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# STAR COLORING OF HELM GRAPH FAMILIES 

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#### Abstract

A star coloring of a graph $G$ is a proper vertex coloring (no two adjacent vertices of $G$ has the same color) such that the induced subgraph of any two color classes is a collection of stars. The minimum number of colors needed to star color the vertices of a graph is called its star chromatic number and is denoted by $X_{s}(G)$. In this research paper, we present coloring algorithms and find the exact value of the star chromatic number of Middle, Total and Central graph of Helm graph families.


Keywords: Star coloring, Middle graph, Total graph and Central graph.
Ams Classification Number: 05C15.

## 1. INTRODUCTION

All graphs considered here are simple, finite and undirected. In the whole paper, the term coloring will refers the vertex coloring of graphs. A proper vertex coloring of a graph G means the coloring of the vertices of G such that no two adjacent vertices have the same color.
1.1 Definition: A subgraph $H$ of a graph $G$ is an induced subgraph if it has all the edges that appear in $G$ over the same vertex set. The subgraph induced by the vertex set $\left\{v_{1}, v_{2}, v_{3}, \ldots v_{k}\right\}$ is denoted by $<v_{1}, v_{2}, v_{3}, \ldots, v_{k}>$.
1.2 Definition: A vertex coloring of a graph G is said to be star coloring [9] if the induced subgraph of any two color classes is a collection of stars. In otherwords, the induced subgraph of any two color classes has no bicolored path of length 3.
In the whole paper, let us denote a bicolored path, with colors i and j , of length atleast 3 by $\mathrm{H}_{\mathrm{i}, \mathrm{j}}$.
1.3 Definition: A graph $G$ is said to be $\mathrm{H}_{\mathrm{ij}}$-free graph if it does not contain any bicolored ( $\mathrm{i}, \mathrm{j}$ )-path of length 3 .
1.4 Definition: The minimum number of colors required for star coloring of a graph is said to be its star chromatic number and is denoted by $\mathrm{X}_{\mathrm{s}}(\mathrm{G})$.
1.5 Definition: The Helm $H_{n}$, is the graph obtained from a Wheel graph $W_{n}$, by attaching a pendent edge at each vertex of the n-cycle.'

In this paper, we obtain the exact value of the star chromatic number of the Helm graph families.

## 2. STAR COLORING OF M( $\mathbf{H}_{\mathbf{n}}$ )

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$.
2.1 Definition: The Middle graph [2], denoted by $M(G)$, of a graph $G$ is the graph obtained from $G$ by inserting a new vertex into every edge of $G$ and by joining those pairs of these new vertices with edges which lie on adjacent edges of G.

In Helm $\mathrm{H}_{\mathrm{n}}$, let $v$ be the root vertex and $v_{1}, v_{2}, v_{3}, \ldots, v_{n}$ be the vertices of $n$-cycle. Let $w_{1}, w_{2}, w_{3}, \ldots w_{n}$ be the n pendent vertices of $\mathrm{H}_{\mathrm{n}}$. Let $e_{k}(k=1$ to $n)$ be the newly added vertex on the edge joining $v$ and $v_{k}$ and $f_{k}(k=1$ to $n$ ) be the newly added vertex on the edge joining $v_{k}$ and $v_{k+1}$. Let $g_{k}\left(k=1\right.$ to $n$ ) be the newly added vertex on the edge joining $v_{k}$ and $w_{k}$.

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We use these notations for sections 3 and 4 also.

### 2.2 Structural properties of $\mathbf{M}\left(\mathbf{H}_{\mathbf{n}}\right)$

By definition 2.1, $\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)$ has the following structural properties.
(i) $<v, e_{k} ; k=1$ to $n>$ form a clique of order $n+1$.
(ii) For each $\mathrm{k}=2$ to n , the neighbors of $v_{k}$ are $\left\{e_{k}, f_{k}, f_{k-1}, g_{k}\right\}$ and the neighbors of $v_{1}$ are $\left\{e_{1}, f_{1}, f_{n}, g_{1}\right\}$.
(iii) The neighbors of $w_{k}$ is $\left\{g_{k}\right\}, k=1$ to $n$.
(iv) For each $k=2$ to $n-1$, the neighbors of $f_{k}$ are $\left\{f_{k-1}, f_{k+1}, e_{k}, e_{k+1}, v_{k}, v_{k+1}, g_{k}, g_{k+1}\right\}$ and the neighbors of $f_{1}$ and $f_{n}$ are respectively $\left\{f_{n}, f_{2}, e_{1}, e_{2}, v_{1}, v_{2}, g_{1}, g_{2}\right\}$ and $\left\{f_{n-1}, f_{1}, e_{n}, e_{1}, v_{n}, v_{1}, g_{n}, g_{1}\right\}$.
(v) For each $k=1$ to $n, e_{k}$ and $g_{k}$ are adjacent.

We use these structural properties for coloring the vertices of $M\left(H_{n}\right)$. First, we present the structure and coloring algorithm of $\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)$ and then we prove that the coloring is a star coloring in theorem 2.5.

### 2.3 Structure Algorithm of $\mathbf{M}\left(\mathbf{H}_{\mathbf{n}}\right)$

Input: $M\left(H_{n}\right)$
$\mathrm{V} \leftarrow\left\{v, e_{1}, e_{2}, \ldots, e_{n}, v_{1}, v_{2}, \ldots, v_{n}, f_{1}, f_{2}, \ldots, f_{n}, g_{1}, g_{2}, \ldots, g_{n}, w_{1}, w_{2}, \ldots, w_{n}\right\}$
 $g_{1} ", g_{2} ", \ldots, g_{n} ", h_{1}{ }^{\prime}, h_{2}{ }^{\prime}, \ldots, h_{n}{ }^{\prime}, h_{1} ", h_{2} ", \ldots, h_{n} ", d_{1}{ }^{\prime}, d_{2}{ }^{\prime}, \ldots d_{n}{ }^{\prime}, d_{1} ", d_{2} ", \ldots d_{n} "$, $\left.l_{1}{ }^{\prime}, l_{2}^{\prime}, \ldots, l_{n}^{\prime}, l_{1} ", l_{2} ", \ldots, l_{n} "\right\}$
for $k=1$ to $n$
\{
$v e_{k} \leftarrow e_{k}$;
\}
end for
for $j=1$ to $n-1$
for $k=1$ to $n$
\{
if $j<k$,
$\mathrm{e}_{\mathrm{j}} \mathrm{e}_{\mathrm{k}} \leftarrow \mathrm{e}_{\mathrm{jk}}{ }^{\prime}$;
\}
\}
end for
end for
for $k=1$ to $n$
\{
$e_{k} v_{k} \leftarrow e_{k}{ }^{\prime} ; e_{k} f_{k} \leftarrow f_{k}{ }^{\prime} ; v_{k} g_{k} \leftarrow g_{k}{ }^{\prime} ; g_{k} w_{k} \leftarrow g_{k} " ; e_{k} g_{k} \leftarrow d_{k} " ;$
\}
end for
for $k=1$ to $n-1$
\{
$f_{k} e_{k+1} \leftarrow f_{k} " ;$
\}
end for
$f_{n} e_{1} \leftarrow f_{n} " ;$
for $k=1$ to $n-1$
\{
$f_{k} f_{k+1} \leftarrow h_{k}{ }^{\prime} ; f_{k} g_{k+1} \leftarrow h_{k}{ }^{\prime} ; f_{k} v_{k+1} \leftarrow l_{k}{ }^{\prime} ;$
\}
end for
$f_{n} f_{1} \leftarrow h_{n}{ }^{\prime} ; f_{n} g_{1} \leftarrow h_{n}{ }^{\prime} ; \mathrm{f}_{\mathrm{n}} \mathrm{v}_{1} \leftarrow \mathrm{l}_{\mathrm{n}}{ }^{\prime} ;$
for $k=1$ to $n$
\{
$\mathrm{g}_{\mathrm{k}} \mathrm{f}_{\mathrm{k}} \leftarrow \mathrm{d}_{\mathrm{k}}{ }^{\prime} ; \mathrm{v}_{\mathrm{k}} \mathrm{f}_{\mathrm{k}} \leftarrow \mathrm{l}_{\mathrm{k}}{ }^{\prime} ;$
\}
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end for
Output: edge labeled $\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)$.

### 2.4 Coloring Algorithm of $M\left(H_{n}\right), n \geq 9$

```
Input: \(\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)\)
\(v \leftarrow n+1\);
for \(k=1\) to \(n\)
    \{
        \(v_{k} \leftarrow n+1 ; w_{k} \leftarrow n+1 ;\)
        \}
end for
for \(k=1\) to \(n\)
    \{
    \(e_{k} \leftarrow k ;\)
    \}
end for
for \(\mathrm{k}=1\) to n
\{
    \(r \leftarrow k+3\);
if \(r \leq n\),
    \(f_{k} \leftarrow r\);
else
    \(f_{k} \leftarrow r-n\);
    \}
end for
for \(k=1\) to \(n\)
    \{
    \(s \leftarrow k+4\);
if \(s \leq n\),
    \(g_{k} \leftarrow s ;\)
else
    \(g_{k} \leftarrow s\) - \(n\);
    \}
end for
Output: colored M( \(\left.\mathrm{H}_{\mathrm{n}}\right)\).
```

2.5 Theorem: The star chromatic number of $M\left(H_{n}\right)$ is $X_{s}\left[M\left(H_{n}\right)\right]=n+1, n \geq 9$.

## Proof:

Case (i): Consider the colors $n+1$ and $k, k=1$ to $n$. The color class of $n+1$ is $\left\{v, v_{k}, w_{k} ; k=1\right.$ to $\left.n\right\}$ whereas the color class of $k$ is $\left\{\mathrm{e}_{\mathrm{k}}, \mathrm{f}_{\mathrm{x}}, g_{y}\right\}$ (where $\mathrm{x}=\mathrm{n}+\mathrm{k}-3$, if $\mathrm{k} \leq 3$ and $\mathrm{x}=\mathrm{k}-3$, if $\mathrm{k}>3$.Similarly, $\mathrm{y}=\mathrm{n}+\mathrm{k}-4$, if $\mathrm{k} \leq 4$ and $\mathrm{y}=\mathrm{k}-4$, if $\mathrm{k}>4$ ).

The induced subgraph of these color classes contain the star graphs $v e_{k} v_{k}$, $v_{x} f_{x} v_{z}$ (where $\mathrm{z}=\mathrm{n}+\mathrm{k}-2$, if $\mathrm{k} \leq 2$ and $\mathrm{z}=\mathrm{k}-2$, if $\mathrm{k}>2$ ) and $w_{y} g_{y} v_{y}$ and the isolated vertices. Thus, $\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{k, n+1}$-free graph.

Case (ii): Consider the colors $k$ and $k+1, k=1$ to $n-1$. The color class of $k$ is $\left\{e_{k}, f_{x}, g_{y}\right\}$ and that of $k+1$ is $\left\{e_{k+1}, f_{z}, g_{\chi}\right\}$. The induced subgraph of these color classes contains the bicolored disjoint paths $g_{x} f_{x} f_{z}$ (where $\mathrm{x}=\mathrm{n}+\mathrm{k}-3$, if $\mathrm{k} \leq 3$ and $\mathrm{x}=\mathrm{k}-3$, if $\mathrm{k}>3$ and $\mathrm{z}=\mathrm{n}+\mathrm{k}-2$, if $\mathrm{k} \leq 2$ and $\mathrm{z}=\mathrm{k}-2$, if $\mathrm{k}>2$ ) and $e_{k} e_{k+1}$ and an isolated vertex. Thus, $\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{k, k+1}$-free graph.

Case (iii): Consider the colors $j$ and $k, 1 \leq j<k \leq n$ and $k \neq j+1$. The induced subgraph of their color classes contains bicolored paths of length 2 and 1 and isolated vertices (the paths varies with $|j-k|)$. Thus, $M\left(H_{n}\right)$ is $H_{j, k}$-free graph.

Thus, the coloring is a star coloring and as $M\left(H_{n}\right)$ has a clique of order $n+1$, we need minimum $n+1$ colors for proper coloring. Therefore,

$$
\mathrm{X}_{\mathrm{s}}\left[\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+1, \mathrm{n} \geq 9 .
$$



Fig.1. $X_{S}\left(M\left(H_{9}\right)\right)=10$

### 2.6. Remark:

(i) $\mathrm{X}_{\mathrm{s}}\left[\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=7, \mathrm{n}=2,3$
(ii) $X_{s}\left[M\left(H_{n}\right)\right]=8, n=4,5$ and (iii) $X_{s}\left[M\left(H_{n}\right)\right]=9, n=6,7$ and $X_{s}\left[M\left(H_{8}\right)\right]=10$.

## 3. STAR COLORING OF T ( $\mathrm{H}_{\mathrm{n}}$ )

3.1 Definition: The Total graph [2] of a graph, denoted by $T(G)$, is a graph such that the vertex set of $T$ is $V(G) U E(G)$ and two vertices are adjacent in T iff their corresponding elements are either adjacent or incident in G .

### 3.2 Structural properties of $T\left(H_{n}\right)$

By the definition of Total graph, $\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)$ has the following properties.
(i) $<v, e_{k} ; k=1$ to $n>$ form a clique of order $n+1$.
(ii) The neighbors of $v_{k}(k=2$ to $n-1)$ is $\left\{v, e_{k}, v_{k-1}, v_{k+1}, f_{k-1}, f_{k}, g_{k}, w_{k}\right\}$. The neighbors of $v_{1}$ and $v_{n}$ are respectively $\left\{v, e_{1}, v_{2}, v_{n}\right.$ $\left., f_{1}, f_{n}, g_{1}, w_{1}\right\}$ and $\left\{v, e_{n}, v_{n-1}, v_{1}, f_{n-1}, f_{n}, g_{n}, w_{n}\right\}$.
(iii) The neighbors of $f_{k}(k=2$ to $n-1)$ is $\left\{e_{k}, v_{k}, e_{k+1}, v_{k+1}, f_{k-1}, f_{k+1}, g_{k}, g_{k+1}\right\}$. The neighbors of $f_{1}$ and $f_{n}$ are respectively $\left\{e_{1}, v_{1}, g_{1}, e_{2}, v_{2}, g_{2}, f_{n}, f_{2}\right\}$ and $\left\{e_{n}, v_{n}, g_{n}, e_{1}, v_{1}, g_{1}, f_{n-1}, f_{1}\right\}$.
(iv) The neighbors of $g_{k}(k=2$ to $n-1)$ is $\left\{e_{k}, v_{k}, w_{k}, f_{k-1}, f_{k}\right\}$. The neighbors of $g_{1}$ and $g_{n}$ are respectively $\left\{e_{1}, v_{1}, w_{1}, f_{n}, f_{1}\right\}$ and $\left\{e_{n}, v_{n}, w_{n}, f_{n-1}, f_{n}\right\}$.
(v) The neighbors of $w_{k}$ is $\left\{g_{k}, v_{k}\right\}, k=1$ to $n$.

Now, we present the structure and coloring algorithm of $\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)$ and then we prove that the coloring is a star coloring of $\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)$ in the immediate following theorem.

### 3.3 Structure Algorithm of $\mathbf{T}\left(\mathbf{H}_{\mathrm{n}}\right)$

```
Input: T(Hn)
V}\leftarrow{v,\mp@subsup{e}{1}{},\mp@subsup{e}{2}{},\ldots,\mp@subsup{e}{n}{},\mp@subsup{v}{1}{},\mp@subsup{v}{2}{},\ldots,\mp@subsup{v}{n}{},\mp@subsup{f}{1}{},\mp@subsup{f}{2}{},\ldots,\mp@subsup{f}{n}{},\mp@subsup{g}{1}{},\mp@subsup{g}{2}{},\ldots,\mp@subsup{g}{n}{},\mp@subsup{w}{1}{},\mp@subsup{w}{2}{},\ldots,\mp@subsup{w}{n}{}
```





```
for }k=1\mathrm{ to }
    {
    ve
```


## end for

for $j=1$ to $n-1$
\{
for $k=1$ to $n$
\{
if $j<k$,
$\mathrm{e}_{\mathrm{j}} \mathrm{e}_{\mathrm{k}} \leftarrow \mathrm{e}_{\mathrm{jk}} ;$
\}
\}
end for
end for
for $k=1$ to $n$
\{
$e_{k} v_{k} \leftarrow e_{k}{ }^{\prime} ; e_{k} f_{k} \leftarrow f_{k}{ }^{\prime} ; v_{k} g_{k} \leftarrow g_{k}{ }^{\prime} ;$
$v_{k} w_{k} \leftarrow x_{k} " ; g_{k} w_{k} \leftarrow g_{k} " ; e_{k} g_{k} \leftarrow d_{k} " ;$
\}
end for
for $k=1$ to $n-1$
\{
$f_{k} e_{k+1} \leftarrow f_{k} " ;$
\}
end for
$f_{n} e_{1} \leftarrow f_{n} " ;$
for $k=1$ to $n-1$
\{

$$
v_{k} v_{k+1} \leftarrow y_{k}^{\prime} ; f_{k} f_{k+1} \leftarrow h_{k}^{\prime}
$$

$f_{k} g_{k+1} \leftarrow h_{k} " ; f_{k} v_{k+1} \leftarrow l_{k} " ;$
\}
end for
$v_{n} v_{1} \leftarrow y_{n}{ }^{\prime} ; f_{n} f_{1} \leftarrow h_{n}{ }^{\prime} ;$
$f_{n} g_{1} \leftarrow h_{n} " ; f_{n} v_{1} \leftarrow l_{n} " ;$
for $k=1$ to $n$
\{
$g_{k} f_{k} \leftarrow d_{k}{ }^{\prime} ; v_{k} f_{k} \leftarrow l_{k}{ }^{\prime} ;$
end for
Output: edge labeled $T\left(\mathrm{H}_{\mathrm{n}}\right)$.

### 3.4 Coloring Algorithm of $\mathbf{T}\left(\mathbf{H}_{\mathbf{n}}\right)$

Input : $\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right), n \geq 11$
$v \leftarrow n+1$;
for $k=1$ to $n$
\{
$e_{k} \leftarrow k ;$
end for
for $k=1$ to $n$
\{
$r \leftarrow n+k-2$;
if $r \leq n$,
$v_{k} \leftarrow r$;
else
$v_{k} \leftarrow r-n$;
\}
end for
for $k=1$ to $n$
\{
$s \leftarrow k+4$;
if $s \leq n$,
$f_{k} \leftarrow s ;$
else
$f_{k} \leftarrow s-n ;$
end for
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```
for k=1 to n
{
    t\leftarrowk+5;
```

if $t \leq n$,
$g_{k} \leftarrow t ;$
else
$g_{k} \leftarrow t-n ;$
end for
for $k=1$ to $n$
\{
$p \leftarrow k+1 ;$
if $p \leq n$,
$w_{k} \leftarrow p ;$
else
$w_{k} \leftarrow p-n ;$
\}
end for
Output : colored $T\left(\mathrm{H}_{\mathrm{n}}\right)$.
3.4 Theorem: For any Helm graph $\mathrm{H}_{\mathrm{n}}$,

$$
\mathrm{X}_{\mathrm{s}}\left[\mathrm{~T}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+1, \mathrm{n} \geq 11 .
$$

Proof: The color class of each color in a bicolored path of length 3, should contain atleast two vertices. As $v$ is the only vertex with color $n+1, T\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{\mathrm{k}, \mathrm{n}+1}$-free graph, $\mathrm{k}=1$ to n . So, we discuss the following cases.

Case (i): Consider the colors $k$ and $k+1, k=1$ to $n-1$. The color class of $k$ is $\left\{e_{k}, v_{m}, f_{k-4}, g_{k-5}\right\}$ (where $\mathrm{m}=\mathrm{k}+2$, if $\mathrm{k}+2 \leq \mathrm{n}$, else $m=k+2-n$ ) and that of $k+1$ is $\left\{e_{k+1}, v_{u}, f k-3, g_{k-4}\right\}$ (where $u=k+3$, if $k+3 \leq n$, else $u=k+3-n$ ). The subgraph induced by these color classes is a collection of stars, $e_{k} e_{k+1}$ and $v_{m} v_{u}$ and $g_{k-4} f_{k-4} f_{k-3}$. Thus, $\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{k, k+1}$-free graph.

Case (ii): Consider j and $\mathrm{k}, 1 \leq \mathrm{j}<\mathrm{k} \leq \mathrm{n}, \mathrm{k} \neq \mathrm{j}+1$. The color class of j is $\left.\left\{\mathrm{v}_{\mathrm{m}^{\prime}}, \mathrm{e}_{\mathrm{j}}, \mathrm{f}_{\mathrm{j}-4}, \mathrm{~g}_{\mathrm{j}-5}\right\}\right\}$ (where $\mathrm{m}^{\prime}=\mathrm{j}+2$, if $\mathrm{j}+2 \leq \mathrm{n}, \mathrm{else} \mathrm{m}^{\prime}=$ $\mathrm{j}+2-\mathrm{n}$ ) and that of k is $\left\{e_{k}, v_{m}, f_{k-4}, g_{k-5}\right\}$. The subgraph induced by these color classes contain bicolored paths of length 2 and 1. Therefore, $\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{\mathrm{j}, \mathrm{k}}$-free graph.

Thus, the coloring given in the coloring algorithm 3.3, is a star coloring. As $T\left(H_{n}\right)$ has a clique of order $n+1$, we need minimum $\mathrm{n}+1$ colors for proper coloring.

Therefore,

$$
\mathrm{X}_{\mathrm{s}}\left[\mathrm{~T}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+1, \mathrm{n} \geq 11 .
$$



### 3.4 Remark

(i) $X_{s}\left[T\left(H_{n}\right)\right]=n+2, n=9,10$
(ii) $\mathrm{X}_{\mathrm{s}}\left[\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+4, \mathrm{n}=5,6,7$,
(iii) $\mathrm{X}_{\mathrm{s}}\left[\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+6, \mathrm{n}=3,4$

## 4. STAR COLORING OF C( $\mathbf{H}_{n}$ )

4.1 Definition: Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The central graph of $G$, denoted by $C(G)[11]$, is obtained from $G$ by subdividing each edge exactly once and joining all the non adjacent vertices of $G$.

### 4.2 Structural properties of $\mathbf{C}\left(\mathrm{H}_{\mathrm{n}}\right)$

(i) $<\mathrm{v}, \mathrm{w}_{\mathrm{k}} ; \mathrm{k}=1$ to $\mathrm{n}>$ form a clique of order $\mathrm{n}+1$.
(ii) $\left\{v, f_{k}, g_{k} ; k=1\right.$ to $\left.n\right\}$ form an independent set.
(iii) The neighbors of $\mathrm{v}_{\mathrm{k}}$ is $\left\{\mathrm{e}_{\mathrm{k}}, \mathrm{f}_{\mathrm{k}}, \mathrm{g}_{\mathrm{k}}\right\} \cup\left\{\mathrm{v}_{\mathrm{j}} ; \mathrm{j}=1\right.$ to n and $\left.\mathrm{j} \neq \mathrm{k}-1, \mathrm{k}+1\right\} \cup\left\{\mathrm{w}_{\mathrm{j}} ; \mathrm{j}=1\right.$ to n and $\left.\mathrm{j} \neq \mathrm{k}\right\}$.
(iv) The neighbors of $e_{k}$ is $\left\{v, v_{k}\right\}, k=1$ to $n$.
(v) The neighbors of $\mathrm{g}_{\mathrm{k}}$ is $\left\{\mathrm{v}_{\mathrm{k}}, \mathrm{w}_{\mathrm{k}}\right\}, \mathrm{k}=1$ to n .

Now, we present the structure and coloring algorithm of $C\left(H_{n}\right)$ and then we show that the coloring is a star coloring of $\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)$ in the immediate following theorem .

### 4.3 Structure Algorithm of $\mathbf{C}\left(\mathrm{H}_{\mathrm{n}}\right)$

```
Input: C (H)
V}\leftarrow{v,\mp@subsup{e}{1}{},\mp@subsup{e}{2}{},\ldots,\mp@subsup{e}{n}{},\mp@subsup{v}{1}{},\mp@subsup{v}{2}{},\ldots,\mp@subsup{v}{n}{},\mp@subsup{f}{1}{},\mp@subsup{f}{2}{},\ldots,\mp@subsup{f}{n}{},\mp@subsup{g}{1}{},\mp@subsup{g}{2}{},\ldots,\mp@subsup{g}{n}{},\mp@subsup{w}{1}{},\mp@subsup{w}{2}{},\ldots,\mp@subsup{w}{n}{}}
```




```
    i<j\leqn,j\not=i+l)};
for k= 1 to n
    {
    ve}\mp@subsup{e}{k}{\leftarrow}\mp@subsup{e}{k}{\prime};\mp@subsup{e}{k}{}\mp@subsup{v}{k}{}\leftarrow\mp@subsup{\textrm{e}}{k}{\prime}";v\mp@subsup{w}{k}{}\leftarrow\mp@subsup{l}{k}{\prime};
    }
end for
for }k=1\mathrm{ to n
    {
    vk}\mp@subsup{f}{k}{}\leftarrow\mp@subsup{f}{k}{\prime};\mp@subsup{v}{k}{}\mp@subsup{g}{k}{}\leftarrow\mp@subsup{g}{k}{\prime};\mp@subsup{g}{k}{}\mp@subsup{w}{k}{}\leftarrow\mp@subsup{g}{k}{\prime\prime}
    }
end for
for k= 1 to n-1
    {
if k<n,
    fk}\mp@subsup{v}{k+1}{}\leftarrow\mp@subsup{f}{k}{\prime\prime
        }
end for
fn}\mp@subsup{v}{1}{}\leftarrow\mp@subsup{f}{n}{\prime\prime}
for j=1 to n
    {
for k= 1 to n
    {
    if j\not=k,
    vj}\mp@subsup{w}{k}{}\leftarrow\mp@subsup{d}{jk}{}
        }
        }
end for
end for
for j=1 to n
    {
for }k=1\mathrm{ to n
    {
    if j<k,
    w}\mp@subsup{w}{k}{}\leftarrow\mp@subsup{l}{jk}{}
        }
        }
end for
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```

```
end for
for }k=3\mathrm{ to n-1
    {
    \mp@subsup{v}{1}{}}\mp@subsup{\textrm{v}}{\textrm{k}}{}\leftarrow\mp@subsup{\textrm{h}}{1\textrm{k}}{}
    }
end for
for j= 2 to n-2
    {
for }k=j+2 to n
    {
    v}\mp@subsup{v}{j}{}\mp@subsup{v}{k}{}\leftarrow\mp@subsup{h}{jk}{}
        }
    }
end for
end for
Output: edge labeled C( }\mp@subsup{\textrm{H}}{n}{})\mathrm{ .
```


### 4.4 Coloring Algorithm of $\mathbf{C}\left(\mathrm{H}_{\mathrm{n}}\right)$

```
Input: \(\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right), n \geq 2\).
\(v \leftarrow n+1\);
for \(k=1\) to \(n\)
\{
    \(f_{k} \leftarrow n+1 ; g_{k} \leftarrow n+1 ;\)
    \}
end for
for \(k=1\) to \(n\)
    \{
        \(w_{k} \leftarrow k ;\)
        \}
end for
for \(k=1\) to \(n\)
    \{
        \(v_{k} \leftarrow n+k+1 ;\)
            \}
end for
for \(k=1\) to \(n\)
    \{
        \(r \leftarrow n+k+2\);
if \(r \leq 2 n+1\),
\(e_{k} \longleftarrow r\);
    else
\(\mathrm{e}_{k} \leftarrow r-(n-1)\);
        \}
end for
Output: colored \(\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)\)
```

4.4 Theorem: The star chromatic number of central graph of Helm is

$$
\mathrm{X}_{\mathrm{s}}\left[\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=2 \mathrm{n}+1, \mathrm{n} \geq 2 .
$$

## Proof:

Case (i): Consider the colors $j$ and $k$ where $1 \leq j, k \leq n$. The color class of $j$ is $\left\{w_{j}\right\}$ and that of $k$ is $\left\{w_{k}\right\}$. So, the induced subgraph is $w_{j} w_{k}$ and therefore $\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{j, k}$ - free graph.

Case (ii): Consider the colors $n+1$ and $k, k=1$ to $n$. The induced subgraph of the color classes contain the bicolored path $v w_{k} g_{k}$ of length 2 and therefore, $\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{k, n+1}$-free graph.

Case (iii): Consider the colors $k$ and $k+1, n+2 \leq k \leq 2 n+1$. The color class of $k$ is $\left\{v_{k-n-1}, e_{k-n-2}\right\}$ whereas the color class of $k+1$ is $\left\{v_{k-n}, e_{k-n-1}\right\}$. The induced subgraph contains the edge $e_{k-n-1} v_{k-n-1}$ and isolated vertices. Thus, $\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{k, k+1}$-free graph.

Case (iv): Consider the colors $n+1$ and $k$, $k=(n+2)$ to $(2 n+1)$. The induced subgraph contain the bicolored paths $g_{1} \mathrm{v}_{1} \mathrm{f}_{1}$ and $\mathrm{ve}_{\mathrm{n}}$ when $\mathrm{k}=\mathrm{n}+2$, and bicolored star $\mathrm{g}_{\mathrm{n}} \mathrm{v}_{\mathrm{n}} \mathrm{f}_{\mathrm{n}-1} \mathrm{f}_{\mathrm{n}}$ and $\mathrm{ve}_{\mathrm{n}-1}$ when $\mathrm{k}=2 \mathrm{n}+1$. When $\mathrm{n}+3 \leq \mathrm{k} \leq 2 \mathrm{n}$, the induced subgraph contains the bicolored star $f_{k-n-2} \mathrm{v}_{\mathrm{k}-\mathrm{n}-1} \mathrm{f}_{\mathrm{k}-\mathrm{n}-1} \mathrm{~g}_{\mathrm{k}-\mathrm{n}-1}$ and the edge $\mathrm{ve}_{\mathrm{k}-\mathrm{n}-2}$. Thus, the induced subgraph is a collection of stars and hence, $\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{\mathrm{k}, \mathrm{n}+1}$-free graph.

Case (v): Consider the colors j and $\mathrm{k}, \mathrm{n}+2 \leq \mathrm{j}<\mathrm{k} \leq 2 \mathrm{n}+1$. The induced subgraph contains the edge $\mathrm{v}_{\mathrm{j}-\mathrm{n}-1} \mathrm{v}_{\mathrm{k}-\mathrm{n}-1}$ and isolated vertices. Therefore, $\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)$ is $\mathrm{H}_{\mathrm{j}, \mathrm{k}}$-free graph.

By (i) of sec 4.2, $X_{s}\left(C\left(H_{n}\right) \geq n+1\right.$. The vertices $v_{1}$ to $v_{n}$ are properly colored with colors $n+2$ to $2 n+1$. If, suppose, we assign the same color, say $k$, to two vertices $v_{i}$ and $v_{i+1}$, then $g_{i} v_{i} f_{i} v_{i+1}$ will become a bicolored ( $k, n+1$ )-path of length 3 .

So, we need minimum $2 \mathrm{n}+1$ colors for star coloring. Therefore,

$$
\mathrm{X}_{\mathrm{s}}\left[\mathrm{C}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=2 \mathrm{n}+1, \mathrm{n} \geq 2 .
$$



Fig.3. $\mathbf{X}_{\mathrm{S}}\left[\mathrm{C}\left(\mathbf{H}_{4}\right)\right]=\mathbf{9}$

## CONCLUSION

We have shown that
(i) $\mathrm{X}_{\mathrm{s}}\left[\mathrm{M}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+1, \mathrm{n} \geq 9$.
(ii) $\mathrm{X}_{\mathrm{s}}\left[\mathrm{T}\left(\mathrm{H}_{\mathrm{n}}\right)\right]=\mathrm{n}+1, \mathrm{n} \geq 11$.
(i) $X_{s}\left[C\left(H_{n}\right)\right]=2 n+1, n \geq 2$.

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