

## INVENTORY MODELS FOR DETERIORATING ITEMS WITH STOCK DEPENDENT PRODUCTION RATE AND WEIBULL DECAY

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*(Received on: 03-09-12; Accepted on: 09-10-12)*

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### ABSTRACT

*In this paper a production inventory model for deteriorating items is developed and analyzed with the assumption that the production rate is dependent on stock on hand. It is further assumed that lifetime of the commodity is random and follows a three parameter Weibull distribution and demand rate is a function of both selling price and time. Using the differential equations the instantaneous state of inventory is derived. With suitable cost considerations the total cost function and profit rate function are obtained. By maximizing the profit rate function the optimal ordering policies are derived. Through numerical illustrations the sensitivity analysis is carried. It is observed that the demand rate parameters and stock dependent production rate parameters have significant influence on optimal production scheduling and profit rate.*

<sup>1</sup>*2000 Mathematics Subject Classification: Primary 90B05.*

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### 1. INTRODUCTION

Much work have been reported on inventory models for deteriorating items for varying assumptions on the demand rate, production (replenishment) rate and lifetime of the commodity. It is customary to consider that the replenishment in many inventory models is having finite or infinite rate. Urban [28], Teng and Yang [24], Dye, *et al.* [2], Sana and Chaudhuri [16] and others have studied inventory models having infinite production rate. Sana, *et al.* [17], Srinivasa Rao and Begum [22], Manna and Chiang [9], Uma Maheswara Rao, *et al.* [27], Sarkar and Moon [18] and others considered the finite rate of production. Two different rates of replenishment in one inventory system were also studied [12, 19].

In classical inventory models demand rate is assumed to be a constant. Widyadana and Wee [30] developed an economic production quantity (EPQ) model for deteriorating items where production, rework, deteriorating and demand rate are assumed constant. In reality, the demand rate of any product may vary with time or with price or with the instantaneous level of inventory displayed in a supermarket. Inventory problems involving time dependent demand patterns have received the attention of several researchers in recent years. Ritchie [13], Giri, *et al.* [4], Manna, *et al.* [8], Mahata and Goswami [7] and Skouri, *et al.* [20] are among those who studied inventory models for deteriorating items having time dependent demand. Roy and Chaudhuri [14] and Sana [15] have studied inventory models with demand rates depending on selling price of the item. It has been observed that for certain types of inventories, particularly consumer goods, heaps of stock will attract customers. Taking due account to this fact, Venkata Subbaiah, *et al.* [29], Dye, *et al.* [1], Panda, *et al.* [11], Mahata and Goswami [6], Lee and Dye [32] and others have developed inventory models where demand rate is a function of on-hand inventory. Srinivasa Rao, *et. al* [23] developed a production inventory system with demand rate a function of production quantity, Weibull decay and finite rate of production.

Inventory models for deteriorating items having multivariate demand functions were also studied by several authors. You [31] and Tsao and Sheen [26] have dealt with time and selling price dependent demand. Models for deteriorating items having stock level and selling price dependent demand rate were studied by Teng and Chang [25] and Khanra, *et al.* [5]. Pal, *et al.* [10] considered a single deteriorating item with the demand rate dependent on displayed stock level, selling price of an item and frequency of advertisement.

In all of the above models replenishment was assumed to be infinite or finite with a constant rate. Sridevi, *et al.* [21] developed and analyzed an inventory model with the assumption that the rate of production is random and follows a Weibull distribution and the demand is a function of selling price. Recently, Muluneh and Srinivasa Rao [3] develop an

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inventory model for deteriorating items with the assumption that the production rate is dependent on stock on hand and demand is a power function of time. Very little work has been reported regarding EPQ models with stock dependent production rate and demand as a function of both selling price and time which is a common phenomenon in many production processes. Hence, in this paper we develop and analyze a production level inventory model for deteriorating items with the assumption that the replenishment is a function of on-hand inventory and demand is a function of both selling price and time.

Using the differential equations the instantaneous state of inventory is derived. With suitable cost considerations the total cost function and profit rate function are derived. By maximizing the profit rate function the optimal ordering policies of the model are obtained. The sensitivity of the model with respect to the parameters and costs is also discussed. This model includes some of the earlier models as particular cases for specific and limiting values of the parameters.

## 2. NOTATIONS AND ASSUMPTIONS

### 2.1. Notations

The following notations are used to develop the model under study.

$I(t)$  = Inventory level at any time  $t$

$R(t)$  = Production rate at any time  $t$

$D(s,t)$  = Demand rate at any time  $t$

$Q$  = Total production quantity

$TC$  = Total cost of the system per unit time

$TP$  = Total profit per unit time of the system

$A$  = Set up cost of the item

$p$  = Per unit production cost of the item

$h$  = Inventory holding cost per unit per unit time

$c_2$  = Shortage cost per unit per unit time

$T$  = Length of the cycle

$(\alpha, \beta, \gamma)$  = Deterioration rate parameters

$(\tau, \phi, \eta, n)$  = Demand rate parameters

$(\phi, \theta)$  = Production rate parameters

### 2.2. Assumptions

The model under study is based on the following assumptions.

- i. Lifetime of the commodity is random and follows a three parameter Weibull distribution with probability density function of the form

$$f(t) = \alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^\beta}, \alpha, \beta > 0, t \geq \gamma$$

where,  $\alpha$  is scale parameter,  $\beta\alpha$  is shape parameter and  $\gamma$  is location parameter.

The instantaneous failure rate (hazard rate) at time  $t$  is therefore,

$$h(t) = \frac{f(t)}{1-F(t)} = \alpha\beta(t-\gamma)^{\beta-1}$$

- ii. Demand is a function of time and selling price. Let  $n$  be the demand pattern index and  $s$  be the unit selling price

of the item. Then demand rate is assumed to have the functional form  $D(s,t) = \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{1/n}}$ , where,

$\tau, \phi$  and  $\eta$  are constants. This demand function includes several patterns for specific values of the parameters. For example if  $\eta=0$ , this demand is a function of selling price. If  $\phi=0$ , this demand includes time dependent demand. If  $\phi=0$  and  $\eta=0$  this demand includes constant rate.

- iii. Production rate is assumed to be finite and a linear function of the instantaneous inventory level at time  $t$ ,  $I(t)$  i.e.,

$$R(t) = \begin{cases} \theta - \phi I(t), & 0 \leq t \leq t_1 \\ \theta, & t_3 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

where  $\theta$  is a constant such that  $\theta > 0$ ,  $\phi$  is the stock-dependent production rate parameter,  $0 \leq \phi \leq 1$ . It is assumed that  $R(t) \geq D(t)$  at any time where replenishment takes place. If  $\phi=0$ , then it includes the finite rate of production.

- iv. Shortages are allowed and completely backlogged.
- v. There is no repair or replacement of deteriorated items.
- vi. The planning horizon is infinite. Each cycle will have length  $T$
- vii. The inventory holding cost per unit per unit time  $h$ , the shortage cost per unit per unit time  $c_2$ , the unit production cost per unit time  $p$  and set up cost  $A$  per cycle are fixed and known,

### 3. MODEL FORMULATION

In this model the stock level for the item is initially zero. Then production starts at time  $t = 0$  and continues adding items to stock until the on-hand inventory reaches its maximum level,  $Q_1$  at time  $t = t_1$ . During the time  $(0, \gamma)$  production will continuously satisfy the current demand and the excess production will be accumulated in stock. At time  $t = \gamma$  deterioration of the item starts and stock is depleted by consumption and deterioration while production is continuously adding to it. At  $t = t_1$  production is stopped and stock will be depleted by deterioration and demand until it reaches zero at time  $t = t_2$ . As demand is assumed to occur continuously, at this point shortage begin to accumulate until it reaches its maximum level of  $Q_2$  at  $t = t_3$ . At this point production will resume meeting the current demand and clearing the backlog. Finally, shortages will be cleared at time  $t = T$ . Then the cycle will be repeated indefinitely.

The schematic diagram representing the inventory system is shown in Fig. 1.

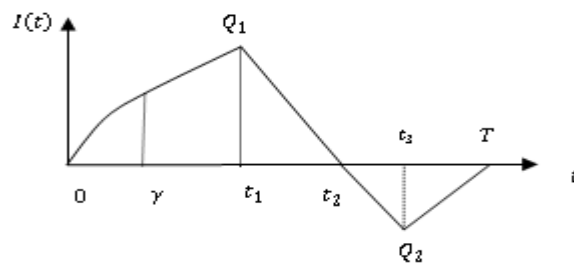


Fig. 1. Schematic diagram representing the inventory level of the system

The differential equations governing the system in the cycle time  $[0, T]$  are:

$$\frac{dI(t)}{dt} = \{\theta - \phi I(t)\} - \left\{ \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}} \right\}, 0 \leq t \leq \gamma \quad (1)$$

$$\frac{dI(t)}{dt} = \{\theta - \phi I(t)\} - \left\{ \alpha \beta (t - \gamma)^{\beta-1} I(t) \right\} - \left\{ \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}} \right\}, \gamma \leq t \leq t_1 \quad (2)$$

$$\frac{dI(t)}{dt} = -\left\{ \alpha \beta (t - \gamma)^{\beta-1} I(t) \right\} - \left\{ \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}} \right\}, t_1 \leq t \leq t_2 \quad (3)$$

$$\frac{dI(t)}{dt} = -\left\{ \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}} \right\}, t_2 \leq t \leq t_3 \quad (4)$$

$$\frac{dI(t)}{dt} = \theta - \left\{ \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}} \right\}, t_3 \leq t \leq T \quad (5)$$

with boundary conditions;

$$I(0) = 0, I(t_2) = 0 \text{ and } I(T) = 0$$

Solving the differential equations (1) to (5) after applying the boundary conditions given above, we get:

The instantaneous state of inventory at any time  $t$  in  $(0, \gamma)$  is

$$I(t) = e^{-\phi t} \left\{ (\theta - \tau + \phi s) \int_0^t e^{\phi u} du - \frac{\eta}{nT^{1/n}} \int_0^t u^{(1/n)-1} e^{\phi u} du \right\}, \quad 0 \leq t \leq \gamma \quad (6)$$

The instantaneous state of inventory at any time  $t$  in  $(\gamma, t_1)$  is

$$I(t) = e^{-\phi t - \alpha(t-\gamma)^\beta} \left\{ (\theta - \tau + \phi s) \int_0^\gamma e^{\phi u} du - \frac{\eta}{nT^{1/n}} \int_0^\gamma u^{(1/n)-1} e^{\phi u} du + \int_\gamma^t (\theta - \tau + \phi s) e^{\phi u + \alpha(u-\gamma)^\beta} du - \frac{\eta}{nT^{1/n}} \int_\gamma^t u^{(1/n)-1} e^{\phi u + \alpha(u-\gamma)^\beta} du \right\}, \quad \gamma \leq t \leq t_1 \quad (7)$$

The instantaneous state of inventory at any time  $t$  in  $(t_1, t_2)$  is

$$I(t) = e^{-\alpha(t-\gamma)^\beta} \left\{ (\tau - \phi s) \int_t^{t_2} e^{\alpha(u-\gamma)^\beta} du + \frac{\eta}{nT^{1/n}} \int_t^{t_2} u^{(1/n)-1} e^{\alpha(u-\gamma)^\beta} du \right\}, \quad t_1 \leq t \leq t_2 \quad (8)$$

The instantaneous state of inventory at any time  $t$  in  $(t_2, t_3)$  is

$$I(t) = -(\tau - \phi s) \int_{t_2}^t du - \frac{\eta}{nT^{1/n}} \int_{t_2}^t u^{(1/n)-1} du, \quad t_2 \leq t \leq t_3 \quad (9)$$

And the instantaneous state of inventory at any time  $t$  in  $(t_3, T)$  is

$$I(t) = -(\theta - \tau + \phi s) \int_t^T du - \frac{\eta}{nT^{1/n}} \int_t^T u^{(1/n)-1} du, \quad t_3 \leq t \leq T \quad (10)$$

Since  $I(t)$  is continuous at  $t_1$  evaluating (7) and (8) and equating  $I(t)$  at  $t = t_1$  we get the equation.

$$e^{-\phi t_1 - \alpha(t_1-\gamma)^\beta} \left\{ (\theta - \tau + \phi s) \int_0^\gamma e^{\phi u} du - \frac{\eta}{nT^{1/n}} \int_0^\gamma u^{(1/n)-1} e^{\phi u} du + \int_\gamma^{t_1} (\theta - \tau + \phi s) e^{\phi u + \alpha(u-\gamma)^\beta} du - \frac{\eta}{nT^{1/n}} \int_\gamma^{t_1} u^{(1/n)-1} e^{\phi u + \alpha(u-\gamma)^\beta} du \right\} = e^{-\alpha(t_1-\gamma)^\beta} \left\{ (\tau - \phi s) \int_{t_1}^{t_2} e^{\alpha(u-\gamma)^\beta} du + \frac{\eta}{nT^{1/n}} \int_{t_1}^{t_2} u^{(1/n)-1} e^{\alpha(u-\gamma)^\beta} du \right\} \quad (11)$$

Let  $\beta$  be a positive integer. Equation (11) is evaluated using the Taylor series approximation of the exponential function and ignoring terms of higher order and then taking the binomial expansion of  $(u - \gamma)^\beta$ . The equality in (11) is used to establish the relationship between  $t_1$  and  $t_2$ . Also the maximum inventory level  $Q_1 = I(t_1)$  is

$$Q_1 = e^{-\alpha(t_1-\gamma)^\beta} \left\{ (\tau - \phi s) \left( t_2 - t_1 + \frac{\alpha}{\beta+1} \left[ (t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \right] \right) + \frac{\eta}{T^{1/n}} \left( t_2^{1/n} - t_1^{1/n} + \alpha \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{t_2^{(1/n)+j} - t_1^{(1/n)+j}}{(jn+1)} \right) \right\} \quad (12)$$

Similarly, since  $I(t)$  is continuous at  $t_3$  evaluating  $I(t)$  at  $t = t_3$  and equating (9) and (10) we get the equation.

$$(\tau - \phi s)(t_3 - t_2) + \frac{\eta}{T^{1/n}} (t_3^{1/n} - t_2^{1/n}) = (\theta - \tau + \phi s)(T - t_3) - \eta \left( 1 - \frac{t_3^{1/n}}{T^{1/n}} \right) \quad (13)$$

This equality can be used to establish the relationship between  $t_2$  and  $t_3$ . Therefore we have,

$$t_3 = T - \frac{1}{\theta}(\tau - \phi s)(T - t_2) - \frac{\eta}{\theta} \left( 1 - \frac{t_2^{1/n}}{T^{1/n}} \right) = f(s, t_2), \text{ say} \quad (14)$$

The maximum shortage level  $Q_2 = I(t_3)$  is

$$Q_2 = -(\tau - \phi s)(t_3 - t_2) + \frac{\eta}{T^{1/n}} (t_3^{1/n} - t_2^{1/n}) \quad (15)$$

Backlogged demand,  $B(t)$  is

$$\begin{aligned} B(t) &= \int_{t_2}^{t_3} \left\{ \tau - \phi s + \frac{\eta t^{(1/n)-1}}{nT^{1/n}} \right\} dt \\ &= (\tau - \phi s)(t_3 - t_2) - \frac{\eta}{T^{1/n}} (t_3^{1/n} - t_2^{1/n}) \end{aligned} \quad (16)$$

Stock loss due to deterioration at any time  $t$ ,  $L(t)$  is defined as

$$L(t) = \int_0^t R(t)dt - \int_0^t D(t)dt - I(t)$$

This implies

$$L(t) = \begin{cases} \theta t - \phi \int_0^\gamma I(t)dt - \phi \int_\gamma^t I(t)dt - (\tau - \phi s)t - \frac{\eta t^{1/n}}{T^{1/n}} - I(t), & \gamma \leq t \leq t_1 \\ \theta t_1 - \phi \int_0^\gamma I(t)dt - \phi \int_\gamma^{t_1} I(t)dt - (\tau - \phi s)t - \frac{\eta t^{1/n}}{T^{1/n}} - I(t), & t_1 \leq t \leq t_2 \\ 0, & \text{elsewhere} \end{cases} \quad (17)$$

Total Production in the cycle time  $(0, T)$  is

$$\begin{aligned} Q &= \int_0^\gamma R(t)dt + \int_\gamma^{t_1} R(t)dt + \int_{t_3}^T R(t)dt \\ &= \theta(t_1 + T - t_3) - \phi \int_0^\gamma I(t)dt - \phi \int_\gamma^{t_1} I(t)dt \end{aligned}$$

Substituting the values of  $I(t)$  in equations (6) and (7), using the Taylor series approximation of the exponential functions and ignoring terms of higher order, taking the binomial expansion of  $(u - \gamma)^\beta$  and then integrating we get

$$\begin{aligned} Q &= \theta \left\{ t_1 + \frac{1}{\theta}(\tau - \phi s)(T - t_2) + \frac{\eta}{\theta} \left( \frac{t_2^{1/n}}{T^{1/n}} - 1 \right) \right\} \\ &\quad - \phi \left\{ \frac{(\theta - \tau + \phi s)}{\phi^2} (e^{-\phi\gamma} + \phi\gamma - 1) + \frac{\eta n \gamma^{(1/n)+1}}{(n+1)T^{1/n}} \left( \frac{\gamma^2 \phi^2}{(3n+1)} + \frac{n\gamma\phi}{(2n+1)} - 1 \right) \right\} \\ &\quad - \phi B \left\{ (t_1 - \gamma) - \frac{\phi}{2}(t_1^2 - \gamma^2) - \frac{\alpha}{\beta+1}(t_1 - \gamma)^{\beta+1} \right\} \end{aligned}$$

$$\begin{aligned}
 & -\phi(\theta - \tau + \phi s) \left\{ \frac{1}{2}(t_1^2 - \gamma^2) - \frac{\phi}{6}(t_1^3 - \gamma^3) - \frac{\phi^2}{8}(t_1^4 - \gamma^4) \right. \\
 & \left. - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1} \left( t_1 + \frac{1}{2}\phi t_1^2 \right) + \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2(t_1 - \gamma)^{2(\beta+1)}}{2(\beta+1)^2} \right\} \\
 & - \frac{\eta n \phi}{T^{1/n}} \left\{ \frac{t_1^{(1/n)+1}}{(n+1)} \left( \frac{\phi^2 t_1^2}{(3n+1)} + \frac{\phi n t_1}{(2n+1)} - 1 \right) - \frac{\gamma^{(1/n)+1}}{(n+1)} \left( \frac{\phi^2 \gamma^2}{(3n+1)} + \frac{\phi n \gamma}{(2n+1)} - 1 \right) \right. \\
 & + \alpha n \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{j \cdot (t_1^{(1/n)+j+1} - \gamma^{(1/n)+j+1})}{(jn+1)(jn+n+1)} \\
 & + \alpha \phi \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{(t_1^{(1/n)+j+2} - \gamma^{(1/n)+j+2})}{(jn+2n+1)} \left( \frac{1}{jn+1} + \frac{1}{n+1} \right) \\
 & \left. + \alpha^2 \sum_{j=0}^{\beta} \sum_{k=0}^{\beta} \binom{\beta}{j} \binom{\beta}{k} (-\gamma)^{2\beta-j-k} \frac{(t_1^{(1/n)+j+k+1} - \gamma^{(1/n)+j+k+1})}{(jn+1)(jn+kn+n+1)} \right\} \quad (18)
 \end{aligned}$$

where,

$$B = \frac{(\theta - \tau + \phi s)}{\phi} \left( e^{\phi \gamma} - \frac{1}{2} \gamma^2 \phi^2 - \phi \gamma - 1 \right) + \frac{\eta \alpha}{T^{1/n}} \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{\gamma^{(1/n)+j}}{jn+1} \quad (19)$$

Total cost is the sum of setup cost per unit time, the production cost per unit time, inventory holding cost per unit time and the shortage cost per unit time. Let  $TC(t_1, t_2, s)$  be the total cost per unit time. Therefore we have:

$$TC(t_1, t_2, s) = \frac{A}{T} + \frac{pQ}{T} + \frac{h}{T} \left\{ \int_0^{\gamma} I(t) dt + \int_{\gamma}^{t_1} I(t) dt + \int_{t_1}^{t_2} I(t) dt \right\} + \frac{c_2}{T} \left\{ \int_{t_2}^{t_3} -I(t) dt + \int_{t_3}^T -I(t) dt \right\} \quad (20)$$

Substituting the expressions for  $I(t)$  from equations (6) to (10) in equation (20), using the Taylor series expansion of the exponential functions and neglecting higher order terms, using binomial expansion of  $(u - \gamma)^{\beta}$  and then integrating we get

$$\begin{aligned}
 TC(t_1, t_2, s) = & \frac{A}{T} + \frac{p\theta}{T} \left\{ t_1 + \frac{1}{\theta} (\tau - \phi s)(T - t_2) + \frac{\eta}{\theta} \left( \frac{t_2^{1/n}}{T^{1/n}} - 1 \right) \right\} \\
 & + \frac{h - p\phi}{T} \left\{ \frac{(\theta - \tau + \phi s)}{\phi^2} (e^{-\phi \gamma} + \phi \gamma - 1) + \frac{\eta n \gamma^{(1/n)+1}}{(n+1)T^{1/n}} \left( \frac{\gamma^2 \phi^2}{(3n+1)} + \frac{n \gamma \phi}{(2n+1)} - 1 \right) \right\} \\
 & + \frac{(h - p\phi)}{T} B \left\{ (t_1 - \gamma) - \frac{\phi}{2}(t_1^2 - \gamma^2) - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1} \right\} \\
 & + \frac{h - p\phi}{T} (\theta - \tau + \phi s) \left\{ \frac{1}{2}(t_1^2 - \gamma^2) - \frac{\phi}{6}(t_1^3 - \gamma^3) - \frac{\phi^2}{8}(t_1^4 - \gamma^4) \right. \\
 & \left. - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1} \left( t_1 + \frac{1}{2}\phi t_1^2 \right) + \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2(t_1 - \gamma)^{2(\beta+1)}}{2(\beta+1)^2} \right\} \\
 & + \frac{h(\tau - \phi s)}{T} \left\{ \left[ t_2 + \frac{\alpha(t_2 - \gamma)^{\beta+1}}{\beta+1} \right] \left[ (t_2 - t_1) - \frac{\alpha \{ (t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \}}{(\beta+1)} \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{2}(t_2^2 - t_1^2) + \frac{\alpha \{t_2(t_2 - \gamma)^{\beta+1} - t_1(t_1 - \gamma)^{\beta+1}\}}{(\beta+1)} \\
 & - \frac{2\alpha \{(t_2 - \gamma)^{\beta+2} - (t_1 - \gamma)^{\beta+2}\}}{(\beta+1)(\beta+2)} + \frac{\alpha^2 \{(t_2 - \gamma)^{2(\beta+1)} - (t_1 - \gamma)^{2(\beta+1)}\}}{2(\beta+1)^2} \Bigg\} \\
 & + \frac{(h - p\phi)\eta n}{T^{(1/n)+1}} \left\{ \frac{t_1^{(1/n)+1}}{(n+1)} \left( \frac{\phi^2 t_1^2}{(3n+1)} + \frac{\phi n t_1}{(2n+1)} - 1 \right) - \frac{\gamma^{(1/n)+1}}{(n+1)} \left( \frac{\phi^2 \gamma^2}{(3n+1)} + \frac{\phi n \gamma}{(2n+1)} - 1 \right) \right. \\
 & + \alpha n \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{j \cdot (t_1^{(1/n)+j+1} - \gamma^{(1/n)+j+1})}{(jn+1)(jn+n+1)} \\
 & + \alpha \phi \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{(t_1^{(1/n)+j+2} - \gamma^{(1/n)+j+2})}{(jn+2n+1)} \left( \frac{1}{jn+1} + \frac{1}{n+1} \right) \\
 & + \alpha^2 \sum_{j=0}^{\beta} \sum_{k=0}^{\beta} \binom{\beta}{j} \binom{\beta}{k} (-\gamma)^{2\beta-j-k} \frac{(t_1^{(1/n)+j+k+1} - \gamma^{(1/n)+j+k+1})}{(jn+1)(jn+kn+n+1)} \Bigg\} \\
 & + \frac{h\eta}{T^{(1/n)+1}} \left\{ \left[ t_2^{1/n} + \alpha \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{t_2^{(1/n)+j}}{(jn+1)} \right] \left[ (t_2 - t_1) - \frac{\alpha \{(t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1}\}}{(\beta+1)} \right] \right. \\
 & - \frac{n}{n+1} (t_2^{(1/n)+1} - t_1^{(1/n)+1}) + \alpha n^2 \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{j \cdot (t_2^{(1/n)+j+1} - t_1^{(1/n)+j+1})}{(jn+1)(jn+n+1)} \\
 & + \alpha^2 n \sum_{j=0}^{\beta} \sum_{k=0}^{\beta} \binom{\beta}{j} \binom{\beta}{k} (-\gamma)^{2\beta-j-k} \frac{(t_2^{(1/n)+j+k+1} - t_1^{(1/n)+j+k+1})}{(jn+1)(jn+kn+n+1)} \Bigg\} \\
 & + \frac{c_2}{T} \left\{ \frac{1}{2} (\tau - \phi s) (T - t_2) (2f(t_2, s) - T - t_2) + \frac{1}{2} \theta (T - f(t_2, s))^2 \right. \\
 & + \eta \left( f(t_2, s) - \frac{T}{n+1} \right) + \frac{\eta t_2^{1/n}}{T^{1/n}} \left( \frac{t_2}{n+1} - f(t_2, s) \right) \Bigg\} \quad (21)
 \end{aligned}$$

Let  $TR(t_1, t_2, s)$  be the total revenue per unit time. Then

$$TR(t_1, t_2, s) = \frac{s}{T} \left\{ \int_0^T (\tau - \phi s) dt + \frac{\eta}{nT^{1/n}} \int_0^T t^{(1/n)-1} dt \right\}$$

This implies

$$TR(t_1, t_2, s) = s(\tau - \phi s) + \frac{\eta s}{T} \quad (22)$$

Let  $TP(t_1, t_2, s)$  be the total profit per unit time. Therefore we have

$$TP(t_1, t_2, s) = TR(t_1, t_2, s) - TC(t_1, t_2, s) \quad (23)$$

where,  $TC(t_1, t_2, s)$  is as defined in (21)

#### 4. OPTIMAL POLICIES OF THE MODEL

The problem here is to find the optimal values of production down-time  $t_1$ , time to begin shortages,  $t_2$  and selling price  $s$  that maximize the total profit  $TP(t_1, t_2, s)$  over  $(0, T)$ . To obtain these values we differentiate  $TP(t_1, t_2, s)$  in equation (23) with respect to  $t_1$ ,  $t_2$  and  $s$  and equate the resulting equations to zero.

Differentiating  $TP(t_1, t_2, s)$  with respect to  $t_1$  and equating to zero we get

$$\begin{aligned} & \frac{p\theta}{T} + \frac{(h-p\phi)}{T} B \{1 - \phi t_1 - \alpha(t_1 - \gamma)^\beta\} \\ & + \frac{h-p\phi}{T} \left\{ (\theta - \tau + \phi s) \left\{ t_1 - \frac{1}{2} \phi t_1^2 - \frac{1}{2} \phi^2 t_1^3 - \alpha(t_1 - \gamma)^\beta \left( t_1 + \frac{1}{2} \phi t_1^2 \right) \right. \right. \\ & + \frac{\alpha(t_1 - \gamma)^{\beta+1}}{(\beta+1)} (1 - \phi t_1) - \frac{\alpha^2(t_1 - \gamma)^{2\beta+1}}{(\beta+1)} \left. \right\} + \frac{\eta t_1^{(1/n)}}{T^{(1/n)}} \left( \frac{\phi^2 t_1^2}{n+1} + \frac{\phi n t_1}{n+1} - 1 \right) \\ & + \frac{\eta \alpha n}{T^{1/n}} \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{j t_1^{(1/n)+j}}{(jn+1)} + \frac{\eta \alpha \phi}{T^{1/n}} \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} t_1^{(1/n)+j+1} \left( \frac{1}{jn+1} + \frac{1}{n+1} \right) \\ & + \frac{\eta \alpha^2}{T^{1/n}} \sum_{j=0}^{\beta} \sum_{k=0}^{\beta} \binom{\beta}{j} \binom{\beta}{k} (-\gamma)^{2\beta-j-k} \frac{t_1^{(1/n)+j+k}}{(jn+1)} \left. \right\} \\ & + \frac{h\eta}{T^{(1/n)+1}} \left\{ \left[ t_2^{1/n} + \alpha \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{t_2^{(1/n)+j}}{(jn+1)} \right] \left[ \alpha(t_1 - \gamma)^{\beta+1} - 1 \right] + t_1^{1/n} \right. \\ & - \alpha n \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{j t_1^{(1/n)+j}}{(jn+1)} - \alpha^2 \sum_{j=0}^{\beta} \sum_{k=0}^{\beta} \binom{\beta}{j} \binom{\beta}{k} (-\gamma)^{2\beta-j-k} \frac{t_1^{(1/n)+j+k}}{(jn+1)} \left. \right\} \\ & + \frac{h(\tau - \phi s)}{T} \left\{ \left[ t_2 + \frac{\alpha(t_2 - \gamma)^{\beta+1}}{\beta+1} \right] \left[ \alpha(t_1 - \gamma)^\beta - 1 \right] + t_1 \left[ 1 - \alpha(t_1 - \gamma)^\beta \right] \right. \\ & + \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1} - \frac{\alpha^2(t_1 - \gamma)^{2\beta+1}}{\beta+1} \left. \right\} = 0 \end{aligned} \quad (24)$$

Differentiating  $TP(t_1, t_2, s)$  with respect to  $t_2$  and equating to zero we get

$$\begin{aligned} & - \frac{p(\tau - \phi s)}{T} - \frac{p\eta t_2^{(1/n)-1}}{nT^{(1/n)+1}} + \frac{h(\tau - \phi s)}{T} \left\{ \frac{\alpha^2(t_2 - \gamma)^{2\beta+1}}{(\beta+1)} - \frac{\alpha(t_2 - \gamma)^{\beta+1}}{(\beta+1)} \right\} \\ & + \frac{h(\tau - \phi s)}{T} \left\{ \left[ 1 + \alpha(t_2 - \gamma)^\beta \right] \left[ (t_2 - t_1) - \frac{\alpha \{ (t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \}}{\beta+1} \right] \right. \\ & + \left. \left[ \frac{\alpha(t_2 - \gamma)^{\beta+1}}{\beta+1} + t_2 - 1 \right] \left[ 1 - \alpha(t_2 - \gamma)^\beta \right] \right\} \\ & + \frac{\eta}{T^{(1/n)+1}} \left\{ \left[ \frac{1}{n} t_2^{(1/n)-1} + \frac{\alpha}{n} \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} t_2^{(1/n)+j-1} \right] \left[ (t_2 - t_1) - \frac{\alpha \{ (t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \}}{\beta+1} \right] \right. \\ & + \left. \left[ t_2^{(1/n)} + \alpha \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{t_2^{(1/n)+j}}{(jn+1)} \right] \left[ 1 - \alpha(t_2 - \gamma)^\beta \right] - t_2^{(1/n)} \right\} \end{aligned}$$



$$\begin{aligned} & \alpha n \sum_{j=0}^{\beta} \binom{\beta}{j} (-\gamma)^{\beta-j} \frac{j t_2^{(1/n)+j}}{(jn+1)} + \alpha^2 \sum_{j=0}^{\beta} \sum_{k=0}^{\beta} \binom{\beta}{j} \binom{\beta}{k} (-\gamma)^{2\beta-j-k} \frac{t_2^{(1/n)+j+k}}{(jn+1)} \Bigg\} \\ & + \frac{c_2}{T} \left\{ (\tau - \phi s) \left[ (T - t_2) f'(t_2, s) + t_2 - f(t_2, s) \right] - \theta (T - f(t_2, s)) f'(t_2, s) \right. \\ & \left. + \frac{\eta t_2^{(1/n)-1}}{n T^{1/n}} (t_2 - f(t_2, s)) + \eta f'(t_2, s) \left( 1 - \frac{t_2^{(1/n)}}{T^{(1/n)}} \right) \right\} = 0 \end{aligned} \quad (25)$$

Differentiating  $TP(t_1, t_2, s)$  with respect to  $s$  and equating to zero we get

$$\begin{aligned} & \tau - 2\phi s + \frac{\eta}{T} + \frac{\phi p(T - t_2)}{T} - \frac{\phi(h - p\phi)}{\phi^2 T} (e^{-\phi\gamma} + \phi\gamma - 1) \\ & - \frac{\phi(h - p\phi)}{\phi T} \left\{ e^{\phi\gamma} - \frac{1}{2} \phi^2 \gamma^2 - \phi\gamma - 1 \right\} \left\{ (t_1 - \gamma) - \frac{1}{2} \phi(t_1^2 - \gamma^2) - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1} \right\} \\ & - \frac{\phi(h - p\phi)}{T} \left\{ \frac{1}{2} (t_1^2 - \gamma^2) - \frac{\phi}{6} (t_1^3 - \gamma^3) - \frac{\phi^2}{8} (t_1^4 - \gamma^4) \right. \\ & \left. - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{(\beta+1)} \left( t_1 + \frac{1}{2} \phi t_1^2 \right) + \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2(t_1 - \gamma)^{2(\beta+1)}}{2(\beta+1)^2} \right\} \\ & + \frac{h\phi}{T} \left\{ \left[ t_2 + \frac{\alpha(t_2 - \gamma)^{\beta+1}}{\beta+1} \right] \left[ (t_2 - t_1) - \frac{\alpha \{ (t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \}}{(\beta+1)} \right] \right. \\ & \left. - \frac{1}{2} (t_2^2 - t_1^2) + \frac{\alpha [t_2(t_2 - \gamma)^{\beta+1} - t_1(t_1 - \gamma)^{\beta+1}]}{(\beta+1)} \right. \\ & \left. - \frac{2\alpha [(t_2 - \gamma)^{\beta+2} - (t_1 - \gamma)^{\beta+2}]}{(\beta+1)(\beta+2)} + \frac{\alpha^2 [(t_2 - \gamma)^{2(\beta+1)} - (t_1 - \gamma)^{2(\beta+1)}]}{2(\beta+1)^2} \right\} \\ & - \frac{c_2}{T} \left\{ \phi(T - t_2)^2 \left( \frac{(\tau - \phi s)}{\theta} - \frac{1}{2} \right) + \frac{\phi\eta(T - t_2)}{\theta} \left( 1 - \frac{t_2^{(1/n)}}{T^{(1/n)}} \right) \right\} = 0 \end{aligned} \quad (26)$$

where,

$$f'(t_2, s) = + \frac{\partial f(t_2, s)}{\partial t_2} = \frac{(\tau - \phi s)}{\theta} + \frac{\eta t_2^{(1/n)-1}}{n \theta T^{(1/n)}}$$

The solutions  $t_1^*$ ,  $t_2^*$  and  $s^*$  of  $t_1$ ,  $t_2$  and  $s$  respectively are obtained by solving the equations (24), (25) and (26) using numerical methods. Substituting these optimal values of  $t_1$ ,  $t_2$  and  $s$  in to equations (14), (12), (15), (18), (21) and (23) we obtain the optimal values for  $t_3$ ,  $Q_1$ ,  $Q_2$ ,  $Q$ ,  $TC(t_1, t_2, s)$  and  $TP(t_1, t_2, s)$  respectively.

## 5. NUMERICAL ILLUSTRATION

Consider the case of deriving an EPQ, production downtime and production uptime for a petrochemical industry viz., fertilizer manufacturing plant. Here the product is of a deteriorating type and has a random lifetime which is assumed to follow a three parameter Weibull distribution. Records and discussions held with the production and marketing personnel suggested the values of various parameters. The deterioration parameters  $\alpha$ ,  $\beta$  and  $\gamma$  are estimated to vary over 0.01 to 0.07, 1 to 4 and 0.2 to 0.8 respectively. Stock dependent production rate parameters  $\phi$  and  $\theta$  vary over 0.3 to 0.6 and 60 to 90 respectively and demand parameters  $\tau$ ,  $\phi$ ,  $\eta$  and  $n$  vary over 50 to 80, 0.6 to 0.9, 15 to 24 and 1 to 4 respectively. Let the values for other parameters be  $p=10$ ,  $h=7$ ,  $c_2=3$  and  $A=75$  in appropriate units. The cycle length is taken to be  $T=6$  units and the values of the parameters are varied to observe the trend in the optimal policies. The

optimal values of production downtime ( $t_1$ ), production uptime ( $t_3$ ), selling price(s) production quantity ( $Q$ ) and total profit ( $TP$ ) are obtained and presented in Table 1.

**Table 1:** Optimal values of the decision variables for different values of parameters

| $\tau$ | $\phi$ | $\nu$ | $n$ | $\phi$ | $\theta$ | $\alpha$ | $\beta$ | $\gamma$ | $p$ | $h$ | $c_2$ | $A$ | $t_1^*$ | $t_3^*$ | $s^*$  | $Q^*$   | $TP^*$   |
|--------|--------|-------|-----|--------|----------|----------|---------|----------|-----|-----|-------|-----|---------|---------|--------|---------|----------|
| 50     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 75  | 2.724   | 5.740   | 35.425 | 298.427 | 321.796  |
| 60     |        |       |     |        |          |          |         |          |     |     |       |     | 2.676   | 5.611   | 41.702 | 296.389 | 553.730  |
| 70     |        |       |     |        |          |          |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
| 80     |        |       |     |        |          |          |         |          |     |     |       |     | 2.551   | 5.306   | 54.226 | 293.260 | 971.056  |
| 70     | 0.6    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 75  | 2.612   | 5.453   | 63.165 | 295.040 | 1311.761 |
|        | 0.7    |       |     |        |          |          |         |          |     |     |       |     | 2.614   | 5.460   | 54.489 | 294.969 | 1005.291 |
|        | 0.8    |       |     |        |          |          |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        | 0.9    |       |     |        |          |          |         |          |     |     |       |     | 2.621   | 5.475   | 42.894 | 295.161 | 604.295  |
| 70     | 0.8    | 15    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 75  | 2.624   | 5.466   | 47.641 | 295.653 | 740.625  |
|        |        | 18    |     |        |          |          |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        | 21    |     |        |          |          |         |          |     |     |       |     | 2.611   | 5.469   | 48.287 | 294.482 | 815.990  |
|        |        | 24    |     |        |          |          |         |          |     |     |       |     | 2.605   | 5.470   | 48.61  | 294.029 | 852.566  |
| 70     | 0.8    | 18    | 1   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 75  | 2.556   | 5.419   | 48.038 | 289.461 | 775.366  |
|        |        |       | 2   |        |          |          |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       | 3   |        |          |          |         |          |     |     |       |     | 2.643   | 5.486   | 47.936 | 298.078 | 783.448  |
|        |        |       | 4   |        |          |          |         |          |     |     |       |     | 2.657   | 5.496   | 47.922 | 299.974 | 786.658  |
| 70     | 0.8    | 18    | 2   | 0.3    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 75  | 2.815   | 5.690   | 47.214 | 270.284 | 968.600  |
|        |        |       |     | 0.4    |          |          |         |          |     |     |       |     | 2.695   | 5.528   | 47.684 | 281.748 | 837.473  |
|        |        |       |     | 0.5    |          |          |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     | 0.6    |          |          |         |          |     |     |       |     | 2.697   | 5.534   | 47.969 | 332.430 | 868.651  |
| 70     | 0.8    | 18    | 2   | 0.5    | 60       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 75  | 2.443   | 5.078   | 48.011 | 223.097 | 622.478  |
|        |        |       |     |        | 70       |          |         |          |     |     |       |     | 2.549   | 5.315   | 47.985 | 258.017 | 726.789  |
|        |        |       |     |        | 80       |          |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        | 90       |          |         |          |     |     |       |     | 2.665   | 5.574   | 47.934 | 333.336 | 801.985  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.01     | 2       | 0.4      | 10  | 7   | 3     | 75  | 3.263   | 5.562   | 47.632 | 368.731 | 867.613  |
|        |        |       |     |        |          | 0.03     |         |          |     |     |       |     | 2.980   | 5.497   | 47.782 | 338.953 | 802.127  |
|        |        |       |     |        |          | 0.05     |         |          |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        |          | 0.07     |         |          |     |     |       |     | 2.162   | 5.394   | 48.345 | 250.679 | 690.339  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 1       | 0.4      | 10  | 7   | 3     | 75  | 3.355   | 5.625   | 47.568 | 377.521 | 931.259  |
|        |        |       |     |        |          |          | 2       |          |     |     |       |     | 3.263   | 5.562   | 47.632 | 368.731 | 867.613  |
|        |        |       |     |        |          |          | 3       |          |     |     |       |     | 2.955   | 5.469   | 47.783 | 332.500 | 759.390  |
|        |        |       |     |        |          |          | 4       |          |     |     |       |     | 2.025   | 5.246   | 48.653 | 245.444 | 423.521  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.2      | 10  | 7   | 3     | 75  | 2.469   | 5.459   | 48.066 | 260.212 | 774.835  |
|        |        |       |     |        |          |          |         | 0.4      |     |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        |          |          |         | 0.6      |     |     |       |     | 2.742   | 5.473   | 47.888 | 327.871 | 778.368  |
|        |        |       |     |        |          |          |         | 0.8      |     |     |       |     | 2.846   | 5.476   | 47.834 | 358.486 | 777.620  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 9   | 7   | 3     | 75  | 2.487   | 5.315   | 47.908 | 307.111 | 579.876  |
|        |        |       |     |        |          |          |         |          | 10  |     |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        |          |          |         |          | 11  |     |       |     | 2.686   | 5.636   | 48.011 | 307.009 | 919.575  |
|        |        |       |     |        |          |          |         |          | 12  |     |       |     | 2.646   | 5.786   | 48.162 | 289.707 | 986.162  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 6   | 3     | 75  | 2.654   | 5.729   | 47.768 | 297.003 | 1058.934 |
|        |        |       |     |        |          |          |         |          |     | 7   |       |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        |          |          |         |          |     | 8   |       |     | 2.483   | 5.283   | 48.167 | 292.619 | 476.508  |
|        |        |       |     |        |          |          |         |          |     | 9   |       |     | 2.372   | 5.570   | 48.289 | 289.663 | 221.045  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 2     | 75  | 2.687   | 5.461   | 47.882 | 305.174 | 896.673  |
|        |        |       |     |        |          |          |         |          |     |     | 3     |     | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        |          |          |         |          |     |     | 4     |     | 2.547   | 5.471   | 48.055 | 285.532 | 660.454  |
|        |        |       |     |        |          |          |         |          |     |     | 5     |     | 2.475   | 5.472   | 48.157 | 276.473 | 540.918  |
| 70     | 0.8    | 18    | 2   | 0.5    | 80       | 0.05     | 2       | 0.4      | 10  | 7   | 3     | 50  | 2.617   | 5.468   | 47.971 | 294.976 | 783.002  |
|        |        |       |     |        |          |          |         |          |     |     |       | 75  | 2.617   | 5.468   | 47.971 | 294.976 | 778.836  |
|        |        |       |     |        |          |          |         |          |     |     |       | 100 | 2.617   | 5.468   | 47.971 | 294.976 | 774.669  |
|        |        |       |     |        |          |          |         |          |     |     |       | 125 | 2.617   | 5.468   | 47.971 | 294.976 | 770.502  |

From Table 1, it is observed that as the values of the parameters  $n$ ,  $\phi$ ,  $\theta$ ,  $\gamma$  and  $p$  increase, the optimal values of the production downtime  $t_1^*$  and the production uptime  $t_3^*$  increase. The optimal values will decrease when the values of the parameters  $\tau$ ,  $\phi$ ,  $\alpha$ ,  $\beta$  and  $h$  increase.

The selling price  $s^*$  increases when the values of the parameters  $\nu$ ,  $\tau$ ,  $\phi$ ,  $\alpha$ ,  $\beta$ ,  $p$ ,  $h$  and  $c_2$  and  $A$  increase and decrease if the values of other parameters increase. The effect of the demand parameters is high as compared to other parameters.

Increasing the values of the parameters  $n$ ,  $\phi$ ,  $\theta$  and  $\gamma$  increases the optimal production quantity  $Q^*$ . When the holding cost  $h$  increases keeping other parameters fixed, the optimal value of  $Q$  is decreasing (production is discouraged).

The optimal value of total profit per unit time  $TP^*$  increases when the parameters  $\nu$ ,  $n$ ,  $\tau$ ,  $\theta$ ,  $\gamma$  and  $p$  increase. Increasing other parameters decreases the optimal profit. The effect of changes in the parameters  $\phi$ ,  $p$  and  $h$  on the optimal profit is very high as compared to the changes in other parameters. If  $\phi$  increases, then the demand decreases and hence revenue decreases, which ultimately decreases profit. The increase in the parameter values which increase the total demand or decrease the hazard function or increase the production rate will increase the profit. In this regard increasing  $\tau$ ,  $\nu$ ,  $\gamma$  and  $\theta$  increase the total profit and the increase in  $\phi$ ,  $\alpha$  and  $\beta$  decrease the total profit.

## 6. SENSITIVITY ANALYSIS

In order to study how the parameters affect the optimal solution sensitivity analysis is carried out taking the values  $\tau = 70$ ,  $\phi = 0.8$ ,  $\nu = 18$ ,  $n = 2$ ,  $\theta = 80$ ,  $\phi = 0.5$ ,  $\alpha = 0.05$ ,  $\beta = 2$ ,  $\gamma = 0.4$ ,  $p = 10$ ,  $h = 7$  and  $c_2 = 3$  in appropriate units. Sensitivity analysis is performed by decreasing and increasing these parameter values by 5%, 10% and 15%, first changing the value of one parameter at a time while keeping all the rest at their true values and then changing the values of all the parameters simultaneously. The result of this analysis is given in Table 2. The relationships between parameters, costs and the optimal values are shown in Figure 2.

From Table 2 we observe that the production downtime  $t_1^*$  is moderately sensitive to  $\alpha$  and  $p$  and slightly sensitive to the changes in other parameter values. The production uptime  $t_3^*$  is moderately sensitive to  $h$  and less sensitive to all other parameters. The optimal production quantity  $Q^*$  is highly sensitive to the production parameter  $\theta$  moderately sensitive to the production rate parameter  $\phi$  and deterioration distribution parameter  $\alpha$  and less sensitive to others. For example, decreasing  $\theta$  by 15% results in 16.921% decrease in the quantity produced.

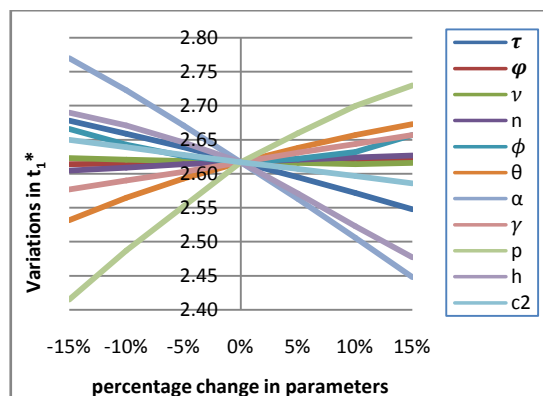
The optimal selling price and the optimal total profit are highly sensitive to the demand parameters  $\tau$  and  $\phi$ . A 15% decrease in  $\tau$  results in 13.733% and 30.381% decrease in  $s^*$  and  $TP^*$  respectively and a 15% decrease in  $\phi$  results in 16.674% and 36.004% increase in  $s^*$  and  $TP^*$  respectively. The total profit is highly sensitive to the changes in  $p$  and  $h$  also. For example profit will decrease by 40.586% if holding cost is increased by 15%.

**Table 2.** Sensitivity Analysis of the model without shortages

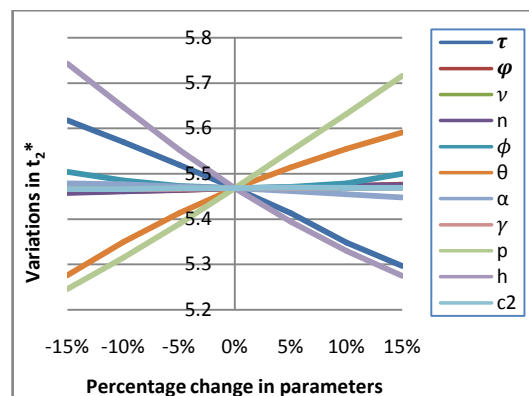
| Parameter Values | Variable | Percentage Change in the parameter Values |         |         |         |         |         |         |
|------------------|----------|---|---------|---------|---------|---------|---------|---------|
|                  |          | -15%                                      | -10%    | -5%     | 0%      | +5%     | +10%    | +15%    |
| $\tau = 70$      | $t_1^*$  | 2.678                                     | 2.659   | 2.639   | 2.617   | 2.595   | 2.572   | 2.548   |
|                  | $t_3^*$  | 5.618                                     | 5.570   | 5.520   | 5.468   | 5.413   | 5.348   | 5.297   |
|                  | $s^*$    | 41.383                                    | 43.578  | 45.771  | 47.971  | 50.157  | 52.346  | 54.539  |
|                  | $Q^*$    | 296.369                                   | 295.814 | 295.387 | 294.976 | 294.929 | 295.736 | 295.404 |
|                  | $TP^*$   | 542.215                                   | 622.833 | 702.015 | 778.836 | 845.849 | 903.844 | 968.795 |
|                  |          |   |         |         |         |         |         |         |
| $\phi = 0.8$     | $t_1^*$  | 2.614                                     | 2.615   | 2.616   | 2.617   | 2.619   | 2.620   | 2.621   |
|                  | $t_3^*$  | 5.459                                     | 5.462   | 5.464   | 5.468   | 5.470   | 5.473   | 5.476   |
|                  | $s^*$    | 56.013                                    | 53.034  | 50.365  | 47.971  | 45.791  | 43.816  | 42.012  |
|                  | $Q^*$    | 295.026                                   | 295.019 | 295.015 | 294.976 | 295.111 | 295.110 | 295.109 |
|                  | $TP^*$   | 1045.250                                  | 954.731 | 861.569 | 778.836 | 703.562 | 635.611 | 573.880 |
|                  |          |   |         |         |         |         |         |         |
| $\nu = 18$       | $t_1^*$  | 2.623                                     | 2.621   | 2.619   | 2.617   | 2.616   | 2.614   | 2.616   |
|                  | $t_3^*$  | 5.467                                     | 5.467   | 5.467   | 5.468   | 5.468   | 5.468   | 5.471   |
|                  | $s^*$    | 47.673                                    | 47.770  | 47.867  | 47.971  | 48.079  | 48.158  | 48.361  |
|                  | $Q^*$    | 294.524                                   | 295.341 | 295.159 | 294.976 | 294.927 | 294.787 | 294.980 |
|                  | $TP^*$   | 744.789                                   | 756.112 | 767.415 | 778.836 | 790.312 | 800.659 | 816.068 |
|                  |          |   |         |         |         |         |         |         |
| $n = 2$          | $t_1^*$  | 2.605                                     | 2.609   | 2.614   | 2.617   | 2.621   | 2.624   | 2.627   |
|                  | $t_3^*$  | 5.458                                     | 5.461   | 5.465   | 5.468   | 5.470   | 5.473   | 5.475   |
|                  | $s^*$    | 47.979                                    | 47.973  | 47.966  | 47.971  | 47.960  | 47.956  | 47.953  |
|                  | $Q^*$    | 293.740                                   | 294.154 | 294.681 | 294.976 | 295.452 | 295.762 | 296.109 |
|                  | $TP^*$   | 776.836                                   | 777.046 | 778.013 | 778.836 | 778.938 | 779.877 | 780.387 |
|                  |          |   |         |         |         |         |         |         |

Table 2. Sensitivity Analysis of the model without shortages

| Parameter Values | Variable | Percentage Change in the parameter Values |         |         |         |         |         |         |
|------------------|----------|---|---------|---------|---------|---------|---------|---------|
|                  |          | -15%                                      | -10%    | -5%     | 0%      | +5%     | +10%    | +15%    |
| $\phi=0.5$       | $t_1^*$  | 2.666                                     | 2.643   | 2.626   | 2.617   | 2.622   | 2.632   | 2.657   |
|                  | $t_3^*$  | 5.504                                     | 5.485   | 5.473   | 5.468   | 5.471   | 5.479   | 5.500   |
|                  | $s^*$    | 47.776                                    | 47.853  | 47.916  | 47.971  | 48.048  | 48.008  | 48.001  |
|                  | $Q^*$    | 284.193                                   | 287.102 | 290.575 | 294.976 | 301.268 | 308.693 | 316.814 |
|                  | $TP^*$   | 813.062                                   | 794.552 | 782.749 | 778.836 | 785.843 | 798.674 | 826.000 |
| $\theta=120$     | $t_1^*$  | 2.532                                     | 2.565   | 2.593   | 2.617   | 2.638   | 2.657   | 2.673   |
|                  | $t_3^*$  | 5.276                                     | 5.350   | 5.413   | 5.468   | 5.514   | 5.555   | 5.591   |
|                  | $s^*$    | 47.989                                    | 47.982  | 47.972  | 47.971  | 47.955  | 47.949  | 47.942  |
|                  | $Q^*$    | 245.063                                   | 261.317 | 280.026 | 294.976 | 310.181 | 325.636 | 341.116 |
|                  | $TP^*$   | 706.222                                   | 734.221 | 762.052 | 778.836 | 790.598 | 799.258 | 804.853 |
| $\alpha=0.05$    | $t_1^*$  | 2.770                                     | 2.723   | 2.671   | 2.617   | 2.563   | 2.506   | 2.448   |
|                  | $t_3^*$  | 5.479                                     | 5.477   | 5.472   | 5.468   | 5.462   | 5.455   | 5.448   |
|                  | $s^*$    | 47.878                                    | 47.936  | 47.935  | 47.971  | 48.000  | 48.040  | 48.079  |
|                  | $Q^*$    | 313.312                                   | 307.540 | 301.311 | 294.976 | 288.901 | 282.668 | 276.565 |
|                  | $TP^*$   | 787.789                                   | 786.957 | 782.823 | 778.836 | 773.003 | 766.202 | 758.149 |
| $\gamma=0.5$     | $t_1^*$  | 2.577                                     | 2.590   | 2.603   | 2.617   | 2.631   | 2.644   | 2.657   |
|                  | $t_3^*$  | 5.466                                     | 5.466   | 5.467   | 5.468   | 5.468   | 5.469   | 5.469   |
|                  | $s^*$    | 48.020                                    | 47.985  | 47.988  | 47.971  | 47.955  | 47.947  | 47.939  |
|                  | $Q^*$    | 284.935                                   | 288.259 | 291.542 | 294.976 | 298.451 | 301.764 | 305.118 |
|                  | $TP^*$   | 779.182                                   | 778.315 | 778.765 | 778.836 | 778.371 | 778.636 | 778.347 |
| $p=10$           | $t_1^*$  | 2.415                                     | 2.487   | 2.552   | 2.617   | 2.660   | 2.700   | 2.730   |
|                  | $t_3^*$  | 5.246                                     | 5.315   | 5.389   | 5.468   | 5.551   | 5.632   | 5.716   |
|                  | $s^*$    | 47.865                                    | 47.908  | 47.941  | 47.971  | 47.983  | 48.011  | 48.071  |
|                  | $Q^*$    | 288.226                                   | 290.377 | 293.209 | 294.976 | 294.808 | 291.481 | 287.784 |
|                  | $TP^*$   | 480.577                                   | 584.710 | 684.817 | 778.836 | 861.814 | 925.065 | 984.589 |
| $h=7$            | $t_1^*$  | 2.690                                     | 2.671   | 2.646   | 2.617   | 2.571   | 2.523   | 2.477   |
|                  | $t_3^*$  | 5.743                                     | 5.647   | 5.553   | 5.468   | 5.394   | 5.330   | 5.275   |
|                  | $s^*$    | 47.765                                    | 47.807  | 47.880  | 47.971  | 48.044  | 48.114  | 48.176  |
|                  | $Q^*$    | 277.100                                   | 283.969 | 289.466 | 294.976 | 294.807 | 293.781 | 292.476 |
|                  | $TP^*$   | 1068.753                                  | 989.300 | 888.718 | 778.836 | 668.555 | 562.178 | 462.740 |
| $c_2=3$          | $t_1^*$  | 2.650                                     | 2.639   | 2.628   | 2.617   | 2.607   | 2.597   | 2.586   |
|                  | $t_3^*$  | 5.466                                     | 5.466   | 5.468   | 5.468   | 5.470   | 5.469   | 5.469   |
|                  | $s^*$    | 47.943                                    | 47.938  | 47.951  | 47.971  | 48.065  | 47.990  | 48.004  |
|                  | $Q^*$    | 299.651                                   | 298.096 | 296.480 | 294.976 | 293.492 | 292.710 | 290.709 |
|                  | $TP^*$   | 832.394                                   | 814.026 | 797.114 | 778.836 | 763.866 | 743.189 | 725.578 |
| All Parameters   | $t_1^*$  | 2.790                                     | 2.721   | 2.664   | 2.617   | 2.580   | 2.550   | 2.513   |
|                  | $t_3^*$  | 5.497                                     | 5.481   | 5.471   | 5.468   | 5.470   | 5.478   | 5.492   |
|                  | $s^*$    | 47.107                                    | 47.400  | 47.685  | 47.971  | 48.234  | 48.496  | 48.777  |
|                  | $Q^*$    | 247.455                                   | 262.476 | 278.326 | 294.976 | 312.728 | 331.337 | 348.769 |
|                  | $TP^*$   | 804.366                                   | 799.565 | 790.035 | 778.836 | 767.881 | 762.170 | 763.436 |



(a)



(b)

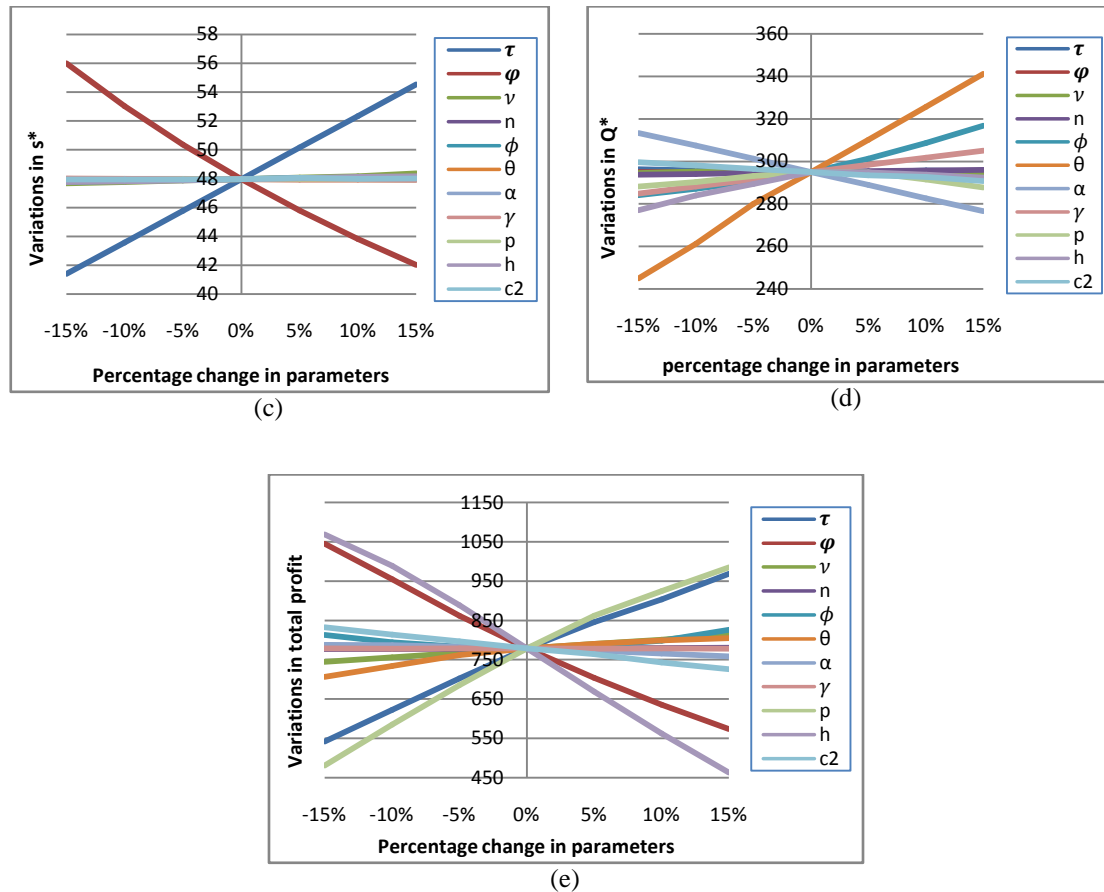


Fig. 2. Sensitivity analysis of the system variables with respect to the parameters and costs

## PARTICULAR CASE

In some other production processes shortages are not allowed. That is, the production starts as and when the inventory level reaches zero. For this sort of situations an inventory system for deteriorating items having stock dependent production rate and time and selling price dependent demand in which the lifetime of the commodity is random and follows three parameter Weibull distribution may be deduced as a limiting case of the model developed in section 3 above when the cost of incurring shortages increases indefinitely ( $c_2 \rightarrow \infty$ ) and  $t_2 \rightarrow T$ . Then in this system the inventory level changes during  $(0, \gamma)$  due to demand and production, during  $(\gamma, t_1)$  due to deterioration, demand and production and during  $(t_1, T)$  due to demand and deterioration.

## 7. CONCLUSIONS

Production inventory models play a dominant role in manufacturing and production industries like cement, food processing, petrochemical, pharmaceutical and paint manufacturing units. In this paper, a production inventory model for deteriorating items with stock dependent production rate, time and selling price dependent demand and Weibull decay has been developed and analyzed in the light of various parameters and costs and with the objective of maximizing the total system profit. The optimal production schedule is derived. The model was illustrated with numerical examples and sensitivity analysis of the model with respect to costs and parameters was also carried out. It can be concluded from the numerical examples and sensitivity analysis that the stock dependent nature of production rate is having significant influence on the optimal production quantity and profit rate and the demand parameters tremendously influence the optimal values of the unit selling price, production quantity and profit rate. This model also includes the exponential decay model as a particular case for specific values of the parameters. The proposed model can further be enriched by incorporating salvage of deteriorated units, inflation, quantity discount, and trade credits etc. It can also be extended to a multi-commodity model with constraints on budget, shelf space, etc., These models may also be formulated in fuzzy environments.

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**Source of support: Nil, Conflict of interest: None Declared**