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INVENTORY MODELS FOR DETERIORATING ITEMS WITH STOCK DEPENDENT PRODUCTION RATE AND WEIBULL DECAY

K. Srinivasa Rao
Department of Statistics, Andhra University, Visakhapatnam 530 003, India

Essey Kebede Muluneh[2*]
Department of Statistics, Bahir Dar University, Bahir Dar, Ethiopia

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ABSTRACT

In this paper a production inventory model for deteriorating items is developed and analyzed with the assumption that the production rate is dependent on stock on hand. It is further assumed that lifetime of the commodity is random and follows a three parameter Weibull distribution and demand rate is a function of both selling price and time. Using the differential equations the instantaneous state of inventory is derived. With suitable cost considerations the total cost function and profit rate function are obtained. By maximizing the profit rate function the optimal ordering policies are derived. Through numerical illustrations the sensitivity analysis is carried. It is observed that the demand rate parameters and stock dependent production rate parameters have significant influence on optimal production scheduling and profit rate.

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1. INTRODUCTION

Much work have been reported on inventory models for deteriorating items for varying assumptions on the demand rate, production (replenishment) rate and lifetime of the commodity. It is customary to consider that the replenishment in many inventory models is having finite or infinite rate. Urban [28], Teng and Yang [24], Dye, *et al.* [2], Sana and Chaudhuri [16] and others have studied inventory models having infinite production rate. Sana, *et al.* [17], Srinivasa Rao and Begum [22], Manna and Chiang [9], Uma Maheswara Rao, *et al.* [27], Sarkar and Moon [18] and others considered the finite rate of production. Two different rates of replenishment in one inventory system were also studied [12, 19].

In classical inventory models demand rate is assumed to be a constant. Widyadana and Wee [30] developed an economic production quantity (EPQ) model for deteriorating items where production, rework, deteriorating and demand rate are assumed constant. In reality, the demand rate of any product may vary with time or with price or with the instantaneous level of inventory displayed in a supermarket. Inventory problems involving time dependent demand patterns have received the attention of several researchers in recent years. Ritchie [13], Giri, *et al*. [4], Manna, *et al*. [8], Mahata and Goswami [7] and Skouri, *et al*. [20] are among those who studied inventory models for deteriorating items having time dependent demand. Roy and Chaudhuri [14] and Sana [15] have studied inventory models with demand rates depending on selling price of the item. It has been observed that for certain types of inventories, particularly consumer goods, heaps of stock will attract customers. Taking due account to this fact, Venkata Subbaiah, *et al.* [29], Dye, *et al.* [1], Panda, *et al.* [11], Mahata and Goswami [6], Lee and Dye [32] and others have developed inventory models where demand rate is a function of on-hand inventory. Srinivasa Rao, *et. al* [23] developed a production inventory system with demand rate a function of production quantity, Weibull decay and finite rate of production.

Inventory models for deteriorating items having multivariate demand functions were also studied by several authors. You [31] and Tsao and Sheen [26] have dealt with time and selling price dependent demand. Models for deteriorating items having stock level and selling price dependent demand rate were studied by Teng and Chang [25] and Khanra, *et al.* [5]. Pal, *et al.* [10] considered a single deteriorating item with the demand rate dependent on displayed stock level, selling price of an item and frequency of advertisement.

In all of the above models replenishment was assumed to be infinite or finite with a constant rate. Sridevi, *et al.* [21] developed and analyzed an inventory model with the assumption that the rate of production is random and follows a Weibull distribution and the demand is a function of selling price. Recently, Muluneh and Srinivasa Rao [3] develop an

Corresponding author: *Essey Kebede Muluneh[2*]*
Department of Statistics, Bahir Dar University, Bahir Dar, Ethiopia

inventory model for deteriorating items with the assumption that the production rate is dependent on stock on hand and demand is a power function of time. Very little work has been reported regarding EPQ models with stock dependent production rate and demand as a function of both selling price and time which is a common phenomenon in many production processes. Hence, in this paper we develop and analyze a production level inventory model for deteriorating items with the assumption that the replenishment is a function of on-hand inventory and demand is a function of both selling price and time.

Using the differential equations the instantaneous state of inventory is derived. With suitable cost considerations the total cost function and profit rate function are derived. By maximizing the profit rate function the optimal ordering policies of the model are obtained. The sensitivity of the model with respect to the parameters and costs is also discussed. This model includes some of the earlier models as particular cases for specific and limiting values of the parameters.

2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

The following notations are used to develop the model under study.

$I(t)$ = Inventory level at any time t
$R(t)$ = Production rate at any time t
$D(s,t)$= Demand rate at any time t
Q = Total production quantity
TC = Total cost of the system per unit time
TP = Total profit per unit time of the system
A = Set up cost of the item
p = Per unit production cost of the item
h = Inventory holding cost per unit per unit time
c_2 = Shortage cost per unit per unit time
T = Length of the cycle
(α, β, γ) = Deterioration rate parameters
(τ, φ, η, n) = Demand rate parameters
(ϕ, θ) = Production rate parameters

2.2. Assumptions

The model under study is based on the following assumptions.

i. Lifetime of the commodity is random and follows a three parameter Weibull distribution with probability density function of the form

$$f(t) = \alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}},\ \alpha,\beta > 0,\ t \geq \gamma$$

where, α is scale parameter, βα is shape parameter and γ is location parameter.

The instantaneous failure rate (hazard rate) at time t is therefore,

$$h(t) = \frac{f(t)}{1-F(t)} = \alpha\beta(t-\gamma)^{\beta-1}$$

ii. Demand is a function of time and selling price. Let n be the demand pattern index and s be the unit selling price of the item. Then demand rate is assumed to have the functional form $D(s,t) = \tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{1/n}}$, where, τ, φ and η are constants. This demand function includes several patterns for specific values of the parameters. For example if η=0, this demand is a function of selling price. If φ φ=0, this demand includes time dependent demand. If φ=0 and η=0 this demand includes constant rate.

iii. Production rate is assumed to be finite and a linear function of the instantaneous inventory level at time t, $I(t)$ i.e.,

$$R(t) = \begin{cases} \theta - \phi I(t), & 0 \leq t \leq t_1 \\ \theta, & t_3 \leq t \leq T \\ 0, & otherwise \end{cases}$$

where θ is a constant such that $\theta > 0$, ϕ is the stock-dependent production rate parameter, $0 \le \phi \le 1$. It is assumed that $R(t) \ge D(t)$ at any time where replenishment takes place. If ϕ=0, then it includes the finite rate of production.

iv. Shortages are allowed and completely backlogged.

v. There is no repair or replacement of deteriorated items.

vi. The planning horizon is infinite. Each cycle will have length T

vii. The inventory holding cost per unit per unit time h, the shortage cost per unit per unit time c_2, the unit production cost per unit time p and set up cost A per cycle are fixed and known,

3. MODEL FORMULATION

In this model the stock level for the item is initially zero. Then production starts at time $t = 0$ and continues adding items to stock until the on-hand inventory reaches its maximum level, Q_1 at time $t = t_1$. During the time $(0, \gamma)$ production will continuously satisfy the current demand and the excess production will be accumulated in stock. At time $t = \gamma$ deterioration of the item starts and stock is depleted by consumption and deterioration while production is continuously adding to it. At $t = t_1$ production is stopped and stock will be depleted by deterioration and demand until it reaches zero at time $t = t_2$. As demand is assumed to occur continuously, at this point shortage begin to accumulate until it reaches its maximum level of Q_2 at $t = t_3$. At this point production will resume meeting the current demand and clearing the backlog. Finally, shortages will be cleared at time $t = T$. Then the cycle will be repeated indefinitely.

The schematic diagram representing the inventory system is shown in Fig. 1.

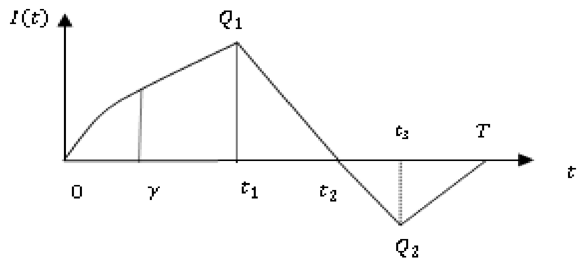


Fig. 1. Schematic diagram representing the inventory level of the system

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{dI(t)}{dt} = \{\theta - \phi I(t)\} - \left\{\tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}}\right\}, 0 \le t \le \gamma \tag{1}$$

$$\frac{dI(t)}{dt} = \{\theta - \phi I(t)\} - \{\alpha\beta(t-\gamma)^{\beta-1} I(t)\} - \left\{\tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}}\right\}, \gamma \le t \le t_1 \tag{2}$$

$$\frac{dI(t)}{dt} = -\{\alpha\beta(t-\gamma)^{\beta-1} I(t)\} - \left\{\tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}}\right\}, t_1 \le t \le t_2 \tag{3}$$

$$\frac{dI(t)}{dt} = -\left\{\tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}}\right\}, t_2 \le t \le t_3 \tag{4}$$

$$\frac{dI(t)}{dt} = \theta - \left\{\tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}}\right\}, t_3 \le t \le T \tag{5}$$

with boundary conditions;

$I(0) = 0$, $I(t_2) = 0$ and $I(T) = 0$

Solving the differential equations (1) to (5) after applying the boundary conditions given above, we get:

The instantaneous state of inventory at any time t in $(0,\gamma)$ is

$$I(t)=e^{-\phi t}\left\{(\theta-\tau+\varphi s)\int_0^t e^{\phi u}du-\frac{\eta}{nT^{1/n}}\int_0^t u^{(1/n)-1}e^{\phi u}du\right\},\ 0\le t\le\gamma \tag{6}$$

The instantaneous state of inventory at any time t in (γ,t_1) is

$$I(t)=e^{-\phi t-\alpha(t-\gamma)^\beta}\left\{(\theta-\tau+\varphi s)\int_0^\gamma e^{\phi u}du-\frac{\eta}{nT^{1/n}}\int_0^\gamma u^{(1/n)-1}e^{\phi u}du\right.$$
$$\left.+\int_\gamma^t(\theta-\tau+\varphi s)e^{\phi u+\alpha(u-\gamma)^\beta}du-\frac{\eta}{nT^{1/n}}\int_\gamma^t u^{(1/n)-1}e^{\phi u+\alpha(u-\gamma)^\beta}du\right\},\ \gamma\le t\le t_1 \tag{7}$$

The instantaneous state of inventory at any time t in (t_1,t_2) is

$$I(t)=e^{-\alpha(t-\gamma)^\beta}\left\{(\tau-\varphi s)\int_t^{t_2}e^{\alpha(u-\gamma)^\beta}du+\frac{\eta}{nT^{1/n}}\int_t^{t_2}u^{(1/n)-1}e^{\alpha(u-\gamma)^\beta}du\right\},\ t_1\le t\le t_2 \tag{8}$$

The instantaneous state of inventory at any time t in (t_2,t_3) is

$$I(t)=-(\tau-\varphi s)\int_{t_2}^t du-\frac{\eta}{nT^{1/n}}\int_{t_2}^t u^{(1/n)-1}du,\ t_2\le t\le t_3 \tag{9}$$

And the instantaneous state of inventory at any time t in (t_3,T) is

$$I(t)=-(\theta-\tau+\varphi s)\int_t^T du-\frac{\eta}{nT^{1/n}}\int_t^T u^{(1/n)-1}du,\ t_3\le t\le T \tag{10}$$

Since $I(t)$ is continuous at t_1 evaluating (7) and (8) and equating $I(t)$ at $t=t_1$ we get the equation.

$$e^{-\phi t_1-\alpha(t_1-\gamma)^\beta}\left\{(\theta-\tau+\varphi s)\int_0^\gamma e^{\phi u}du-\frac{\eta}{nT^{1/n}}\int_0^\gamma u^{(1/n)-1}e^{\phi u}du+\int_\gamma^{t_1}(\theta-\tau+\varphi s)e^{\phi u+\alpha(u-\gamma)^\beta}du\right.$$
$$\left.-\frac{\eta}{nT^{1/n}}\int_\gamma^{t_1}u^{(1/n)-1}e^{\phi u+\alpha(u-\gamma)^\beta}du\right\}=e^{-\alpha(t_1-\gamma)^\beta}\left\{(\tau-\varphi s)\int_{t_1}^{t_2}e^{\alpha(u-\gamma)^\beta}du+\frac{\eta}{nT^{1/n}}\int_t^{t_2}u^{(1/n)-1}e^{\alpha(u-\gamma)^\beta}du\right\} \tag{11}$$

Let β be a positive integer. Equation (11) is evaluated using the Taylor series approximation of the exponential function and ignoring terms of higher order and then taking the binomial expansion of $(u-\gamma)^\beta$. The equality in (11) is used to establish the relationship between t_1 and t_2. Also the maximum inventory level $Q_1=I(t_1)$ is

$$Q_1=e^{-\alpha(t_1-\gamma)^\beta}\left\{(\tau-\varphi s)\left(t_2-t_1+\frac{\alpha}{\beta+1}\left[(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right]\right)\right.$$
$$\left.+\frac{\eta}{T^{1/n}}\left(t_2^{1/n}-t_1^{1/n}+\alpha\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{t_2^{(1/n)+j}-t_1^{(1/n)+j}}{(jn+1)}\right)\right\} \tag{12}$$

Similarly, since $I(t)$ is continuous at t_3 evaluating $I(t)$ at $t=t_3$ and equating (9) and (10) we get the equation.

$$(\tau-\varphi s)(t_3-t_2)+\frac{\eta}{T^{1/n}}\left(t_3^{1/n}-t_2^{1/n}\right)=(\theta-\tau+\varphi s)(T-t_3)-\eta\left(1-\frac{t_3^{1/n}}{T^{1/n}}\right) \tag{13}$$

This equality can be used to establish the relationship between t_2 and t_3. Therefore we have,

$$t_3 = T - \frac{1}{\theta}(\tau - \varphi s)(T - t_2) - \frac{\eta}{\theta}\left(1 - \frac{t_2^{1/n}}{T^{1/n}}\right) = f(s, t_2), \ \ say \tag{14}$$

The maximum shortage level $Q_2 = I(t_3)$ is

$$Q_2 = -(\tau - \varphi s)(t_3 - t_2) + \frac{\eta}{T^{1/n}}\left(t_3^{1/n} - t_2^{1/n}\right) \tag{15}$$

Backlogged demand, $B(t)$ is

$$B(t) = \int_{t_2}^{t_3} \left\{ \tau - \varphi s + \frac{\eta t^{(1/n)-1}}{nT^{(1/n)}} \right\} dt$$

$$= (\tau - \varphi s)(t_3 - t_2) - \frac{\eta}{T^{1/n}}\left(t_3^{1/n} - t_2^{1/n}\right) \tag{16}$$

Stock loss due to deterioration at any time *t*, $L(t)$ is defined as

$$L(t) = \int_0^t R(t)dt - \int_0^t D(t)dt - I(t)$$

This implies

$$L(t) = \begin{cases} \theta t - \phi \int_0^{\gamma} I(t)d\,t - \phi \int_{\gamma}^{t} I(t)d\,t - (\tau - \varphi s)t - \dfrac{\eta t^{1/n}}{T^{1/n}} - I(t), & \gamma \le t \le t_1 \\ \theta t_1 - \phi \int_0^{\gamma} I(t)d\,t - \phi \int_{\gamma}^{t_1} I(t)d\,t - (\tau - \varphi s)t - \dfrac{\eta t^{1/n}}{T^{1/n}} - I(t), & t_1 \le t \le t_2 \\ 0, & elsewhere \end{cases} \tag{17}$$

Total Production in the cycle time $(0, T)$ is

$$Q = \int_0^{\gamma} R(t)dt + \int_{\gamma}^{t_1} R(t)dt + \int_{t_3}^{T} R(t)dt$$

$$= \theta(t_1 + T - t_3) - \phi \int_0^{\gamma} I(t)dt - \phi \int_{\gamma}^{t_1} I(t)dt$$

Substituting the values of I(t) in equations (6) and (7), using the Taylor series approximation of the exponential functions and ignoring terms of higher order, taking the binomial expansion of $(u-\gamma)^{\beta}$ and then integrating we get

$$Q = \theta\left\{ t_1 + \frac{1}{\theta}(\tau - \varphi s)(T - t_2) + \frac{\eta}{\theta}\left(\frac{t_2^{1/n}}{T^{1/n}} - 1\right)\right\}$$

$$-\phi\left\{ \frac{(\theta - \tau + \varphi s)}{\phi^2}\left(e^{-\phi\gamma} + \phi\gamma - 1\right) + \frac{\eta n \gamma^{(1/n)+1}}{(n+1)T^{1/n}}\left(\frac{\gamma^2\phi^2}{(3n+1)} + \frac{n\gamma\phi}{(2n+1)} - 1\right)\right\}$$

$$-\phi B\left\{ (t_1 - \gamma) - \frac{\phi}{2}(t_1^2 - \gamma^2) - \frac{\alpha}{\beta + 1}(t_1 - \gamma)^{\beta+1}\right\}$$

$$-\phi(\theta-\tau+\varphi s)\left\{\frac{1}{2}(t_1^2-\gamma^2)-\frac{\phi}{6}(t_1^3-\gamma^3)-\frac{\phi^2}{8}(t_1^4-\gamma^4)\right.$$
$$\left.-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\left(t_1+\frac{1}{2}\phi t_1^2\right)+\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}\right\}$$
$$-\frac{\eta n\phi}{T^{1/n}}\left\{\frac{t_1^{(1/n)+1}}{(n+1)}\left(\frac{\phi^2 t_1^2}{(3n+1)}+\frac{\phi n t_1}{(2n+1)}-1\right)-\frac{\gamma^{(1/n)+1}}{(n+1)}\left(\frac{\phi^2\gamma^2}{(3n+1)}+\frac{\phi n\gamma}{(2n+1)}-1\right)\right.$$
$$+\alpha n\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{j.\left(t_1^{(1/n)+j+1}-\gamma^{(1/n)+j+1}\right)}{(jn+1)(jn+n+1)}$$
$$+\alpha\phi\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{\left(t_1^{(1/n)+j+2}-\gamma^{(1/n)+j+2}\right)}{(jn+2n+1)}\left(\frac{1}{jn+1}+\frac{1}{n+1}\right)$$
$$\left.+\alpha^2\sum_{j=0}^{\beta}\sum_{k=0}^{\beta}\binom{\beta}{j}\binom{\beta}{k}(-\gamma)^{2\beta-j-k}\frac{\left(t_1^{(1/n)+j+k+1}-\gamma^{(1/n)+j+k+1}\right)}{(jn+1)(jn+kn+n+1)}\right\} \tag{18}$$

where,

$$B=\frac{(\theta-\tau+\varphi s)}{\phi}\left(e^{\phi\gamma}-\frac{1}{2}\gamma^2\phi^2-\phi\gamma-1\right)+\frac{\eta\alpha}{T^{1/n}}\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{\gamma^{(1/n)+j}}{jn+1} \tag{19}$$

Total cost is the sum of setup cost per unit time, the production cost per unit time, inventory holding cost per unit time and the shortage cost per unit time. Let $TC(t_1,t_2,s)$ be the total cost per unit time. Therefore we have:

$$TC(t_1,t_2,s)=\frac{A}{T}+\frac{pQ}{T}+\frac{h}{T}\left\{\int_0^{\gamma}I(t)dt+\int_{\gamma}^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt\right\}+\frac{c_2}{T}\left\{\int_{t_2}^{t_3}-I(t)dt+\int_{t_3}^{T}-I(t)dt\right\} \tag{20}$$

Substituting the expressions for I(t) from equations (6) to (10) in equation (20), using the Taylor series expansion of the exponential functions and neglecting higher order terms, using binomial expansion of $(u-\gamma)^{\beta}$ and then integrating we get

$$TC(t_1,t_2,s)=\frac{A}{T}+\frac{p\theta}{T}\left\{t_1+\frac{1}{\theta}(\tau-\varphi s)(T-t_2)+\frac{\eta}{\theta}\left(\frac{t_2^{1/n}}{T^{1/n}}-1\right)\right\}$$
$$+\frac{h-p\phi}{T}\left\{\frac{(\theta-\tau+\varphi s)}{\phi^2}\left(e^{-\phi\gamma}+\phi\gamma-1\right)+\frac{\eta n\gamma^{(1/n)+1}}{(n+1)T^{1/n}}\left(\frac{\gamma^2\phi^2}{(3n+1)}+\frac{n\gamma\phi}{(2n+1)}-1\right)\right\}$$
$$+\frac{(h-p\phi)}{T}B\left\{(t_1-\gamma)-\frac{\phi}{2}(t_1^2-\gamma^2)-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right\}$$
$$+\frac{h-p\phi}{T}(\theta-\tau+\varphi s)\left\{\frac{1}{2}(t_1^2-\gamma^2)-\frac{\phi}{6}(t_1^3-\gamma^3)-\frac{\phi^2}{8}(t_1^4-\gamma^4)\right.$$
$$\left.-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\left(t_1+\frac{1}{2}\phi t_1^2\right)+\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}\right\}$$
$$+\frac{h(\tau-\varphi s)}{T}\left\{\left[t_2+\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right]\left[(t_2-t_1)-\frac{\alpha\left\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{(\beta+1)}\right]\right.$$

$$-\frac{1}{2}\left(t_2{}^2-t_1{}^2\right)+\frac{\alpha\left\{t_2(t_2-\gamma)^{\beta+1}-t_1(t_1-\gamma)^{\beta+1}\right\}}{(\beta+1)}$$

$$\left.-\frac{2\alpha\left\{(t_2-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right\}}{(\beta+1)(\beta+2)}+\frac{\alpha^2\left\{(t_2-\gamma)^{2(\beta+1)}-(t_1-\gamma)^{2(\beta+1)}\right\}}{2(\beta+1)^2}\right\}$$

$$+\frac{(h-p\phi)\eta n}{T^{(1/n)+1}}\left\{\frac{t_1{}^{(1/n)+1}}{(n+1)}\left(\frac{\phi^2 t_1{}^2}{(3n+1)}+\frac{\phi n t_1}{(2n+1)}-1\right)-\frac{\gamma^{(1/n)+1}}{(n+1)}\left(\frac{\phi^2\gamma^2}{(3n+1)}+\frac{\phi n\gamma}{(2n+1)}-1\right)\right.$$

$$+\alpha n\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\,\frac{j.\left(t_1{}^{(1/n)+j+1}-\gamma^{(1/n)+j+1}\right)}{(jn+1)(jn+n+1)}$$

$$+\alpha\phi\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\,\frac{\left(t_1{}^{(1/n)+j+2}-\gamma^{(1/n)+j+2}\right)}{(jn+2n+1)}\left(\frac{1}{jn+1}+\frac{1}{n+1}\right)$$

$$\left.+\alpha^2\sum_{j=0}^{\beta}\sum_{k=0}^{\beta}\binom{\beta}{j}\binom{\beta}{k}(-\gamma)^{2\beta-j-k}\,\frac{\left(t_1{}^{(1/n)+j+k+1}-\gamma^{(1/n)+j+k+1}\right)}{(jn+1)(jn+kn+n+1)}\right\}$$

$$+\frac{h\eta}{T^{(1/n)+1}}\left\{\left[t_2{}^{1/n}+\alpha\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\,\frac{t_2{}^{(1/n)+j}}{(jn+1)}\right]\left[\left(t_2-t_1\right)-\frac{\alpha\left\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{(\beta+1)}\right]\right.$$

$$-\frac{n}{n+1}\left(t_2{}^{(1/n)+1}-t_1{}^{(1/n)+1}\right)+\alpha n^2\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\,\frac{j.\left(t_2{}^{(1/n)+j+1}-t_1{}^{(1/n)+j+1}\right)}{(jn+1)(jn+n+1)}$$

$$\left.+\alpha^2 n\sum_{j=0}^{\beta}\sum_{k=0}^{\beta}\binom{\beta}{j}\binom{\beta}{k}(-\gamma)^{2\beta-j-k}\,\frac{\left(t_2{}^{(1/n)+j+k+1}-t_1{}^{(1/n)+j+k+1}\right)}{(jn+1)(jn+kn+n+1)}\right\}$$

$$+\frac{c_2}{T}\left\{\frac{1}{2}(\tau-\varphi s)(T-t_2)(2f(t_2,s)-T-t_2)+\frac{1}{2}\theta\left(T-f(t_2,s)\right)^2\right.$$

$$\left.+\eta\left(f(t_2,s)-\frac{T}{n+1}\right)+\frac{\eta t_2{}^{1/n}}{T^{1/n}}\left(\frac{t_2}{n+1}-f(t_2,s)\right)\right\}\qquad(21)$$

Let $TR(t_1,t_2,s)$ be the total revenue per unit time. Then

$$TR(t_1,t_2,s)=\frac{s}{T}\left\{\int_0^T(\tau-\varphi s)dt+\frac{\eta}{nT^{1/n}}\int_0^T t^{(1/n)-1}dt\right\}$$

This implies

$$TR(t_1,t_2,s)=s(\tau-\varphi s)+\frac{\eta s}{T}\qquad(22)$$

Let $TP(t_1,t_2,s)$ be the total profit per unit time. Therefore we have

$$TP(t_1,t_2,s)=TR(t_1,t_2,s)-TC(t_1,t_2,s)\qquad(23)$$

where, $TC(t_1,t_2,s)$ is as defined in (21)

4. OPTIMAL POLICIES OF THE MODEL

The problem here is to find the optimal values of production down-time t_1, time to begin shortages, t_2 and selling price s that maximize the total profit $TP(t_1,t_2,s)$ over $(0,T)$. To obtain these values we differentiate $TP(t_1,t_2,s)$ in equation (23) with respect to t_1, t_2 and s and equate the resulting equations to zero.

Differentiating $TP(t_1,t_2,s)$ with respect to t_1 and equating to zero we get

$$
\begin{aligned}
&\frac{p\theta}{T}+\frac{(h-p\phi)}{T}B\left\{1-\phi t_1-\alpha(t_1-\gamma)^{\beta}\right\}\\
&+\frac{h-p\phi}{T}\left\{(\theta-\tau+\varphi s)\left\{t_1-\frac{1}{2}\phi t_1^{2}-\frac{1}{2}\phi^{2}t_1^{3}-\alpha(t_1-\gamma)^{\beta}\left(t_1+\frac{1}{2}\phi t_1^{2}\right)\right.\right.\\
&\left.\left.+\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}(1-\phi t_1)-\frac{\alpha^{2}(t_1-\gamma)^{2\beta+1}}{(\beta+1)}\right\}+\frac{\eta t_1^{(1/n)}}{T^{(1/n)}}\left(\frac{\phi^{2}t_1^{2}}{n+1}+\frac{\phi n t_1}{n+1}-1\right)\right.\\
&+\frac{\eta\alpha n}{T^{1/n}}\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{j.t_1^{(1/n)+j}}{(jn+1)}+\frac{\eta\alpha\phi}{T^{1/n}}\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}t_1^{(1/n)+j+1}\left(\frac{1}{jn+1}+\frac{1}{n+1}\right)\\
&\left.+\frac{\eta\alpha^{2}}{T^{1/n}}\sum_{j=0}^{\beta}\sum_{k=0}^{\beta}\binom{\beta}{j}\binom{\beta}{k}(-\gamma)^{2\beta-j-k}\frac{t_1^{(1/n)+j+k}}{(jn+1)}\right\}\\
&+\frac{h\eta}{T^{(1/n)+1}}\left\{\left[t_2^{1/n}+\alpha\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{t_2^{(1/n)+j}}{(jn+1)}\right]\left[\alpha(t_1-\gamma)^{\beta+1}-1\right]+t_1^{1/n}\right.\\
&\left.-\alpha n\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{j.t_1^{(1/n)+j}}{(jn+1)}-\alpha^{2}\sum_{j=0}^{\beta}\sum_{k=0}^{\beta}\binom{\beta}{j}\binom{\beta}{k}(-\gamma)^{2\beta-j-k}\frac{t_1^{(1/n)+j+k}}{(jn+1)}\right\}\\
&+\frac{h(\tau-\varphi s)}{T}\left\{\left[t_2+\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right]\left[\alpha(t_1-\gamma)^{\beta}-1\right]+t_1\left[1-\alpha(t_1-\gamma)^{\beta}\right]\right.\\
&\left.+\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}-\frac{\alpha^{2}(t_1-\gamma)^{2\beta+1}}{\beta+1}\right\}=0
\end{aligned}
\tag{24}
$$

Differentiating $TP(t_1,t_2,s)$ with respect to t_2 and equating to zero we get

$$
\begin{aligned}
&-\frac{p(\tau-\varphi s)}{T}-\frac{p\eta t_2^{(1/n)-1}}{nT^{(1/n)+1}}+\frac{h(\tau-\varphi s)}{T}\left\{\frac{\alpha^{2}(t_2-\gamma)^{2\beta+1}}{(\beta+1)}-\frac{\alpha(t_2-\gamma)^{\beta+1}}{(\beta+1)}\right\}\\
&+\frac{h(\tau-\varphi s)}{T}\left\{\left[1+\alpha(t_2-\gamma)^{\beta}\right]\left[(t_2-t_1)-\frac{\alpha\left\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{\beta+1}\right]\right.\\
&\left.+\left[\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}+t_2-1\right]\left[1-\alpha(t_2-\gamma)^{\beta}\right]\right\}\\
&+\frac{\eta}{T^{(1/n)+1}}\left\{\left[\frac{1}{n}t_2^{(1/n)-1}+\frac{\alpha}{n}\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}t_2^{(1/n)+j-1}\right]\left[(t_2-t_1)-\frac{\alpha\left\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{\beta+1}\right]\right.\\
&+\left[t_2^{(1/n)}+\alpha\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\frac{t_2^{(1/n)+j}}{(jn+1)}\right]\left[1-\alpha(t_2-\gamma)^{\beta}\right]-t_2^{(1/n)}
\end{aligned}
$$

$$\alpha n\sum_{j=0}^{\beta}\binom{\beta}{j}(-\gamma)^{\beta-j}\,\frac{j.t_2^{(1/n)+j}}{(jn+1)}+\alpha^2\sum_{j=0}^{\beta}\sum_{k=0}^{\beta}\binom{\beta}{j}\binom{\beta}{k}(-\gamma)^{2\beta-j-k}\,\frac{t_2^{(1/n)+j+k}}{(jn+1)}\Bigg\}$$

$$+\frac{c_2}{T}\left\{(\tau-\varphi s)\left[(T-t_2)f'(t_2,s)+t_2-f(t_2,s)\right]-\theta\left(T-f(t_2,s)\right)f'(t_2,s)\right.$$

$$\left.+\frac{\eta t_2^{(1/n)-1}}{nT^{1/n}}\left(t_2-f(t_2,s)\right)+\eta f'(t_2,s)\left(1-\frac{t_2^{(1/n)}}{T^{(1/n)}}\right)\right\}=0 \qquad (25)$$

Differentiating $TP(t_1,t_2,s)$ with respect to s and equating to zero we get

$$\tau-2\varphi s+\frac{\eta}{T}+\frac{\varphi p(T-t_2)}{T}-\frac{\varphi(h-p\phi)}{\phi^2 T}\left(e^{-\phi\gamma}+\phi\gamma-1\right)$$

$$-\frac{\varphi(h-p\phi)}{\phi T}\left\{e^{\phi\gamma}-\frac{1}{2}\phi^2\gamma^2-\phi\gamma-1\right\}\left\{(t_1-\gamma)-\frac{1}{2}\phi(t_1^2-\gamma^2)-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right\}$$

$$-\frac{\varphi(h-p\phi)}{T}\left\{\frac{1}{2}(t_1^2-\gamma^2)-\frac{\phi}{6}(t_1^3-\gamma^3)-\frac{\phi^2}{8}(t_1^4-\gamma^4)\right.$$

$$\left.-\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}\left(t_1+\frac{1}{2}\phi t_1^2\right)+\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}\right\}$$

$$+\frac{h\varphi}{T}\left\{\left[t_2+\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right]\left[(t_2-t_1)-\frac{\alpha\left\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{(\beta+1)}\right]\right.$$

$$-\frac{1}{2}\left(t_2^2-t_1^2\right)+\frac{\alpha\left[t_2(t_2-\gamma)^{\beta+1}-t_1(t_1-\gamma)^{\beta+1}\right]}{(\beta+1)}$$

$$\left.-\frac{2\alpha\left[(t_2-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right]}{(\beta+1)(\beta+2)}+\frac{\alpha^2\left[(t_2-\gamma)^{2(\beta+1)}-(t_1-\gamma)^{2(\beta+1)}\right]}{2(\beta+1)^2}\right\}$$

$$-\frac{c_2}{T}\left\{\varphi(T-t_2)^2\left(\frac{(\tau-\varphi s)}{\theta}-\frac{1}{2}\right)+\frac{\varphi\eta(T-t_2)}{\theta}\left(1-\frac{t_2^{(1/n)}}{T^{(1/n)}}\right)=0\right. \qquad (26)$$

where,

$$f'(t_2,s)=+\frac{\partial f(t_2,s)}{\partial t_2}=\frac{(\tau-\varphi s)}{\theta}+\frac{\eta t_2^{(1/n)-1}}{n\theta T^{(1/n)}}$$

The solutions t_1^*, t_2^* and s^* of t_1, t_2 and s respectively are obtained by solving the equations (24), (25) and (26) using numerical methods. Substituting these optimal values of t_1, t_2 and s in to equations (14), (12), (15), (18), (21) and (23) we obtain the optimal values for t_3, Q_1, Q_2, Q, $TC(t_1,t_2,s)$ and $TP(t_1,t_2,s)$ respectively.

5. NUMERICAL ILLUSTRATION

Consider the case of deriving an EPQ, production downtime and production uptime for a petrochemical industry viz., fertilizer manufacturing plant. Here the product is of a deteriorating type and has a random lifetime which is assumed to follow a three parameter Weibull distribution. Records and discussions held with the production and marketing personnel suggested the values of various parameters. The deterioration parameters α, β and γ are estimated to vary over 0.01 to 0.07, 1 to 4 and 0.2 to 0.8 respectively. Stock dependent production rate parameters ϕ and θ vary over 0.3 to 0.6 and 60 to 90 respectively and demand parameters τ, φ, η and n vary over 50 to 80, 0.6 to 0.9, 15 to 24 and 1 to 4 respectively. Let the values for other parameters be p=10, h=7, c_2=3 and A=75 in appropriate units. The cycle length is taken to be T=6 units and the values of the parameters are varied to observe the trend in the optimal policies. The optimal values of production downtime (t_1), production uptime (t_3), selling price (s) production quantity (Q) and total profit (TP) are obtained and presented in Table 1.

Table 1: Optimal values of the decision variables for different values of parameters

τ	φ	ν	n	ϕ	θ	α	β	γ	p	h	c_2	A	t_1^*	t_3^*	s^*	Q^*	TP^*
50	0.8	18	2	0.5	80	0.05	2	0.4	10	7	3	75	2.724	5.740	35.425	298.427	321.796
60													2.676	5.611	41.702	296.389	553.730
70													2.617	5.468	47.971	294.976	778.836
80													2.551	5.306	54.226	293.260	971.056
70	0.6	18	2	0.5	80	0.05	2	0.4	10	7	3	75	2.612	5.453	63.165	295.040	1311.761
	0.7												2.614	5.460	54.489	294.969	1005.291
	0.8												2.617	5.468	47.971	294.976	778.836
	0.9												2.621	5.475	42.894	295.161	604.295
70	0.8	15	2	0.5	80	0.05	2	0.4	10	7	3	75	2.624	5.466	47.641	295.653	740.625
		18											2.617	5.468	47.971	294.976	778.836
		21											2.611	5.469	48.287	294.482	815.990
		24											2.605	5.470	48.61	294.029	852.566
70	0.8	18	1	0.5	80	0.05	2	0.4	10	7	3	75	2.556	5.419	48.038	289.461	775.366
			2										2.617	5.468	47.971	294.976	778.836
			3										2.643	5.486	47.936	298.078	783.448
			4										2.657	5.496	47.922	299.974	786.658
70	0.8	18	2	0.3	80	0.05	2	0.4	10	7	3	75	2.815	5.690	47.214	270.284	968.600
				0.4									2.695	5.528	47.684	281.748	837.473
				0.5									2.617	5.468	47.971	294.976	778.836
				0.6									2.697	5.534	47.969	332.430	868.651
70	0.8	18	2	0.5	60	0.05	2	0.4	10	7	3	75	2.443	5.078	48.011	223.097	622.478
					70								2.549	5.315	47.985	258.017	726.789
					80								2.617	5.468	47.971	294.976	778.836
					90								2.665	5.574	47.934	333.336	801.985
70	0.8	18	2	0.5	80	0.01	2	0.4	10	7	3	75	3.263	5.562	47.632	368.731	867.613
						0.03							2.980	5.497	47.782	338.953	802.127
						0.05							2.617	5.468	47.971	294.976	778.836
						0.07							2.162	5.394	48.345	250.679	690.339
70	0.8	18	2	0.5	80	0.05	1	0.4	10	7	3	75	3.355	5.625	47.568	377.521	931.259
							2						3.263	5.562	47.632	368.731	867.613
							3						2.955	5.469	47.783	332.500	759.390
							4						2.025	5.246	48.653	245.444	423.521
70	0.8	18	2	0.5	80	0.05	2	0.2	10	7	3	75	2.469	5.459	48.066	260.212	774.835
								0.4					2.617	5.468	47.971	294.976	778.836
								0.6					2.742	5.473	47.888	327.871	778.368
								0.8					2.846	5.476	47.834	358.486	777.620
70	0.8	18	2	0.5	80	0.05	2	0.4	9	7	3	75	2.487	5.315	47.908	307.111	579.876
									10				2.617	5.468	47.971	294.976	778.836
									11				2.686	5.636	48.011	307.009	919.575
									12				2.646	5.786	48.162	289.707	986.162
70	0.8	18	2	0.5	80	0.05	2	0.4	10	6	3	75	2.654	5.729	47.768	297.003	1058.934
										7			2.617	5.468	47.971	294.976	778.836
										8			2.483	5.283	48.167	292.619	476.508
										9			2.372	5.570	48.289	289.663	221.045
70	0.8	18	2	0.5	80	0.05	2	0.4	10	7	2	75	2.687	5.461	47.882	305.174	896.673
											3		2.617	5.468	47.971	294.976	778.836
											4		2.547	5.471	48.055	285.532	660.454
											5		2.475	5.472	48.157	276.473	540.918
70	0.8	18	2	0.5	80	0.05	2	0.4	10	7	3	50	2.617	5.468	47.971	294.976	783.002
												75	2.617	5.468	47.971	294.976	778.836
												100	2.617	5.468	47.971	294.976	774.669
												125	2.617	5.468	47.971	294.976	770.502

From Table 1, it is observed that as the values of the parameters n, φ, θ, γ and p increase, the optimal values of the production downtime $t_1{}^*$ and the production uptime $t_3{}^*$ increase. The optimal values will decrease when the values of the parameters τ, ϕ, α, β and h increase.

The selling price s^* increases when the values of the parameters ν, τ, ϕ, α, β, p, h and c_2 and A increase and decrease if the values of other parameters increase. The effect of the demand parameters is high as compared to other parameters.

Increasing the values of the parameters n, ϕ, θ and γ increases the optimal production quantity Q^*. When the holding cost h increases keeping other parameters fixed, the optimal value of Q is decreasing (production is discouraged).

The optimal value of total profit per unit time TP^* increases when the parameters ν, n, τ, θ, γ and p increase. Increasing other parameters decreases the optimal profit. The effect of changes in the parameters φ, p and h on the optimal profit is very high as compared to the changes in other parameters. If φ increases, then the demand decreases and hence revenue decreases, which ultimately decreases profit. The increase in the parameter values which increase the total demand or decrease the hazard function or increase the production rate will increase the profit. In this regard increasing τ, ν, γ and θ increase the total profit and the increase in φ, ϕ, α and β decrease the total profit.

6. SENSITIVITY ANALYSIS

In order to study how the parameters affect the optimal solution sensitivity analysis is carried out taking the values $\tau = 70$, $\varphi = 0.8$, $\nu = 18$, $n = 2$, $\theta = 80$, $\phi = 0.5$, $\alpha = 0.05$, $\beta = 2$, $\gamma = 0.4$, $p = 10$, $h = 7$ and $c_2 = 3$ in appropriate units. Sensitivity analysis is performed by decreasing and increasing these parameter values by 5%, 10% and 15%, first changing the value of one parameter at a time while keeping all the rest at their true values and then changing the values of all the parameters simultaneously. The result of this analysis is given in Table 2. The relationships between parameters, costs and the optimal values are shown in Figure 2.

From Table 2 we observe that the production downtime $t_1{}^*$ is moderately sensitive to α and p and slightly sensitive to the changes in other parameter values. The production uptime $t_3{}^*$ is moderately sensitive to h and less sensitive to all other parameters. The optimal production quantity Q^* is highly sensitive to the production parameter θ moderately sensitive to the production rate parameter ϕ and deterioration distribution parameter α and less sensitive to others. For example, decreasing θ by 15% results in 16.921% decrease in the quantity produced.

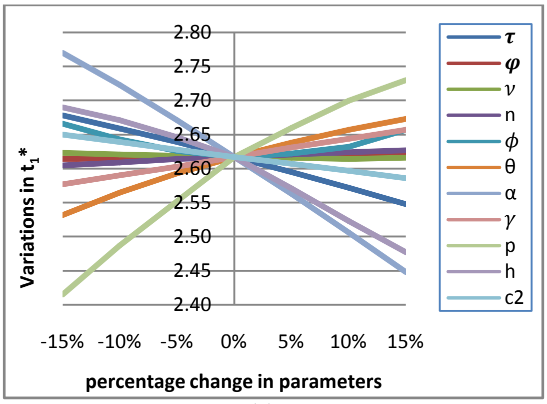
The optimal selling price and the optimal total profit are highly sensitive to the demand parameters τ and φ. A 15% decrease in τ results in 13.733% and 30.381% decrease in $s^*$and TP^* respectively and a 15% decrease in φ results in 16.674% and 36.004% increase in s^* and TP^* respectively. The total profit is highly sensitive to the changes in p and h also. For example profit will decrease by 40.586% if holding cost is increased by 15%.

Table 2. Sensitivity Analysis of the model without shortages

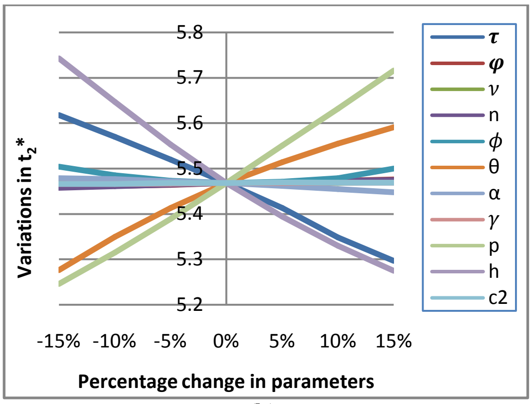
Parameter Values	Variable	Percentage Change in the parameter Values						
		-15%	**-10%**	**-5%**	**0%**	**+5%**	**+10%**	**+15%**
τ= 70	t_1^*	2.678	2.659	2.639	2.617	2.595	2.572	2.548
	t_3^*	5.618	5.570	5.520	5.468	5.413	5.348	5.297
	s^*	41.383	43.578	45.771	47.971	50.157	52.346	54.539
	Q^*	296.369	295.814	295.387	294.976	294.929	295.736	295.404
	TP^*	542.215	622.833	702.015	778.836	845.849	903.844	968.795
φ= 0.8	t_1^*	2.614	2.615	2.616	2.617	2.619	2.620	2.621
	t_3^*	5.459	5.462	5.464	5.468	5.470	5.473	5.476
	s^*	56.013	53.034	50.365	47.971	45.791	43.816	42.012
	Q^*	295.026	295.019	295.015	294.976	295.111	295.110	295.109
	TP^*	1045.250	954.731	861.569	778.836	703.562	635.611	573.880
ν=18	t_1^*	2.623	2.621	2.619	2.617	2.616	2.614	2.616
	t_3^*	5.467	5.467	5.467	5.468	5.468	5.468	5.471
	s^*	47.673	47.770	47.867	47.971	48.079	48.158	48.361
	Q^*	294.524	295.341	295.159	294.976	294.927	294.787	294.980
	TP^*	744.789	756.112	767.415	778.836	790.312	800.659	816.068
n =2	t_1^*	2.605	2.609	2.614	2.617	2.621	2.624	2.627
	t_3^*	5.458	5.461	5.465	5.468	5.470	5.473	5.475
	s^*	47.979	47.973	47.966	47.971	47.960	47.956	47.953
	Q^*	293.740	294.154	294.681	294.976	295.452	295.762	296.109
	TP^*	776.836	777.046	778.013	778.836	778.938	779.877	780.387

Table 2. Sensitivity Analysis of the model without shortages

Parameter Values	Variable	Percentage Change in the parameter Values						
		-15%	**-10%**	**-5%**	**0%**	**+5%**	**+10%**	**+15%**
ϕ=0.5	t_1^*	2.666	2.643	2.626	2.617	2.622	2.632	2.657
	t_3^*	5.504	5.485	5.473	5.468	5.471	5.479	5.500
	s^*	47.776	47.853	47.916	47.971	48.048	48.008	48.001
	Q^*	284.193	287.102	290.575	294.976	301.268	308.693	316.814
	TP^*	813.062	794.552	782.749	778.836	785.843	798.674	826.000
θ=120	t_1^*	2.532	2.565	2.593	2.617	2.638	2.657	2.673
	t_3^*	5.276	5.350	5.413	5.468	5.514	5.555	5.591
	s^*	47.989	47.982	47.972	47.971	47.955	47.949	47.942
	Q^*	245.063	261.317	280.026	294.976	310.181	325.636	341.116
	TP^*	706.222	734.221	762.052	778.836	790.598	799.258	804.853
α=0.05	t_1^*	2.770	2.723	2.671	2.617	2.563	2.506	2.448
	t_3^*	5.479	5.477	5.472	5.468	5.462	5.455	5.448
	s^*	47.878	47.936	47.935	47.971	48.000	48.040	48.079
	Q^*	313.312	307.540	301.311	294.976	288.901	282.668	276.565
	TP^*	787.789	786.957	782.823	778.836	773.003	766.202	758.149
γ=0.5	t_1^*	2.577	2.590	2.603	2.617	2.631	2.644	2.657
	t_3^*	5.466	5.466	5.467	5.468	5.468	5.469	5.469
	s^*	48.020	47.985	47.988	47.971	47.955	47.947	47.939
	Q^*	284.935	288.259	291.542	294.976	298.451	301.764	305.118
	TP^*	779.182	778.315	778.765	778.836	778.371	778.636	778.347
p=10	t_1^*	2.415	2.487	2.552	2.617	2.660	2.700	2.730
	t_3^*	5.246	5.315	5.389	5.468	5.551	5.632	5.716
	s^*	47.865	47.908	47.941	47.971	47.983	48.011	48.071
	Q^*	288.226	290.377	293.209	294.976	294.808	291.481	287.784
	TP^*	480.577	584.710	684.817	778.836	861.814	925.065	984.589
h=7	t_1^*	2.690	2.671	2.646	2.617	2.571	2.523	2.477
	t_3^*	5.743	5.647	5.553	5.468	5.394	5.330	5.275
	s^*	47.765	47.807	47.880	47.971	48.044	48.114	48.176
	Q^*	277.100	283.969	289.466	294.976	294.807	293.781	292.476
	TP^*	1068.753	989.300	888.718	778.836	668.555	562.178	462.740
c_2=3	t_1^*	2.650	2.639	2.628	2.617	2.607	2.597	2.586
	t_3^*	5.466	5.466	5.468	5.468	5.470	5.469	5.469
	s^*	47.943	47.938	47.951	47.971	48.065	47.990	48.004
	Q^*	299.651	298.096	296.480	294.976	293.492	292.710	290.709
	TP^*	832.394	814.026	797.114	778.836	763.866	743.189	725.578
All Parameters	t_1^*	2.790	2.721	2.664	2.617	2.580	2.550	2.513
	t_3^*	5.497	5.481	5.471	5.468	5.470	5.478	5.492
	s^*	47.107	47.400	47.685	47.971	48.234	48.496	48.777
	Q^*	247.455	262.476	278.326	294.976	312.728	331.337	348.769
	TP^*	804.366	799.565	790.035	778.836	767.881	762.170	763.436



(a)



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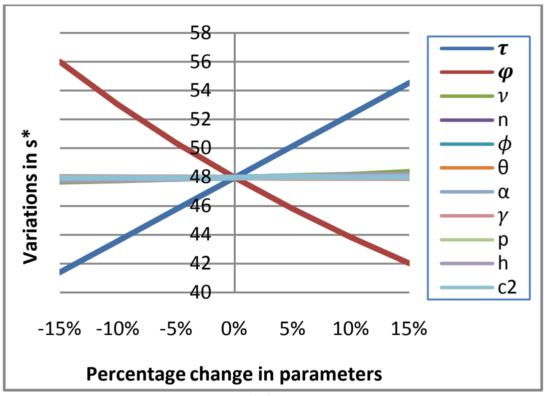

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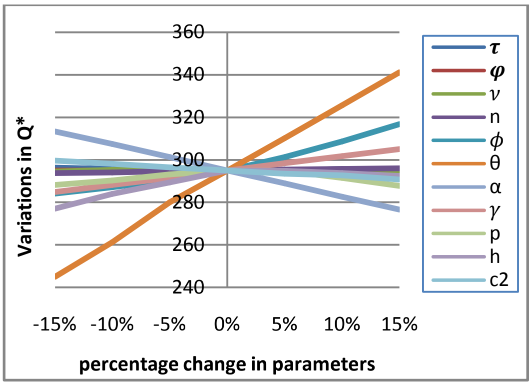
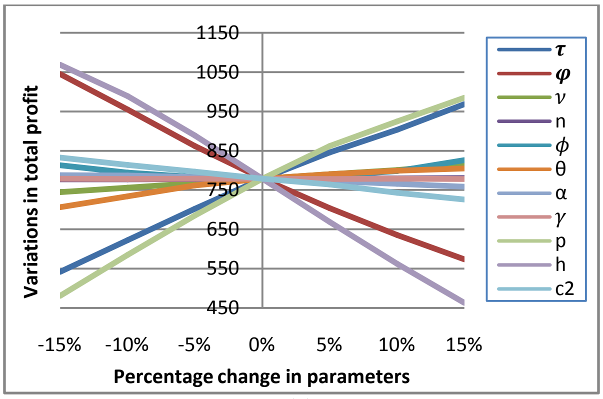


(d)



(e)

Fig. 2. Sensitivity analysis of the system variables with respect to the parameters and costs

PARTICULAR CASE

In some other production processes shortages are not allowed. That is, the production starts as and when the inventory level reaches zero. For this sort of situations an inventory system for deteriorating items having stock dependent production rate and time and selling price dependent demand in which the lifetime of the commodity is random and follows three parameter Weibull distribution may be deduced as a limiting case of the model developed in section 3 above when the cost of incurring shortages increases indefinitely $(c_2 \to \infty)$ and $t_2 \to T$. Then in this system the inventory level changes during $(0,\gamma)$ due to demand and production, during (γ, t_1) due to deterioration, demand and production and during (t_1, T) due to demand and deterioration.

7. CONCLUSIONS

Production inventory models play a dominant role in manufacturing and production industries like cement, food processing, petrochemical, pharmaceutical and paint manufacturing units. In this paper, a production inventory model for deteriorating items with stock dependent production rate, time and selling price dependent demand and Weibull decay has been developed and analyzed in the light of various parameters and costs and with the objective of maximizing the total system profit. The optimal production schedule is derived. The model was illustrated with numerical examples and sensitivity analysis of the model with respect to costs and parameters was also carried out. It can be concluded from the numerical examples and sensitivity analysis that the stock dependent nature of production rate is having significant influence on the optimal production quantity and profit rate and the demand parameters tremendously influence the optimal values of the unit selling price, production quantity and profit rate. This model also includes the exponential decay model as a particular case for specific values of the parameters. The proposed model can further be enriched by incorporating salvage of deteriorated units, inflation, quantity discount, and trade credits etc. It can also be extended to a multi-commodity model with constraints on budget, shelf space, etc., These models may also be formulated in fuzzy environments.

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