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STAR COLORING OF HELM GRAPH FAMILIES

MRS. R. ARUNDHADHI*
Asst. Prof. D. G. Vaishnav College, Chennai, Research Scholar Bharathiar University, Coimbatore, India

Dr. K. THIRUSANGU
Associate Professor, Dept of Mathematics, S.I.V.E.T College, Chennai, India

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ABSTRACT

A star coloring of a graph G is a proper vertex coloring (no two adjacent vertices of G has the same color) such that the induced subgraph of any two color classes is a collection of stars. The minimum number of colors needed to star color the vertices of a graph is called its star chromatic number and is denoted by X_s (G). In this research paper, we present coloring algorithms and find the exact value of the star chromatic number of Middle, Total and Central graph of Helm graph families.

Keywords: *Star coloring, Middle graph, Total graph and Central graph.*

Ams Classification Number: *05C15.*

1. INTRODUCTION

All graphs considered here are simple, finite and undirected. In the whole paper, the term coloring will refers the vertex coloring of graphs. A proper vertex coloring of a graph G means the coloring of the vertices of G such that no two adjacent vertices have the same color.

1.1 Definition: A subgraph H of a graph G is an induced subgraph if it has all the edges that appear in G over the same vertex set. The subgraph induced by the vertex set $\{v_1,v_2,v_3,\ldots v_k\}$ is denoted by $<v_1,v_2,v_3,\ldots,v_k>$.

1.2 Definition: A vertex coloring of a graph G is said to be star coloring [9] if the induced subgraph of any two color classes is a collection of stars. In otherwords, the induced subgraph of any two color classes has no bicolored path of length 3.
In the whole paper, let us denote a bicolored path, with colors i and j, of length atleast 3 by $H_{i,j}$.

1.3 Definition: A graph G is said to be H_{ij}-free graph if it does not contain any bicolored (i, j)-path of length 3.

1.4 Definition: The minimum number of colors required for star coloring of a graph is said to be its star chromatic number and is denoted by X_s (G).

1.5 Definition: The Helm H_n, is the graph obtained from a Wheel graph W_n, by attaching a pendent edge at each vertex of the n-cycle.'

In this paper, we obtain the exact value of the star chromatic number of the Helm graph families.

2. STAR COLORING OF M(H_n)

Let G be a graph with vertex set V(G) and edge set E(G).

2.1 Definition: The Middle graph [2], denoted by M(G), of a graph G is the graph obtained from G by inserting a new vertex into every edge of G and by joining those pairs of these new vertices with edges which lie on adjacent edges of G.

In Helm H_n, let v be the root vertex and $v_1, v_2, v_3,\ldots,v_n$ be the vertices of n-cycle. Let $w_1, w_2, w_3, \ldots w_n$ be the n pendent vertices of H_n. Let $e_k(k=1 \text{ to } n)$ be the newly added vertex on the edge joining v and v_k and $f_k(k=1 \text{ to } n)$ be the newly added vertex on the edge joining v_k and v_{k+1}. Let $g_k(k=1 \text{ to } n)$ be the newly added vertex on the edge joining v_k and w_k.

Corresponding author: *MRS. R. ARUNDHADHI**
Asst. Prof. D. G. Vaishnav College, Chennai, Research Scholar Bharathiar University, Coimbatore, India

We use these notations for sections 3 and 4 also.

2.2 Structural properties of M (H_n)

By definition 2.1, M (H_n) has the following structural properties.

(i) $< v,e_k;\ k=1 \text{ to } n>$ form a clique of order n+1.

(ii) For each k=2 to n, the neighbors of v_k are $\{e_k,f_k,f_{k-1},g_k\}$ and the neighbors of v_1 are $\{e_1, f_1, f_n,g_1\}$.

(iii) The neighbors of w_k is $\{g_k\}$, $k=1 \text{ to } n$.

(iv) For each $k=2$ to $n-1$, the neighbors of f_k are $\{f_{k-1},f_{k+1},e_k,e_{k+1},v_k,v_{k+1},g_k,g_{k+1}\}$ and the neighbors of f_1 and f_n are respectively $\{f_n ,f_2,e_1,e_2,v_1,v_2,g_1,g_2\}$ and $\{f_{n-1},f_1,e_n ,e_1,v_n,v_1,g_n ,g_1\}$.

(v) For each $k=1$ to n, e_k and g_k are adjacent.

We use these structural properties for coloring the vertices of M(H_n). First, we present the structure and coloring algorithm of M(H_n) and then we prove that the coloring is a star coloring in theorem 2.5.

2.3 Structure Algorithm of M (H_n)

Input: M(H_n)

V← $\{v,e_1,e_2,\ldots,e_n,v_1,v_2,\ldots,v_n, f_1,f_2,\ldots,f_n,g_1,g_2,\ldots,g_n,w_1,w_2,\ldots,w_n\}$
E← $\{e_1',e_2',\ldots e_n', e_{ij}'(1\leq i < j < n), e_1'',e_2'',\ldots e_n'', f_1',f_2',\ldots f_n', f_1'',f_2'',\ldots f_n'',g_1',g_2',\ldots,g_n',$
$g_1'',g_2'',\ldots,g_n'', h_1',h_2',\ldots,h_n',h_1'',h_2'',\ldots,h_n'', d_1',d_2',\ldots d_n' , d_1'',d_2'',\ldots d_n'' ,$
$l_1',l_2',\ldots,l_n',l_1'',l_2'',\ldots,l_n''\}$
for $k= 1$ to n
{
$ve_k \leftarrow e_k'$;
}
end for
for $j= 1$ to $n-1$

for $k= 1$ to n
{
if $j < k$,
$e_je_k \leftarrow e_{jk}'$;
}
}
end for
end for
for $k= 1$ to n
{
$e_kv_k\leftarrow e_k''$; $e_kf_k\leftarrow f_k'$; $v_kg_k\leftarrow g_k'$; $g_kw_k\leftarrow g_k''$; $e_kg_k\leftarrow d_k''$;
}
end for
for $k= 1$ to $n-1$
{
$f_ke_{k+1}\leftarrow f_k''$;
}
end for
$f_ne_1\leftarrow f_n''$;
for $k= 1$ to $n-1$
{
$f_k f_{k+1}\leftarrow h_k'$; $f_k g_{k+1}\leftarrow h_k''$; $f_k v_{k+1}\leftarrow l_k''$;
}
end for
$f_nf_1 \leftarrow h_n'$; $f_ng_1 \leftarrow h_n''$; $f_n v_1\leftarrow l_n''$;
for $k= 1$ to n
{
$g_kf_k\leftarrow d_k'$; $v_kf_k\leftarrow l_k'$;
}

end for
Output: edge labeled $M(H_n)$.

2.4 Coloring Algorithm of $M(H_n)$, $n \geq 9$

```
Input: M(Hn)
 v← n+1 ;
for k= 1 to n
   {
     vk← n+1 ; wk← n+1 ;
      }
end for
for k= 1 to n
     {
     ek← k ;
     }
end for
for k= 1 to n
 {
  r← k+3 ;
if r ≤ n,
  fk← r ;
else
 fk← r-n ;
  }
end for
for k= 1 to n
  {
   s← k+4 ;
 if s ≤ n,
  gk← s ;
 else
 gk← s-n ;
  }
end for
Output: colored M (Hn).
```

2.5 Theorem: The star chromatic number of $M(H_n)$ is $X_s[M(H_n)] = n+1$, $n \geq 9$.

Proof:
Case (i): Consider the colors $n+1$ and k, $k=1$ *to* n. The color class of $n+1$ is $\{v, v_k, w_k;\ k=1 \text{ to } n\}$ whereas the color class of k is $\{e_k, f_x, g_y\}$(where x= n+k-3,if k ≤ 3 and x= k-3, if k > 3.Similarly, y= n+k-4, if k ≤ 4 and y= k-4, if k > 4).

The induced subgraph of these color classes contain the star graphs $v\, e_k\, v_k$, $v_x\, f_x v_z$ (where z= n+k-2, if k ≤ 2 and z= k-2, if k > 2) and $w_y\, g_y\, v_y$ and the isolated vertices. Thus, $M(H_n)$ is $H_{k,n+1}$-free graph.

Case (ii): Consider the colors k and $k+1$, $k=1$ *to* $n-1$. The color class of k is $\{e_k, f_x, g_y\}$ and that of $k+1$ is $\{e_{k+1}, f_z, g_x\}$. The induced subgraph of these color classes contains the bicolored disjoint paths $g_x\, f_x\, f_z$(where x= n+k-3,if k≤ 3 and x= k-3, if k > 3 and z= n+k-2,if k ≤ 2 and z= k-2, if k > 2) and $e_k\, e_{k+1}$ and an isolated vertex. Thus, $M(H_n)$ is $H_{k,k+1}$-free graph.

Case (iii): Consider the colors j and k, $1 \leq j < k \leq n$ and $k \neq j+1$. The induced subgraph of their color classes contains bicolored paths of length 2 and 1 and isolated vertices (the paths varies with $|j-k|$). Thus, $M(H_n)$ is $H_{j,k}$-free graph.

Thus, the coloring is a star coloring and as $M(H_n)$ has a clique of order n+1, we need minimum n+1 colors for proper coloring. Therefore,

$$X_s[M(H_n)] = n+1\ ,\ n \geq 9.$$

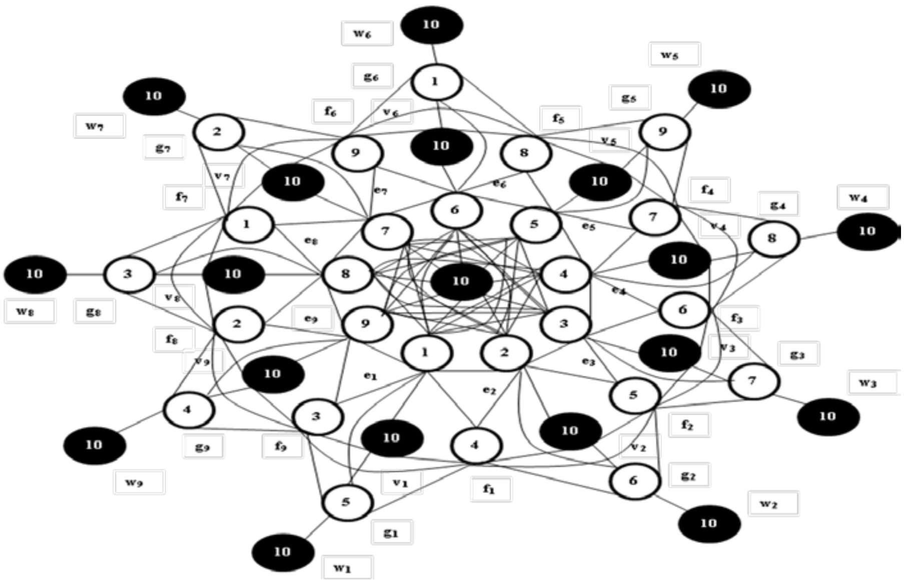

Fig.1.X_S (M (H_9)) = 10

2.6. Remark:

(i) X_s [M(H_n)]=7, n= 2,3
(ii)X_s[M(H_n)]=8, n= 4,5 and (iii) X_s[M(H_n)]=9, n=6,7 and X_s[M(H_8)]=10.

3. STAR COLORING OF T (H_n)

3.1 Definition: The Total graph [2] of a graph, denoted by T(G), is a graph such that the vertex set of T is V(G)UE(G) and two vertices are adjacent in T iff their corresponding elements are either adjacent or incident in G.

3.2 Structural properties of T(H_n)

By the definition of Total graph, T (H_n) has the following properties.

(i) $<v,e_k;\ k=1\ to\ n>$ form a clique of order n+1.

(ii) The neighbors of v_k ($k=2$ to $n-1$) is $\{v,e_k,v_{k-1},v_{k+1},f_{k-1},f_k,g_k,w_k\}$. The neighbors of v_1 and v_n are respectively $\{v,e_1,v_2,v_n,f_1,f_n,g_1,w_1\}$ and $\{v,e_n,v_{n-1},v_1,f_{n-1},f_n,g_n,w_n\}$.

(iii) The neighbors of f_k ($k=2$ to $n-1$) is $\{e_k,v_k,e_{k+1},v_{k+1},f_{k-1},f_{k+1},g_k,g_{k+1}\}$. The neighbors of f_1 and f_n are respectively $\{e_1,v_1,g_1,e_2,v_2,g_2,f_n,f_2\}$ and $\{e_n,v_n,g_n,e_1,v_1,g_1,f_{n-1},f_1\}$.

(iv) The neighbors of g_k ($k=2$ to $n-1$) is $\{e_k,v_k,w_k,f_{k-1},f_k\}$. The neighbors of g_1 and g_n are respectively $\{e_1,v_1,w_1,f_n,f_1\}$ and $\{e_n,v_n,w_n,f_{n-1},f_n\}$.

(v) The neighbors of w_k is $\{g_k,v_k\}$, $k=1$ to n.

Now, we present the structure and coloring algorithm of T(H_n) and then we prove that the coloring is a star coloring of T(H_n) in the immediate following theorem.

3.3 Structure Algorithm of T(H_n)

Input: T(H_n)
V← $\{v,e_1,e_2,\ldots,e_n,v_1,v_2,\ldots,v_n,f_1,f_2,\ldots,f_n,g_1,g_2,\ldots,g_n,w_1,w_2,\ldots,w_n\}$
E← $\{e_1',e_2',\ldots e_n', e_{ij}'(1\le i<j<n), e_1'',e_2'',\ldots e_n'', f_1',f_2',\ldots f_n', f_1'',f_2'',\ldots f_n'',g_1',g_2',\ldots,g_n',$
$g_1'',g_2'',\ldots,g_n'', h_1',h_2',\ldots,h_n',h_1'',h_2'',\ldots,h_n'', d_1',d_2',\ldots d_n', d_1'',d_2'',\ldots d_n'',$
$l_1',l_2',\ldots,l_n',l_1'',l_2'',\ldots,l_n'', x_1',x_2',\ldots,x_n',x_1'',x_2'',\ldots,x_n'', y_1',y_2',\ldots,y_n'\}$

for $k=1$ to n
{
ve_k← e_k' ; vv_k← x_k';}

end for
for $j= 1$ *to* $n-1$
{
for $k= 1$ *to* n
{

if $j < k$,
$e_je_k \leftarrow e_{jk}$;
}
}
end for
end for
for $k= 1$ *to* n
{
$e_kv_k \leftarrow e_k$'' ; $e_kf_k \leftarrow f_k$' ; $v_kg_k \leftarrow g_k$' ;
$v_kw_k \leftarrow x_k$''; $g_kw_k \leftarrow g_k$'' ; $e_kg_k \leftarrow d_k$'' ;
}
end for
for $k= 1$ *to* $n-1$
{
$f_ke_{k+1} \leftarrow f_k$'' ;
}
end for
$f_ne_1 \leftarrow f_n$'' ;
for $k= 1$ *to* $n-1$
{
$v_k v_{k+1} \leftarrow y_k$' ; $f_k f_{k+1} \leftarrow h_k$' ;
$f_k g_{k+1} \leftarrow h_k$'' ; $f_k v_{k+1} \leftarrow l_k$'' ;
}
end for
$v_nv_1 \leftarrow y_n$' ; $f_nf_1 \leftarrow h_n$' ;
$f_ng_1 \leftarrow h_n$''; $f_n v_1 \leftarrow l_n$'' ;
for $k= 1$ *to* n
{
$g_kf_k \leftarrow d_k$' ; $v_kf_k \leftarrow l_k$' ;
}
end for
Output: edge labeled $T(H_n)$.

3.4 Coloring Algorithm of T (H_n)

Input : $T(H_n)$, $n \geq 11$
$v \leftarrow n+1$;
for $k= 1$ *to* n
{
$e_k \leftarrow k$;
}
end for
for $k= 1$ *to* n
{
$r \leftarrow n+k-2$;
if $r \leq n$,
$v_k \leftarrow r$;
else
$v_k \leftarrow r-n$;
}
end for
for $k= 1$ *to* n
{
$s \leftarrow k+4$;
if $s \leq n$,
$f_k \leftarrow s$;
else
$f_k \leftarrow s-n$;
}
end for

for $k = 1$ *to* n
{
$t \leftarrow k+5$;
if $t \leq n$,
$g_k \leftarrow t$;

else
$g_k \leftarrow t-n$;
}
end for
for $k = 1$ *to* n
{
$p \leftarrow k+1$;
if $p \leq n$,
$w_k \leftarrow p$;
else
$w_k \leftarrow p-n$;
}
end for
Output : colored $T(H_n)$.

3.4 Theorem: For any Helm graph H_n,

$$X_s[T(H_n)] = n+1 \text{ , } n \geq 11.$$

Proof: The color class of each color in a bicolored path of length 3, should contain atleast two vertices. As v is the only vertex with color $n+1$, $T(H_n)$ is $H_{k,n+1}$-free graph, k=1 to n. So, we discuss the following cases.

Case (i): Consider the colors k and $k+1$, $k=1$ *to* $n-1$. The color class of k is $\{e_k, v_m, f_{k-4}, g_{k-5}\}$(where m=k+2, if $k+2 \leq n$, else m= k+2-n) and that *of* $k+1$ is $\{e_{k+1}, v_u, f_{k-3}, g_{k-4}\}$(where u= k+3,if $k+3 \leq n$,else u= k+3-n). The subgraph induced by these color classes is a collection of stars, $e_k\ e_{k+1}$ and $v_m\ v_u$ and $g_{k-4}f_{k-4}f_{k-3}$.. Thus, $T(H_n)$ is $H_{k,k+1}$-free graph.

Case (ii): Consider j and k, $1 \leq j < k \leq n$, $k \neq j+1$. The color class of j is $\{v_{m'}, e_j, f_{j-4}, g_{j-5}\}\}$(where m'=j+2, if $j+2 \leq n$,else m'= j+2-n) and that of k is $\{e_k, v_m, f_{k-4}, g_{k-5}\}$. The subgraph induced by these color classes contain bicolored paths of length 2 and 1. Therefore, $T(H_n)$ is $H_{j,k}$-free graph.

Thus, the coloring given in the coloring algorithm 3.3, is a star coloring. As $T(H_n)$ has a clique of order n+1, we need minimum n+1 colors for proper coloring.

Therefore,
$$X_s[T(H_n)]=n+1 \text{ , } n \geq 11.$$

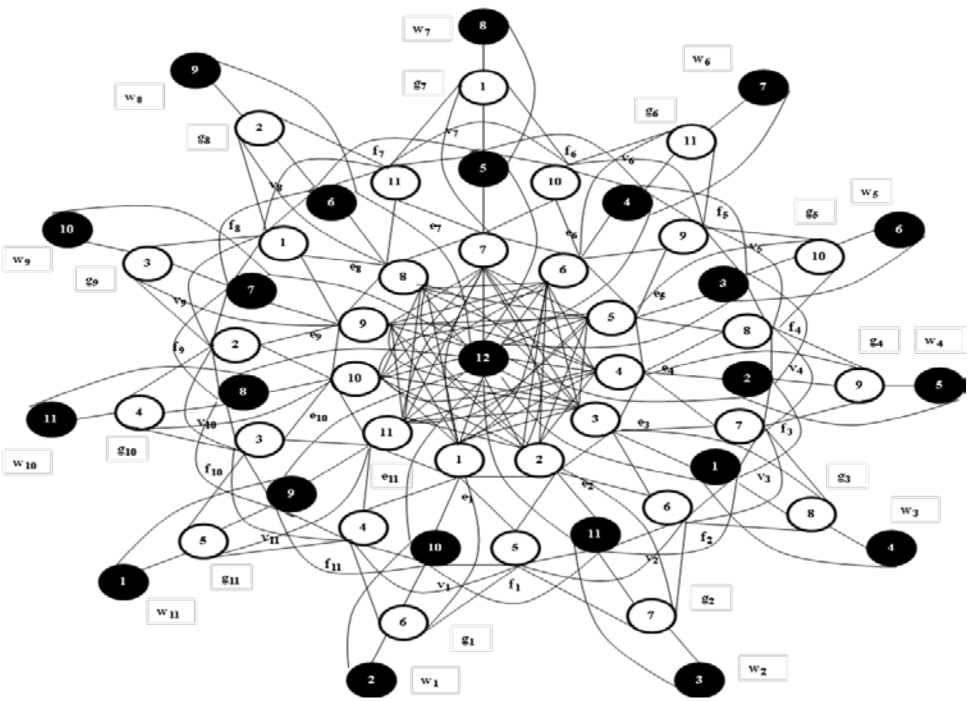

Fig.2. $X_S[\ T(H_{11})\ ] = 12$

3.4 Remark
(i) $X_s[T(H_n)]= n+2$, n=9,10
(ii)$X_s[T(H_n)]=n+4$, n=5,6,7,
(iii) $X_s[T(H_n)]= n+6$, n=3,4

4. STAR COLORING OF $C(H_n)$

4.1 Definition: Let G be a graph with vertex set V(G) and edge set E(G). The central graph of G, denoted by C(G)[11], is obtained from G by subdividing each edge exactly once and joining all the non adjacent vertices of G.

4.2 Structural properties of $C(H_n)$

(i) $< v, w_k$; k=1 to n> form a clique of order n+1.
(ii) $\{v, f_k, g_k;k=1 \text{ to } n\}$ form an independent set.
(iii) The neighbors of v_k is $\{e_k, f_k, g_k\}\cup\{v_j;j=1 \text{ to } n \text{ and } j\neq k-1,k+1\}\cup\{w_j;j=1 \text{ to } n \text{ and } j\neq k\}$.
(iv) The neighbors of e_k is $\{v,v_k\}$,k=1 to n.
(v) The neighbors of g_k is $\{v_k, w_k\}$,k=1 to n.

Now, we present the structure and coloring algorithm of C (H_n) and then we show that the coloring is a star coloring of $C(H_n)$ in the immediate following theorem .

4.3 Structure Algorithm of $C(H_n)$

Input: C (H_n)
V← $\{v,e_1,e_2,\ldots,e_n,v_1,v_2,\ldots,v_n,f_1,f_2,\ldots,f_n,g_1,g_2,\ldots,g_n,w_1,w_2,\ldots,w_n\}$;
E← $\{e_1',e_2',\ldots e_n', e_1'',e_2'',\ldots e_n'',l_1',l_2',\ldots,l_n' f_1',f_2',\ldots f_n',$
$f_1'',f_2'',\ldots f_n'',g_1',g_2',..,g_n',g_1'',g_2'',\ldots,g_n'', d_{ij}\ (1 \le i,j \le n, i\neq j), l_{ij}(1\le i < j \le n), h_{ij}(1\le i \le n-2,$
$i < j \le n, j\neq i+1)\}$;
for $k= 1$ *to* n
{
$v\ e_k\leftarrow e_k'$; $e_k\ v_k\leftarrow e_k''$; $v\ w_k\leftarrow l_k'$;
}
end for
for $k= 1$ *to* n
{
$v_k\ f_k\leftarrow f_k'$; $v_k\ g_k\leftarrow g_k'$; $g_k\ w_k\leftarrow g_k''$;
}
end for
for $k= 1$ *to* $n-1$
{
if $k < n$,
$f_k\ v_{k+1} \leftarrow f_k''$;
}
end for
$f_n\ v_1 \leftarrow f_n''$;
for $j= 1$ *to* n
{
for $k= 1$ *to* n
{
if $j\neq k$,
$v_j\ w_k \leftarrow d_{jk}$;
}
}
end for
end for
for $j= 1$ *to* n
{
for $k= 1$ *to* n
{
if $j < k$,
$w_j\ w_k \leftarrow l_{jk}$;
}
}
end for

end for
for $k = 3$ *to* $n-1$
{
$v_1v_k \leftarrow h_{1k}$;
}
end for
for $j = 2$ to $n-2$
{
for $k = j+2$ *to* n
{
$v_j\, v_k \leftarrow h_{jk}$;
}
}
end for
end for
Output: edge labeled $C(H_n)$.

4.4 Coloring Algorithm of $C(H_n)$

Input : $C(H_n)$, $n \geq 2$.
$v \leftarrow n+1$;
for $k = 1$ *to* n
{
$f_k \leftarrow n+1$; $g_k \leftarrow n+1$;
}
end for
for $k = 1$ *to* n
{
$w_k \leftarrow k$;
}
end for
for $k = 1$ *to* n
{
$v_k \leftarrow n+k+1$;
}
end for
for $k = 1$ *to* n
{
$r \leftarrow n+k+2$;
if $r \leq 2n+1$,
$e_k \leftarrow r$;
else
$e_k \leftarrow r-(n-1)$;
}
end for
Output: colored $C(H_n)$

4.4 Theorem: The star chromatic number of central graph of Helm is

$$X_s[C(H_n)] = 2n+1, \;\; n \geq 2.$$

Proof:

Case (i): Consider the colors *j* and *k* where $1 \leq j,\, k \leq n$. The color class of *j* is $\{w_j\}$ and that of *k* is $\{w_k\}$. So, the induced subgraph is $w_j\, w_k$ and therefore $C(H_n)$ is $H_{j,k}$- free graph.

Case (ii): Consider the colors $n+1$ and k, $k=1$ *to* n. The induced subgraph of the color classes contain the bicolored path $v\, w_k\, g_k$ of length 2 and therefore, $C(H_n)$ is $H_{k,n+1}$-free graph.

Case (iii): Consider the colors k and $k+1, n+2 \leq k \leq 2n+1$. The color class of k is $\{v_{k-n-1}, e_{k-n-2}\}$ whereas the color class of $k+1$ is $\{v_{k-n}, e_{k-n-1}\}$. The induced subgraph contains the edge $e_{k-n-1}v_{k-n-1}$ and isolated vertices. Thus, $C(H_n)$ is $H_{k,k+1}$-free graph.

Case (iv): Consider the colors n+1 and k, k= (n+2) to (2n+1). The induced subgraph contain the bicolored paths $g_1v_1f_1$ and ve_n when k= n+2, and bicolored star $g_nv_nf_{n-1}f_n$ and ve_{n-1} when k=2n+1. When $n+3 \leq k \leq 2n$, the induced subgraph contains the bicolored star $f_{k-n-2}v_{k-n-1}f_{k-n-1}g_{k-n-1}$ and the edge ve_{k-n-2}. Thus, the induced subgraph is a collection of stars and hence,$C(H_n)$ is $H_{k,n+1}$-free graph.

Case (v): Consider the colors j and k, $n+2 \leq j < k \leq 2n+1$. The induced subgraph contains the edge $v_{j-n-1}\ v_{k-n-1}$ and isolated vertices. Therefore, $C(H_n)$ is $H_{j,k}$-free graph.

By (i) of sec 4.2, $X_s(C\ (H_n) \geq n+1$. The vertices v_1 to v_n are properly colored with colors n+2 to 2n+1. If , suppose, we assign the same color, say k, to two vertices v_i and v_{i+1}, then $g_iv_if_iv_{i+1}$ will become a bicolored (k,n+1)-path of length 3.

So, we need minimum 2n+1 colors for star coloring. Therefore,

$$X_s[C\ (H_n)] = 2n+1,\ n \geq 2.$$

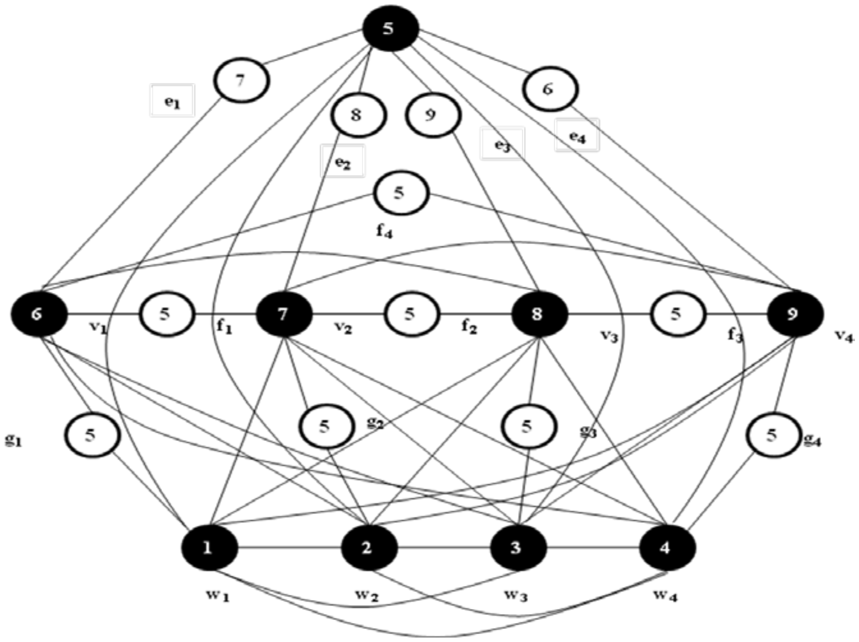

Fig.3. $X_S[C\ (H_4)\ ] = 9$

CONCLUSION

We have shown that
(i) $X_s[M(H_n)]= n+1$, $n \geq 9$.
(ii)$X_s[T(H_n)]= n+1$, $n \geq 11$.
(i) $X_s[C(H_n)]=2\ n+1$, $n \geq 2$.

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