, and body torque, which are important in many fluids, for instance, polymeric liquids crystals, colloidal suspension, animal blood, and fluids containing small amount of polymeric liquids. Eringen [1] presented the theory of non-Newtonian fluids, in which micro-rotation, spin-inertia, couple stress, and body torque were important. Such fluids are called the micropolar fluids. This theory may be applied to explain the flow of colloidal suspensions ((Hadimoto and Tokioka [2]), liquid crystals (Lockwood *et al.* [3]), polymeric fluids, human and animal blood (Ariman *et al.* [4]) and many other situations.

Flow of a viscous fluid past a stretching sheet is a classical problem in dynamics. The development of boundary layer flow induced solely by a stretching sheet was first studied by Crane [5] who first obtained an elegant analytical solution to the boundary layer equations for the problem of steady two-dimensional flow due to a stretching surface in a quiescent incompressible fluid. Flow and heat transfer characteristics due to a stretching sheet in a stationary fluid occur in a number of industrial manufacturing processes and include both metal and polymer sheets, for example, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, paper production, metal spinning, and drawing plastic films. The quality of the final product depends on the rate of heat transfer at the stretching surface. This problem was then extended by Gupta and Gupta [6] to a permeable surface. The flow problem due to a linearly stretching sheet belongs to a class of exact solutions of the Navier-Stokes equations. Thus, the exact solutions reported by Crane [5] and Gupta and Gupta [6] are also the exact solutions to the Navier-Stokes equations. The heat transfer aspects of similar problems were studied by Grubka and Bobba [7], Chen and Char [8], Dutta *et al* [9], Afzal [10], and many others. On the other hand, the effects of buoyancy force on the development of velocity and thermal boundary layer flows over a stretching sheet have been investigated by Chen [11], Ali and Al-Yousef [12], Daskalakis [13], Abd El-Aziz [14], Mahapatra *et al.* [15] and Ishak *et al.* [16], among others.

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The study of flow and heat transfer past a stretching sheet has gained tremendous interest among researchers due to its industrial and engineering applications. This includes extrusion of plastic sheet, annealing and tinning of copper wire, paper production, crystal growing, and glass blowing. The final products depend mainly on the stretching and cooling rates at the surface. Their studies are not only restricted to the Newtonian fluids but also include the non-Newtonian fluids such as micropolar fluids. Such studies have been carried out by Chaim [17], Heruska *et al.* [18], Agarwal *et al.* [19], Hassanian and Gorla [20], Kelson and Farrell [21], Nazar *et al.* [22], and many others. The combined heat and mass transfer problems are of importance in many processes, and therefore have received a considerable amount of attention in recent years. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, the heat and mass transfer occurs simultaneously. Various scenarios in thermoconvective heat and mass transfer for stretching flows were subsequently discussed by many researchers. Gupta and Gupta [6] obtained the heat and mass transfer on a stretching sheet with suction or blowing. Surma Devi *et al.* [23] reported on numerical (finite difference solutions) for the transient three-dimensional boundary layer flow caused by a stretching surface. Takhar *et al.* [24] analyzed the flow dynamics and species mass transfer in a stretching sheet with chemical reaction and magnetic retardation effects, using the Blottner difference scheme. Kai-Long Hsiao [25] found that Heat and mass transfer for micropolar flow with radiation effect past a nonlinearly stretching sheet.

The study of radiation effects on the various types of flows is quite complicated. In the recent years, many authors have studied radiation effects on the boundary layer of radiating fluids past a plate. Raptis [26] studied the flow of a micropolar fluid past a continually moving plate by the presence of radiation. The radiation effect on heat transfer of a micropolar fluid past unmoving horizontal plate through a porous medium was studied by Abo-Eldahab and Ghonaim [27]. Ishak [28] investigated that the thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation effect.

In many situations, there may be an appreciable temperature difference between the surface and the ambient fluid. This necessitates the consideration of temperature-dependent heat source or sinks which may exert strong influence on the heat transfer characteristics. The study of heat generation or absorption in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions (Vajravelu and Hadjinicolaou [29] and Chamkha and Khalid [30]). Gnaneswara Reddy [31] found that radiation effect over a stretching sheet in a micropolar fluid in the presence of heat generation.

The purpose of the present investigation is therefore to study the development of thermal boundary layer flow induced by a stretching sheet immersed in a micropolar fluid with the effect of mass transfer, thermal radiation and heat radiation is taken into consideration. The governing partial differential equations are transformed into ordinary ones using similarity transformation, before solved numerically by the Runge-Kutta fourth order along Shooting method.

MATHEMATICAL MODEL

A steady laminar boundary layer flow over a stretching sheet immersed in a quiescent and incompressible micropolar fluid with uniform surface temperature T_w is considered. It is assumed that the sheet is stretched with a linear velocity $U_w = ax$, where a is a positive constant and x is the distance from the slit where the sheet is issued. The simplified two-dimensional equations governing the flow may be written as

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} \quad (2)$$

Angular Momentum equation,

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j} \left(2N + \frac{\partial N}{\partial y} \right) \quad (3)$$

Energy equation,

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (4)$$

Species equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (5)$$

where u and v are the velocity components in x- and y- direction respectively. T is the fluid temperature and C is the fluid concentration inside the boundary layer. N is a microrotation or angular velocity, and $j, \gamma, \nu, \kappa, \rho, k, c_p$ and D are the microinertia per unit mass, spin gradient viscosity, kinematic viscosity, vortex viscosity, fluid density, thermal conductivity, specific heat at constant pressure and mass diffusivity respectively.

The spin-gradient viscosity γ (Ahmadi [32]) can be defined as,

$$\gamma = \left(\mu + \frac{\kappa}{2} \right) j = \mu \left(1 + \frac{K}{2} \right) j \quad (6)$$

Where μ is the dynamic viscosity, $K = \frac{\kappa}{\mu}$ is the dimensionless viscosity ratio and is called the material parameter, and

we take $j = \frac{\nu}{a}$ as a reference length. The equation (6) is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity.

The boundary conditions for the velocity, angular velocity, temperature and concentration fields are

$$u = U_w, \quad v = 0, \quad N = -m \frac{\partial u}{\partial y}, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad N \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty, \quad (7)$$

Where T_∞ is the ambient fluid temperature and m is the boundary parameter with $0 \leq m \leq 1$.

By using the Rosseland approximation, Brewster [33], the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y}, \quad (8)$$

where σ_s is the Stefan-Boltzmann constant and k_e is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (8) can be liberalized by expanding T^4 into the Taylor series about T_∞ , which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (9)$$

In view of (8) and (9), (4) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha (1 + R) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty), \quad (10)$$

where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity and $R = \frac{16\sigma^* T_\infty^3}{3kk^*}$ is the radiation parameter.

We introduce now the following similarity transformation:

$$\eta = \left(\frac{U_w}{\nu x} \right)^{\frac{1}{2}} y, \quad \psi = (\nu x U_w)^{\frac{1}{2}} f(\eta), \quad N = \left(\frac{U_w}{\nu x} \right)^{\frac{1}{2}} h(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad Q = \frac{Q_0 \nu}{\rho c_p U_w^2}, \quad Sc = \frac{\nu}{D} \quad (11)$$

where η is the similarity variable and ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$, which identically satisfies the mass conservation equation (1). Substituting (11) into (2), (3) and (10) we obtain the following ordinary differential equation:

$$(1 + K) f''' + ff'' - f'^2 + Kh' = 0, \quad (12)$$

$$\left(1 + \frac{K}{2} \right) h'' + fh' - fh - K(2h + f'') = 0, \quad (13)$$

$$(1 + R)\theta'' + Pr f\theta' + Pr Q\theta = 0, \quad (14)$$

$$\phi'' + Scf\phi' = 0, \quad (15)$$

Where prime denote differentiation with respect to η and $Pr = \frac{\mu}{\alpha}$ is the Prandlt number.

The corresponding dimensionless boundary conditions are

$$f = 0, \quad f' = 1, \quad h = -mf''(0), \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0$$

$$f' \rightarrow 0, \quad h \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty. \quad (16)$$

It is note that $K = 0$ corresponding to viscous fluid.

RESULTS AND DISCUSSION

The set of nonlinear ordinary differential equations (12) – (15) with boundary conditions (16) have been solved by using the Runge-Kutta fourth order along with Shooting method. First of all, higher order non-linear differential equations (12) - (15) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al. [34]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta = 0.01$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. To analyze the results, numerical computation has been carried out using the method described in the previous section for various in governing parameters, namely, material parameter K , radiation parameter R , heat generation parameter Q , the Prandlt number Pr , boundary parameter m and Schmidt number Sc . In the present study following default parameter values are adopted for computations: $K = 1.0$, $R = 1.0$, $Q = 0.1$, $Pr = 0.71$, $Sc = 0.6$ and $m = 0.5$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

In order to assess the accuracy of our computed results, the present result has been compared with Ishak [28] for different values of K as shown in Fig. 1 with $Q = 0.0$ and $Sc = 0.0$. It is observed that the agreement with the solution of temperature profiles is excellent.

Figure 2 presents the velocity profiles for various values of K . We note that the parameter R and Pr have no influence on the flow field, which is clear from (12) – (15). It is evident from this figure that the boundary layer thickness increases with

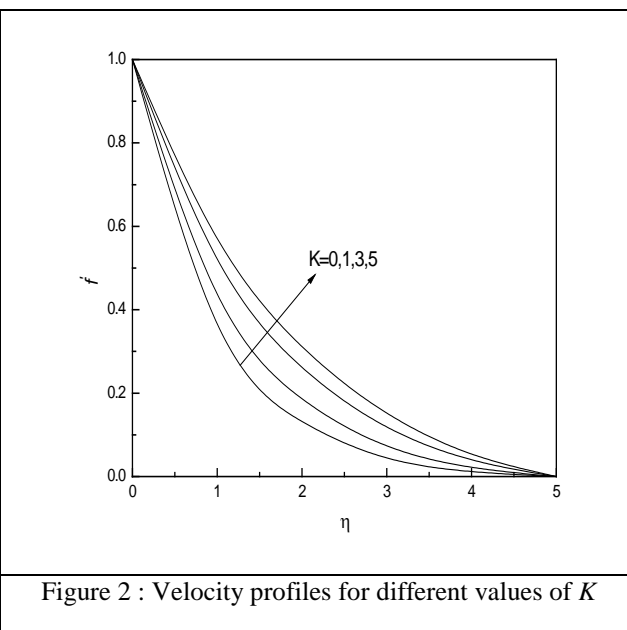
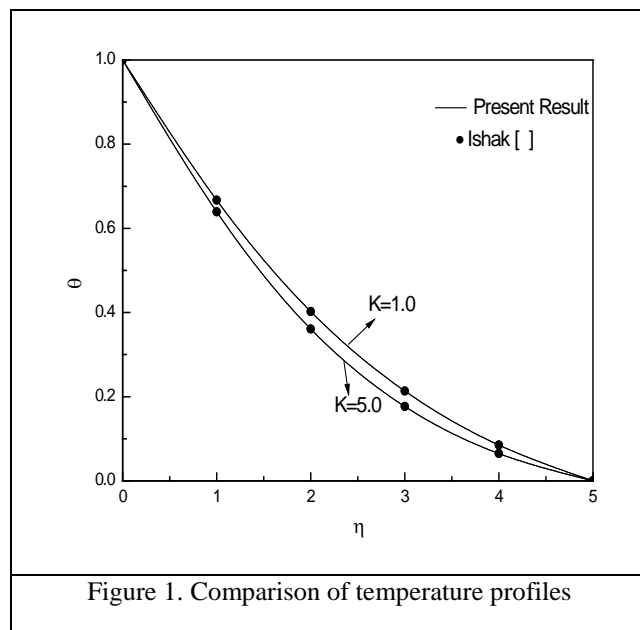
K . The velocity gradient at the surface $f''(0)$ decreases (in absolute sense) as K increases. Thus, micropolar fluids show drag reduction compared to viscous fluids. The negative velocity gradient at the surface $f''(0) < 0$ as shown in Figure 2 means the stretching sheet exerts a drag force on the fluid. This is not surprising since the development of the boundary layer is solely induced by it. The effect of material parameter K on temperature is shown Figure 3. The temperature decrease with an increase in the material parameter. The effect of material parameter K on concentration is shown Figure 4. The concentration decrease with an increase in the material parameter.

The influence of the Prandtl number Pr on temperature field is shown in Figure 5. The numerical results show that the effect of increasing values of the Prandtl number results in a decreasing temperature. It is observed that an increase in Pr results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr . Hence in the case of the smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

Figure 6. shows the temperature profiles for different values of R . The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in increasing temperature within the boundary layer. The effect of heat generation parameter Q on the temperature is shown in Figure 7. From this figure, we observed that when the value of heat generation parameter increases, the temperature distribution also increases along the boundary layer.

The influence of the Schmidt number Sc on concentration is plotted in Figure 8. The Schmidt number Sc embodies the ratio of the momentum to the mass diffusivity. It is noticed that as the Schmidt number Sc increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

The effects of m on velocity, angular velocity, temperature and concentration profiles are depicted in Figures 9, 10, 11 and 12, respectively. Figure 9 shows that the velocity gradient at the surface is larger for larger values of m . Different behaviors are observed for the effect of m on the heat and mass transfer rate at the surface as presented in Figure 10, 11. As expected, the couple stress $h(0)$ is more dominant for larger values of m , as shown in Figure 12.



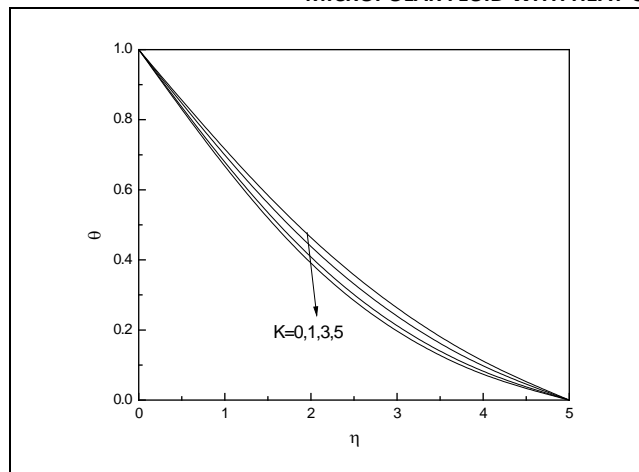


Figure 3: Temperature profiles for different values of K

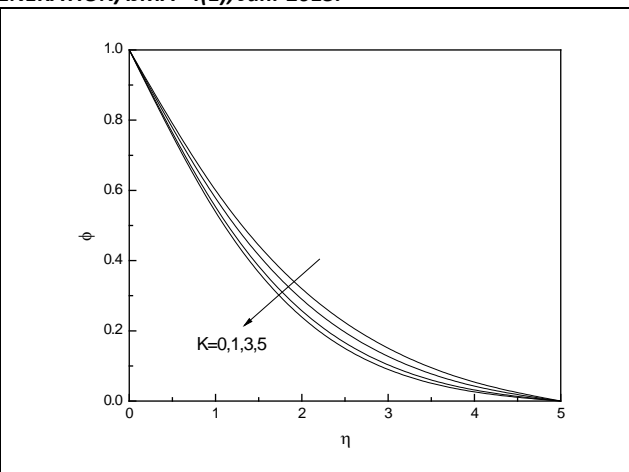


Figure 4: Concentration profiles for different values of K

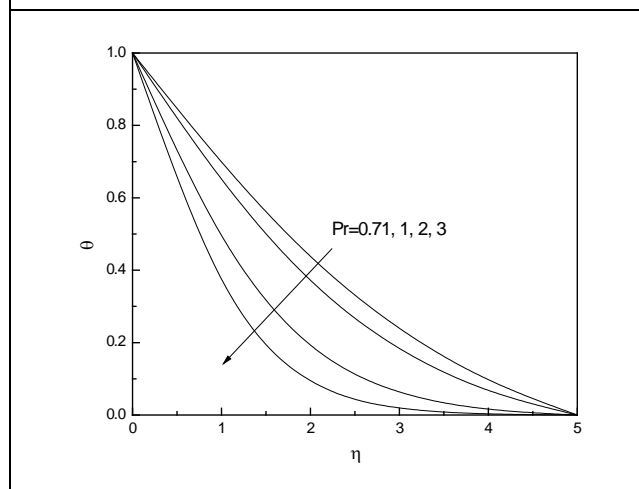


Figure 5: Temperature profiles for different values of Pr

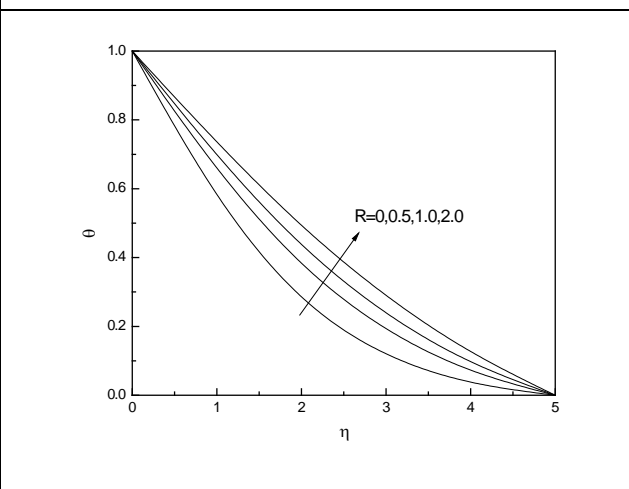


Figure 6: Temperature profiles for different values of R

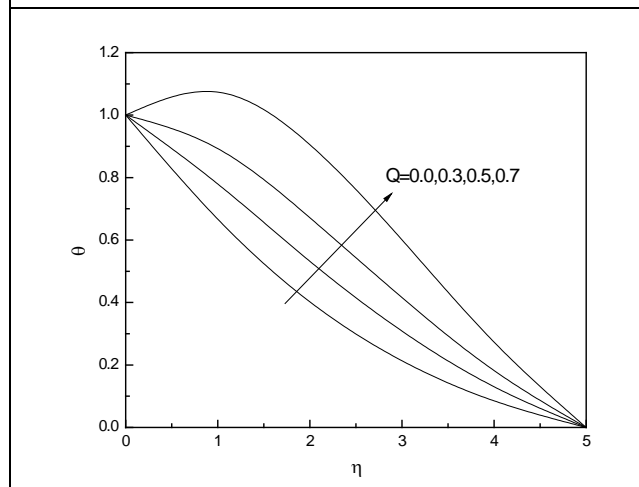


Figure 7: Temperature profiles for different values of Q

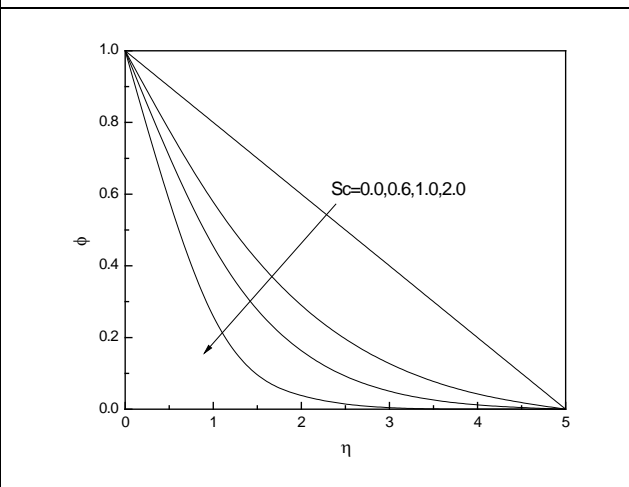


Figure 8: concentration profiles for different values of Sc

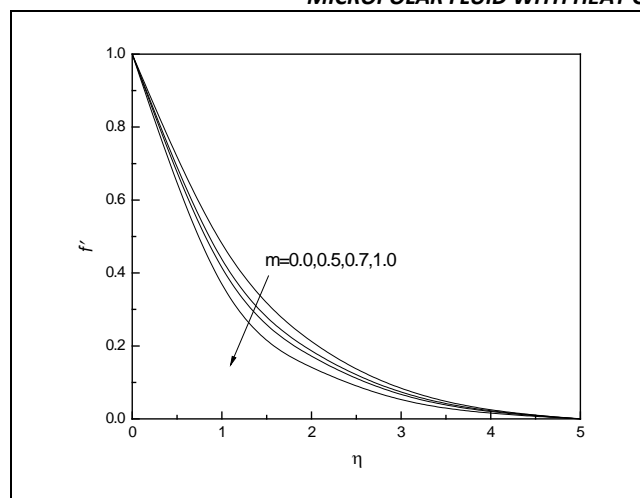


Figure 9 : Velocity profiles for different values of m

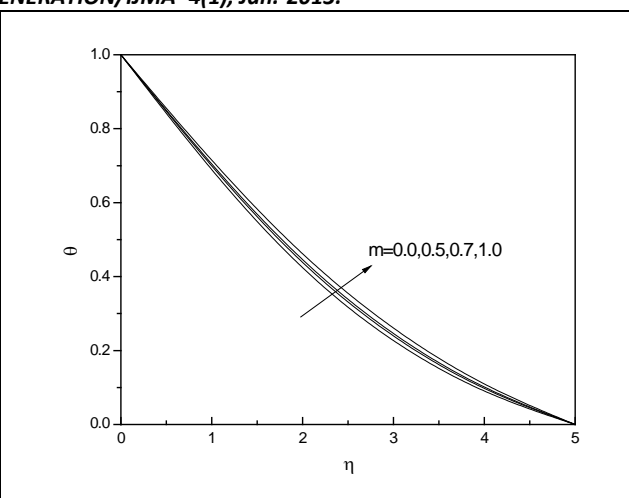


Figure 10: Temperature profiles for different values of m

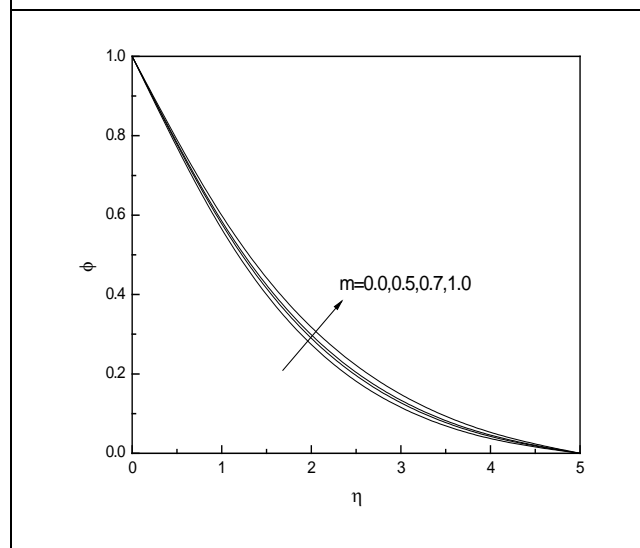


Fig.11 Concentration profiles for different values of m

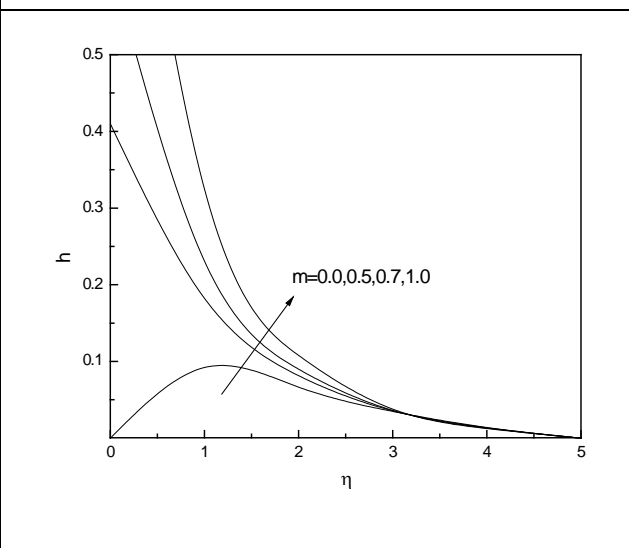


Fig.12. Angular Velocity profile for different values of m

CONCLUSIONS

A steady two-dimensional laminar heat and mass transfer due to a stretching sheet immersed in an incompressible micropolar fluid has been studied. The effects of thermal radiation and heat generation on the development of the thermal boundary layer flow have been taken into consideration. The effects of the governing parameters K , R , Q , Pr , Sc and m on the fluid flow, heat transfer and mass transfer characteristics are discussed. It is observed that the velocity gradient at the surface is larger for larger values of m . It is observed that the velocity increases with an increase in the material parameter. It is seen that the temperature profile is influenced considerably and increases when the value of heat generation parameter increases along the boundary layer. The temperature distribution of the fluid increases with an increase in the radiation parameter. Also concentration distribution of the fluid decreases with an increase in the Schmidt number.

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