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On ω^{μ} - CLOSED SETS AND CONTINUOUS FUNCTIONS IN SUPRA TOPOLOGICAL SPACE

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ABSTRACT

In this paper, we introduce and investigate a new class of sets called ω^{μ} –closed sets. Furthermore, we introduce ω^{μ} –continuous functions and investigate several properties of the new notions.

Key words: *ω^{μ} - closed set, ω^{μ} - continuous and supra topological spaces.*

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1. INTRODUCTION AND PRELIMINARIES

In 1983, Mashhour et al [5] introduced supra topological spaces and studied S-continuous maps and S^{*} - Continuous maps. In 2008, Devi et al [1] introduced the concept of supra α -open set, Sα -continuous functions respectively. In 2010, Sayed et al [7] introduced and investigated several properties of supra b-open sets and supra b- continuity. Ravi et al [6] introduced and investigated a new type of sets called supra g-closed and a new class of maps called supra g-continuous maps.

In this paper, we introduce the concept of ω^{μ} - closed sets and study its basic properties. Also, we introduce the concept of ω^{μ} - continuous functions and investigated several properties for these classes of functions in supra topological spaces.

Definition 1.1: [5, 7] A subfamily of μ of X is said to be a supra topology on X if

(i) $X, \varphi \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$

The pair (X, μ) is called supra topological space .The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition 1.2: [7]

(i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B : B$ is a supra closed and $A \subseteq B\}$.

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(ii) The supra interior of a set A is denoted by $\text{int}^{\mu}(A)$, and defined as $\text{int}^{\mu}(A) = \cup\{B : B$ is a supra open set and $A \supseteq B\}$

Definition 1.3: [5] Let (X,τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition 1.4: [6] Let (X,μ) be a supra topological space. A subset A of X is called

(i) supra semi open set , if $A \subseteq cl^{\mu}(\text{int}^{\mu}(A))$;

(ii) supra pre open set ,if $A \subseteq \text{int}^{\mu}(cl^{\mu}(A))$;
The complement of above mentioned open sets are called their respective closed sets.

Definition 1.5: Let (X,μ) be a supra topological space. A set A of X is called

(i) Supra generalized closed set (simply g^{μ}- closed) [6] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open. The complement of supra generalized closed set is supra generalized open set.

(ii) Supra semi – generalized closed set (simply sg^{μ} - closed [2] if $Scl^{\mu}(A) \subseteq U$ and U is supra semi – open . The complement of supra semi – generalized closed set is supra semi – generalized open set.

(iii) Supra generalized – semi closed set (simply gs^{μ} - closed)[2] if $Scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open. The complement of supra generalized – semi closed set is supra generalized semi – open set.

Definition 1.6: [6] Let A and B be subsets of X. Then the set A and B are said to be supra separated if

$$cl^{\mu}(A) \cap B = A \cap cl^{\mu}(B) = \varphi.$$

2. ω^{μ} - CLOSED SETS

Definition 2.1: A subset A of a supra topological space (X,μ) is called ω^{μ} - closed if $cl^{\mu}(A) \subseteq U$ whenever A ⊆ U and U is supra semi-open in (X,μ) .

The complement of supra ω^{μ} -closed set is called supra ω^{μ} - open if X − A is ω^{μ} – closed. We denote the family of all ω^{μ} – closed sets by $\omega^{\mu}C(X,\mu)$

Theorem 2.2: Every supra closed set is ω^{μ} - closed set in X.

Proof: let A be any supra closed set and U be any supra semi open set such that A ⊆ U. Then $cl^{\mu}(A) \subseteq U$, since $cl^{\mu}(A) = A$ and hence A is ω^{μ} - closed.

Converse of the above theorem need not be true as seen from the following example.

Example 2.3: Let $X = \{a,b,c,d\}$ and $\mu = \{X,\phi,\{b,d\},\{b,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Then the set $\{a,b,c\}$ is ω^{μ} - closed but not supra closed.

Theorem 2.4: Every ω^{μ} closed set is g^{μ} - closed set.

Proof: Let $A \subseteq U$, U is supra open and hence it is supra semi open. Since A is ω^{μ} closed we have $cl^{\mu}(A) \subseteq U$ .Hence g^{μ}-closed. The converse is not true as seen from the following example.

Example 2.5: Let $X = \{a,b,c,d\}$ and $\mu = \{X, \phi, \{a\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ .Then the set $\{c,d\}$ is g^{μ} -closed but not ω^{μ} - closed.

Theorem 2.6: Every ω^{μ} closed set is sg^{μ} - closed set.

Proof: Let A be any supra semi open set containing A. Then $scl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U$ .Hence sg^{μ} -closed. The converse is not true as seen from the following example.

Example 2.7: In example 2.5, the set $\{b,c\}$ is sg^{μ} - closed but not ω^{μ} -closed.

Theorem 2.8: Every ω^{μ} closed set is gs^{μ} - closed set.

Proof: Let $A \subseteq X$ be ω^{μ} closed set and let $A \subseteq U$, where U is supra open .Since A is ω^{μ} closed, then $scl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U$. Hence gs^{μ} -closed. The converse is not true as seen from the following example.

Example 2.9: In example 2.5, the set $\{a,c,d\}$ is sg^{μ} - closed but not ω^{μ} -closed.

Remark 2.10: Union of two ω^{μ} -closed sets need not be a ω^{μ} -closed set as seen from the following example.

Example 2.11: Let $X = \{a,b,c,d\}$ and $\mu = \{X, \phi, \{a\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$ .Then the sets $\{c,d\}$ and $\{a,d\}$ are ω^{μ} -closed sets but their union $\{a,c,d\}$ is not a ω^{μ} -closed set.

Remark 2.12: Intersection of two ω^{μ} -closed sets is generally not an ω^{μ} -closed set as seen from the following example.

Example 2.13: Let $X = \{a,b,c,d\}$ and $\mu = \{X, \varphi, \{a\}, \{a,b\}, \{b\}\}$ be a supra topology on X. Then, $\{a,b,c\}$ and $\{a,b,d\}$ are ω^{μ} closed sets but their intersection $\{a,b\}$ is not ω^{μ} closed set.

Remark 2.14: Intersection of ω^{μ} -closed set and supra open set is neither ω^{μ} -closed nor supra open as seen from the following example.

Example 2.15: Let $X = \{a,b,c,d\}$ and $\mu = \{X, \varphi, \{a\}, \{a,d\}, \{b,c,d\}\}$ be a supra topology on X. Then, $\omega^{\mu}C(X) = \{X, \varphi, \{b,c,d\}, \{b,c\}, \{a\}, \{a,b,c\}\}$ we have $A = \{a,d\}$ is supra open and $B = \{b,c,d\}$ is ω^{μ} closed sets but their intersection $\{d\}$ is neither ω^{μ} -closed nor supra open .

Corollary 2.16: Union of ω^{μ} -open set and supra closed set is neither ω^{μ} -open nor supra closed.

Remark 2.17: Intersection of ω^{μ} -closed set and supra semi open set is neither ω^{μ} -closed nor supra semi open as seen from the following example.

Example 2.18: In example 2.3, we have $A=\{a,c,d\}$ is supra semi open and $B=\{b,c\}$ is ω^{μ} closed sets but their intersection $\{c\}$ is neither ω^{μ} -closed nor supra open.

Corollary 2.19: Union of ω^{μ} -open set and supra semi closed set is neither ω^{μ} -open nor supra semi closed.

Theorem 2.20: A subset A of (X,μ) is ω^{μ} -closed then $cl^{\mu}(A)-A$ does not contain any non empty supra semi closed set.

Proof: Necessity Let A be ω^{μ} -closed set of (X,μ) .Suppose $F\neq\varphi$ is a supra semi closed set of $cl^{\mu}(A)-A$ .Then $F\subseteq cl^{\mu}(A)-A$ implies $F\subseteq cl^{\mu}(A)$ and F^{c} .This implies $A\subseteq F^{c}$. Since A is ω^{μ} closed , $cl^{\mu}(A)\subseteq U^{c}$, Consequently, $F\subseteq\left[cl^{\mu}(A)\right]^{c}$ .Hence $F\subseteq cl^{\mu}(A)\cap\left[cl^{\mu}(A)\right]^{c}=\varphi.$ Therefore F is empty, a contradiction.

Sufficiency Suppose that $A\subseteq U$ and that U is supra semi open. If $cl^{\mu}(A)\not\subset U$, then $cl^{\mu}(A)\cap U^{c}$ is a non empty supra semi closed subset of $cl^{\mu}(A)-A$. Hence, $cl^{\mu}(A)\cap U^{c}=\varphi$ and $cl^{\mu}(A)\subseteq U$. Therefore, A is ω^{μ} -closed.

Corollary 2.21: An ω^{μ} -closed A of X is supra semi closed if and only if $scl^{\mu}(A)-A$ is supra semi-closed.

Proof: If A is ω^{μ} -closed and supra semi closed, then $scl^{\mu}(A)-A=\varphi$ by theorem 2.20. Therefore, $scl^{\mu}(A)-A$ is supra semi- closed.

Conversely, Suppose that $scl^{\mu}(A)-A$ is supra semi closed. Since $scl^{\mu}(A)\subseteq cl^{\mu}(A)$, $cl^{\mu}(A)-A$ contains the semi closed set $scl^{\mu}(A)-A$. Since, A is ω^{μ} -closed, by theorem 2.20, $scl^{\mu}(A)-A=\varphi.$ Hence, $scl^{\mu}(A)=A.$

Therefore, A is supra semi closed.

Theorem 2.22: If A is supra semi-open and ω^{μ} -closed, then A is supra closed.

Proof: Since $A\subseteq A$ and A is supra semi-open and ω^{μ} -closed we have $cl^{\mu}(A)\subseteq A$ therefore we have $cl^{\mu}(A)=A$ and A is supra closed.

Theorem 2.23: If A is an ω^{μ} -closed set of (X,μ) such that $A\subseteq B\subseteq Cl^{\mu}(A)$, then B is ω^{μ} -closed set of (X,μ).

Proof: Let U be a supra semi open of (X,μ) such that $B\subseteq U$ .Then $A\subseteq U$ .since A is ω^{μ} closed, we have $cl^{\mu}(A)\subseteq U$ .Now $B\subseteq Cl^{\mu}(A)$, then $cl^{\mu}(B)\subseteq cl^{\mu}(cl^{\mu}(A))=$

$cl^{\mu}(A)\subseteq U$ .Therefore, B is also an ω^{μ} closed. The converse of the above theorem need not be true from the following example.

Example 2.24: Let $X=\{a,b,c,d\}$ and $\mu=\{X,\phi,\{a\},\{a,b\},\{b,c\},\{a,b,c\}\}$ .Then the set $A=\{d\}$ and $B=\{c,d\}$ are ω^{μ} - closed. But $A\subseteq B\not\subset cl^{\mu}(A)$.

Theorem 2.25: Let $A \subseteq Y \subseteq X$ and suppose that A is ω^{μ} -closed set in X. Then A is ω^{μ} -closed relative to Y.

Proof: Let $A \subseteq Y \cap U$ and suppose that U is supra semi open in X. Then $A \subseteq U$ and hence $cl^{\mu}(A) \subseteq U$. It follows that $Y \cap cl^{\mu}(A) \subseteq Y \cap U$.

Definition 2.26: A subset A of X is called ω^{μ} -open if A^c is ω^{μ} - closed. The collection of all ω^{μ} - open sets in X is denoted by ω^{μ} O(X).

Theorem 2.27: In a supra topological space (X,μ) , $SO(X,\mu)=\{F \subseteq X : F^c \subseteq \mu\}$ if and only if every subset of X is ω^{μ} - closed.

Proof: Suppose that $SO(X,\mu)=\{F \subseteq X : F^c \subseteq \mu\}$. Let A be a subset of (X,μ) such that $A \subseteq U$, where $U \in SO(X,\mu)$ .Then $cl^{\mu}(U)=U$. Also, $cl^{\mu}(A) \subseteq cl^{\mu}(U)=U$. Hence, A is ω^{μ} - closed.

Conversely, suppose that every subset of (X,μ) is ω^{μ} - closed. Let $U \in SO(X,\mu)$. Since $U \subseteq U$, and U is ω^{μ} -closed, we have $cl^{\mu}(U) \subseteq U$. Thus, $cl^{\mu}(U)=U$ and $U \in \{F \subseteq X : F^c \subseteq \mu\}$. Therefore, $SO(X,\mu) \subseteq \{F \subseteq X : F^c \subseteq \mu\}$. If $F \in \{F \subseteq X : F^c \subseteq \mu\}$, then F^c is supra semi –open. Therefore, $F^c \in SO(X,\mu) \subseteq \{F \subseteq X : F^c \subseteq \mu\}$. Hence f is supra open in (X,μ) and so F is supra semi-open in (X,μ). i.e., $F \in SO(X,\mu)$ .Thus, $SO(X,\mu)=\{F \subseteq X : F^c \subseteq \mu\}$.

Theorem 2.28: A subset A of X is ω^{μ} open iff $F \subseteq \operatorname{int}^{\mu}(A)$ whenever F is supra semi closed and $F \subseteq A$.

Proof: Suppose that $F \subseteq \operatorname{int}^{\mu}(A)$, where F is supra semi closed and $F \subseteq A$.Let $A^c \subseteq U$, where U is supra semi open. Then $U^c \subseteq A$ and U^c is supra semi closed. Therefore, $U^c \subseteq \operatorname{int}^{\mu}(A)$. Since $U^c \subseteq \operatorname{int}^{\mu}(A)$, we have $(\operatorname{int}^{\mu}(A))^c \subseteq U$, i.e., $cl^{\mu}(A^c) \subseteq U$,since $cl^{\mu}(A^c)=(\operatorname{int}^{\mu}(A))^c$. Thus A^c is ω^{μ} –closed, i.e. A is ω^{μ} –open.

Conversely, suppose that A is ω^{μ} open. Let $F \subseteq A$ and F be supra semi closed in X. Then F^c is supra semi open and $A^c \subseteq F^c$. Therefore, we obtain $cl^{\mu}(A^c) \subseteq F^c$. But $cl^{\mu}(A^c)=(\operatorname{int}^{\mu}(A))^c$. Hence, $F \subseteq \operatorname{int}^{\mu}(A)$.

3. ω^{μ} CLOSURE AND ω^{μ} INTERIOR

Definition 3.1: Let (X,μ) be a supra topological space and A a subset of X. Then

(i) the ω^{μ} -closure of A , denoted by $cl^{\mu}_{\omega}(A)$ is defined as $cl^{\mu}_{\omega}(A)=\cap\{F : A \subseteq F$ and F is ω^{μ} -closed $\}$.

(ii) the ω^{μ} -interior of A , denoted by $\operatorname{int}^{\mu}_{\omega}(A)$ is defined as $\operatorname{int}^{\mu}_{\omega}(A)=\cup\{G : G \subseteq A$ and G is ω^{μ} -open $\}$.

Theorem 3.2: For the subsets A, B of a supra topological space (X,μ), the following statements hold.

(i) $A \subseteq cl_{\omega}^{\mu}(A) \subseteq cl^{\mu}(A).$

(ii) If A is ω^{μ}-closed, then $A = cl_{\omega}^{\mu}(A).$

(iii) $x \in cl_{\omega}^{\mu}(A)$ if and only if ω^{μ} –open set U containing x, $A \cap U \neq \varphi.$

(iv) If $A \subseteq B,\ cl_{\omega}^{\mu}(A) \subseteq cl_{\omega}^{\mu}(B).$

(v) $cl_{\omega}^{\mu}(A)$ is ω^{μ}-closed.

Proof:

(i) It follows from the fact that every supra semi closed set is ω^{μ}-closed.

(ii) Obvious. But if $A = cl_{\omega}^{\mu}(A)$, then A need not be a ω^{μ}-closed. Let (X,μ) be a supra topological space where

(iii) **Necessity** Suppose that $x \in cl_{\omega}^{\mu}(A).$ Let U be a ω^{μ}-open set containing x such that $A \cap U = \varphi.$ And so, $A \subseteq X \setminus U.$ But X\U is ω^{μ}-closed and hence $cl_{\omega}^{\mu}(A) \subseteq X \setminus U.$ Since $x \notin X \setminus U,$ we obtain $x \notin cl_{\omega}^{\mu}(A)$ which is contrary to the hypothesis.

Sufficiency If $x \notin cl_{\omega}^{\mu}(A)$, then there exists a ω^{μ}-closed set F of X such that $A \subseteq F \quad x \notin A.$ Therefore, $x \in X \setminus F \in \omega^{\mu}O(X).$ Hence X\F is a ω^{μ}-open set of X containing x such that $(X \setminus F) \cap A = \varphi.$ This is contrary to the hypothesis.

(iv) Obvious.

(v) Obvious.

Lemma 3.3: For the subsets A, B of a supra topological space (X,μ), the following statements hold

(i) $int_{\omega}^{\mu}(A)$ is the largest ω^{μ}-open set contained in A.

(ii) $\operatorname{int}_{\omega}^{\mu}(\operatorname{int}_{\omega}^{\mu}(A)) = \operatorname{int}_{\omega}^{\mu}(A).$

(iii) $X \setminus \operatorname{int}_{\omega}^{\mu}(A) = cl_{\omega}^{\mu}(A^{c}).$

(iv) $X \setminus cl_{\omega}^{\mu}(A) = \operatorname{int}_{\omega}^{\mu}(A^{c}).$

(v) If $A \subseteq B,$, then $\operatorname{int}_{\omega}^{\mu}(A) \subseteq \operatorname{int}_{\omega}^{\mu}(B).$

(vi) $\operatorname{int}_{\omega}^{\mu}(A) \cup \operatorname{int}_{\omega}^{\mu}(B) \subseteq \operatorname{int}_{\omega}^{\mu}(A \cup B).$

(vii) $\operatorname{int}_{\omega}^{\mu}(A) \cap \operatorname{int}_{\omega}^{\mu}(B) \supseteq \operatorname{int}_{\omega}^{\mu}(A \cap B).$

Remark 3.4: The equality does not hold in lemma 7.3.3(vi) as per the following example.

Example 3.5: Let $X = \{a,b,c,d\}$ and $\mu = \{X,\phi,\{a,b\},\{a,b,d\},\{b,c,d\}\}$.Consider the sets $A = \{a\}$ and $B = \{b,d\}$ .Then $A \cup B = \{a,b,d\}$ .Now, $\operatorname{int}_{\omega}^{\mu}(A) = \varphi$ and $\operatorname{int}_{\omega}^{\mu}(B) = \{b\}$. Also, $\operatorname{int}_{\omega}^{\mu}(A \cup B) = \{a,b,d\}$ and $\operatorname{int}_{\omega}^{\mu}(A) \cup \operatorname{int}_{\omega}^{\mu}(B) = \{b\}$.

Remark 3.6: The equality does not hold in lemma 3.3 (vii) as per the following example.

Example 3.7: Let $X=\{a,b,c,d\}$ and $\mu=\{X,\phi,\{b,d\},\{a,b,d\},\{b,c,d\},\{a,c,d\}\}$ .Consider the sets $A=\{a,b\}$ and $B=\{b,c\}$. Then $A\cap B=\{b\}$ .Now, $\text{int}_{\omega}^{\mu}(A)=\{a\}$ and $\text{int}_{\omega}^{\mu}(B)=\{b,c\}$. Also, $\text{int}_{\omega}^{\mu}(A\cap B)=\varphi.$ and $\text{int}_{\omega}^{\mu}(A)\cup\text{int}_{\omega}^{\mu}(B)=\{a,b,c\}$.

4. ω^{μ} -CONTINUOUS FUNCTIONS

Definition 4.1: Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ .A function $f:(X,\tau)\to(Y,\sigma)$ is called ω^{μ}-continuous if $f^{-1}(V)$ is ω^{μ} closed in (X,μ) for every closed set V of (Y,σ) .

Definition 4.2: Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ .A function $f:(X,\tau)\to(Y,\sigma)$ is called ω^{μ}-irresolute if $f^{-1}(V)$ is ω^{μ} closed in (X,μ) for every ω^{μ}-closed set V of (Y,σ) .

Theorem 4.3: Every continuous function is ω^{μ}-continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a continuous function and A is closed in Y. Then $f^{-1}(A)$ is a closed set in X. Since μ is associated with τ , then $\tau\subseteq\mu$ .Therefore, $f^{-1}(A)$ is supra closed in X and it is ω^{μ} closed in (X,μ). Hence f is ω^{μ}-continuous.

Remark 4.4: The converse of the above theorem need not be true as seen from the following example**.**

Example 4.5: Let $X=\{a,b,c,d\}$,with topology $\tau=\{X,\phi,\{a,c\},\{b,d\}\}$ and the supra topology is defined as follows: $\mu=\{X,\phi,\{a,c\},\{b,d\},\{a,c,d\}\}$ .Let $f:(X,\tau)\to(X,\tau)$ be a function defined by $f(a)=a,$ $f(b)=c,$ $f(c)=b,$ $f(d)=d.$ The inverse image of the closed set $\{b\}$ is $\{c\}$ which is ω^{μ}-closed but not closed.

Then f is ω^{μ}-continuous but not continuous.

Theorem 4.6: Every supra continuous function is ω^{μ}-continuous.

Proof: Obvious.

Remark 4.7 The converse of the above theorems are not true as seen from the following example.

Example 4.8: In example 4.5, the inverse image of the closed set $\{b,d\}$ is $\{c,d\}$ which is ω^{μ}-closed but not supra closed. Then f is ω^{μ}-continuous but not supra continuous.

Theorem 4.9:

(i) Every ω^{μ}-continuous function is g^{μ} -continuous.

(ii) Every ω^{μ}-continuous function is sg^{μ} -continuous.

(iii) Every ω^{μ}-continuous function is gs^{μ} -continuous.

(iv) Every ω^{μ} -irresolute function is ω^{μ}-continuous.

Proof: obvious.

Remark 4.10: The converse of the above theorem need not be true as seen from the following example.

Example 4.11: Let $X = \{a,b,c,d\}$ with topology $\tau = \{X, \phi, \{a\}\}$ and $\mu = \{X, \phi, \{a\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ be the supra topology on X.

Let $f : (X,\tau) \to (X,\tau)$ be a function defined by $f(a) = a,\ f(b) = d,\ f(c) = b,\ f(d) = c.$

(i) The inverse image of the g^{μ} closed set $\{a,c,d\}$ is $\{a,b,c\}$ which is not ω^{μ}-closed. Then f is g^{μ}-continuous but not ω^{μ} -continuous.

(ii) The inverse image of the sg^{μ} closed set $\{a,b,d\}$ is $\{a,b,c\}$ which is not ω^{μ}-closed. Then f is sg^{μ} -continuous but not ω^{μ} -continuous.

(iii) The inverse image of the gs^{μ} closed set $\{a,d\}$ is $\{a,b\}$ which is not ω^{μ}-closed. Then f is gs^{μ}-continuous but not ω^{μ} -continuous.

(iv) The inverse image of the ω^{μ}-closed set $\{d\}$ is $\{b\}$ which is not ω^{μ}-closed .Then the function on X is ω^{μ}-continuous but not ω^{μ}-irresolute.

Theorem 4.14: Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ .If $f : (X,\mu) \to (Y,\sigma)$ is continuous ω^{μ} -closed and A is an ω^{μ} – closed subset of X , then $f(A)$ is an ω^{μ}-closed set in Y.

Proof: Let U be a semi open set in (Y,σ) such that $f(A) \subseteq U.$ Since f is continuous, $f^{-1}(U)$ is a semi open set containing A. Hence $cl(A) \subseteq f^{-1}(U)$ as A is ω^{μ} -closed in (X,μ). Since f is ω^{μ}- closed, $f(cl(A))$ is an ω^{μ} -closed set contained in the semi open set U, which implies that $cl(f(cl(A))) \subseteq U$ and hence $cl(f(A)) \subseteq U$ .Therefore, $f(A)$ is an ω^{μ}–closed set.

5. APPLICATIONS

Definition 5.1: A supra topological space (X,μ) is called T_{ω}^{μ} - space if every ω^{μ}-closed in it is supra closed

Theorem 5.2: Let (X,μ) be a supra topological space then

(i) $O^{\mu}(\tau) \subset \omega^{\mu}O(\tau)$

(ii) A space (X,μ) is T_{ω}^{μ} iff $O^{\mu}(\tau) = \omega^{\mu}O(\tau)$

Proof: Obvious.

Theorem 5.3: For a space (X,μ), the following are equivalent:

(i) (X,μ) is a T_{ω}^{μ}-space .

(ii) Every singleton of (X,μ) is either supra semi-closed or supra open.

Proof: $(i)\to(ii)$: Assume that for some $x \in X$, the set $\{x\}$ is not a supra semi closed set of (X,μ). Then the only supra semi open set containing $\{x\}^c$ is X and so $\{x\}^c$ is ω^{μ}-closed in (X,μ). By assumption $\{x\}^c$ is supra closed or equivalently $\{x\}$ is supra open in (X,μ) .

$(ii)\to(i)$: Let A be a ω^{μ}-closed subset of (X,μ) and let $x \in cl^{\mu}(A)$ .By assumption, $\{x\}$ is either supra semi-closed or supra open.

Case 1: Suppose $\{x\}$ is supra semi-closed. If $x \notin A$ then $cl^{\mu}(A) - A$ contains a non-empty supra semi-closed set, which is a contradiction to Theorem 7.2.22 .Therefore $x \in A$.

Case 2: Suppose $\{x\}$ is supra open. Since $x \in cl^{\mu}(A)$, $\{x\} \cap A \neq \varphi$ and so $x \in A$. Thus in both cases, and $x \in A$ therefore $cl^{\mu}(A) \subseteq A$ or equivalently A is a supra closed set of (X,μ)

Definition 5.4: A supra topological space (X,μ) is called ${}_gT^{\mu}_{\omega}$ - space if every g^{μ}-closed set of (X,μ) is an ω^{μ} closed.

Theorem 5.5: Let (X,μ) be a supra topological space then

(i) $g^{\mu}O(\tau) \subset \omega^{\mu}O(\tau)$

(ii) A space (X,μ) is ${}_gT^{\mu}_{\omega}$ iff $g^{\mu}O(\tau) = \omega^{\mu}O(\tau)$

Proof: Obvious.

Theorem 5.6: If (X,μ) is a ${}_gT^{\mu}_{\omega}$ space then every singleton subset of (X,μ) is either g^{μ}-closed set or ω^{μ} – open.

Proof: Suppose that for some $x \in X$, the set $\{x\}$ is not g^{μ}-closed .Then $\{x\}$ is not a supra semi closed set, since every supra semi closed is a g^{μ}-closed set. So $\{x\}$ is not supra open and the only supra open set containing $\{x\}^c$ is X itself. Therefore, $\{x\}^c$ is trivially a g^{μ}-closed set and by assumption, $\{x\}^c$ is an ω^{μ} -closed set or equivalently $\{x\}$ is ω^{μ} –open.

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