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STOPPING RULES TO LIMIT INSPECTION EFFORT IN CSP-C CONTINUOUS SAMPLING PLAN

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ABSTRACT

This paper presents the stopping rules to limit the inspection effort during one screening sequence and the derivation or performance measures like OC, AOQ and AFI functions based on simplified Markov- Chain approach for CSP-C continuous sampling plan. Tables of i and S values indexed by AOQL and f are provided to enable the selection and implementation of CSP-C plans. These tables are constructed in order to facilitate the incorporation of CSP-C plans in the subsequent revision of standard table MIL –STD-1235C (1988) of continuous sampling plans. The advantage of the application CSP-C plans is also established.

Key Words and phrases: *Clearance Number; Acceptance Number; Sampling Frequency; Stopping Rule; AOQL.*

INTRODUCTION:

Emanating from the original continuous sampling plan CSP-1 of Dodge (1943) several different continuous sampling plans have been developed to deal with the products consisting of individual units manufactured in bulk quantity by an essentially continuous process where the formation of lots for sampling inspection is impracticable. Dodge's CSP-1 plan requires reversion to screening whenever a non- conforming unit is found during sampling inspection. Continuous sampling plan with acceptance number CSP-C of Govindaraju and Kandasamy (2000) removes this feature and allows for smoother transition between sampling inspection and screening inspection by permitting C non- conforming units during sampling inspection. The introduction of the additional parameter C enables one to pass between sampling phase and screening phase with the required discriminating power hence provides flexibility in administration. The advantage of this approach is to delay the forming of a screening crew until there is more certainty. Balamurali and Jun (2006) studied the application CSP-C plans for short run production processes.

In continuous sampling plans, the amount of screening has an important bearing on the total inspection effort. This aims to bring improvement in the production process.

In this paper a kind of stopping rule to accompany this plan is presented to limit the inspection effort. If the number of inspection units in one screening sequence exceeds some specified value or critical length the inspector has to take special action. This helps in providing prompt action to avoid wastage of materials when changes occur suddenly in incoming quality.

The simplified Markov –Chain approach is followed to derive various measures of performance such as the Average Outgoing Quality (AOQ), the Average Fraction of units inspected (AFI), and the probability of acceptance (P_a) for the CSP-C Plan. Table of i and the values of S for the stopping rule on screening inspection are provided for the selection and implementation of CSP-C plans for C=1 and 2 for a series of f values associated with sample frequency code letters with associated AOQL values and a series of AQL indices (serving to identify the plans only). These tables are constructed for incorporating CSP-C plans along with other continuous sampling plans CSP-1, CSP-2, CSP-3 and CSP-V in the subsequent revision of MIL-STD-1235C (1988) of the standard table of Continuous Sampling Plans.

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OPERATING PROCEDURE:

The operating procedure of CSP-C plan starts with screening inspection of units in the order of production and the inspection continues until the number of consecutive conforming units reaches some preassigned integer i. Then the procedure proceeds to sampling inspection, where only a pre-specified fraction f of the units is inspected. When the number of non-conforming units found during sampling inspection is equal to a preassigned integer (C+1), revert immediately to screening inspection. During sampling inspection, sample units are selected at random such that each unit has a probability f of being selected. Further, all non- conforming units found are corrected or replaced with conforming units.

If manufacturer has established a record for high quality production, it is feasible to introduce CSP-C to allow non-conforming units during sampling inspection. This provides reversion to screening only when the quality is inferior.

DERIVATION OF PERFORMANCE MEASURES:

The simplified Markov-Chain approach described by Brugger (1972) is followed to derive the measures of performance of CSP-C plans since ordinary Markov-Chain approach adopted by Govindaraju and Kandasamy (2000) is time consuming.

The phases of CSP- C plan are SC and SA where

SC = 100% inspection or Screening phase and SA = Sampling phase

The direction of flow from screening phase proceeds only to sampling phase with a probability of one, and flow from sampling phase proceeds only to screening phase with a probability of one. Thus the transitional probability matrix is.

To

From		SC	SA
	SC	0	1
	SA	1	0

The steady-state probabilities are

P”SC = P”SA
P’’SA = P”SC

Solving for the steady-state probabilities interms of any one phase (P”SA), we obtain

P”SC = P”SA
P”SA = P”SA

The expected lengths of time interms of number of units for screening and sampling phases are $\frac{(1-q^i)}{pq^i}$ and $\frac{(C+1)}{fp}$ respectively.

The formulation can be completed by forming the working table.

WORKING TABLE:

Phase	Coefficient	Expected Length	Simplification	AFI Den	AFI Num
SC	1	$(1-q^i)/pq^i$	$f(1-q^i)$	$f(1-q^i)$	$f(1-q^i)$
SA	1	$(C+1)/fp$	$(C+1)q^i$	$(C+1)q^i$	$f(C+1)q^i$

The average fraction of total units inspected (AFI) in the long run is

$$\mathrm{F} = \frac{f(1-q^i) + f(c+1)q^i}{f(1-q^i) + (c+1)q^i}$$

$$= \frac{f(1+cq^i)}{f + (c+1-f)q^i} \tag{1}$$

The average outgoing equality, under the assumption that non-conforming units are replaced with conforming units is

$$\mathrm{AOQ}\ (\mathrm{p}) = \mathrm{p}\ (1-\mathrm{F})$$

$$= \frac{pq^i(c+1)(1-f)}{f + (c+1-f)q^i} \tag{2}$$

The average fraction of total production accepted on a sampling basis (the operating characteristic function) is

P_a = (1-F)/ (1-f)

$$= \frac{(c+1)q^i}{f + q^i(c+1-f)}$$

where p is the probability that a unit is non-confirming and q=1-p.

CONSTRUCTION OF TABLES:

The average outgoing quality (AOQ) for CSP-C plan from (2) is given by

$$\mathrm{p_A} = \frac{pq^i(c+1)(1-f)}{f + q^i(c+1-f)} \tag{4}$$

The Average Outgoing Quality Limit, AOQL (p_L) is the maximum value of p_A for any given value of f, i and c over all possible values of p, the quality of incoming product.

The value of p, for which this maximum value p_L for p_A occurs is designated by p_M, hence,

$$\mathrm{p_L} = \frac{p_M(q_M)^i(c+1)(1-f)}{f + (q_M)^i(c+1-f)} \tag{5}$$

The value of p_M, for which $p_A = p_L$ is determined by differentiating (4) with respect to p, equating to zero ,and solving for p. that is,

$$\frac{dp_A}{dp} = \frac{(c+1)(1-f)}{[f + q^i(c+1-f]}(\mathrm{D_1} + \mathrm{D_2}) \tag{6}$$

where $D_1 = \{q^i - pq^{i-1}i\}\ \{f+q^i(c+1-f)\}$, $D_2 = pq^i\ \{q^{i-1}\ i(c+1-f)\}$

$$\frac{dp_A}{dp} = 0 \text{ implies } fq - fpi + q^{i+1}(c+1-f)=0 \tag{7}$$

Simplifying (7) and using the designation p_M for the maximizing value of p, we obtain

$$(1-\mathrm{p_M})^{\mathrm{i}} = \frac{f(p_M(i+1)-1)}{(c+1-f)(1-p_M)} \tag{8}$$

Substituting in (5) the value of $(1-p_M)^i$ from (8) we obtain

$$p_L = \frac{(i+1)p_M - 1}{i} - \frac{cf\,(p_M(i+1)-1)}{i(c+1-f)} \tag{9}$$

and $$p_M = \frac{ip_L(c+1-f)+(c+1)(c-f)}{(c+1)(1-f)(i+1)} \tag{10}$$

from (8) and (9), we have

$$p_L = \frac{(1-f)(c+1)(1-p_M)^{i+1}}{fi}$$

$$f = \frac{(1-p_M)^{i+1}(c+1)}{ip_L + (1-p_M)^{i+1}(c+1)} \tag{12}$$

CSP-C plans corresponding to C=1 and 2 are constructed using equations (9) and (12) are provided in Tables IA and IIA. In the tables, i-values are given with sample frequency code letters and series of preferred AQL indices (serving to identify the plan only) with associated AOQL values.

Murphy (1958) provided a criterion to limit an excessively long periods of screening by placing a reasonable upper limit (F*) on the fraction of material inspected. Murphy (1959), extended this concept and proposed four stopping rules. The S-stopping rules is modification of Murphy's Rule n* - i. For all the plans in MIL-STD-1235C it was decided to choose p* as the parameter of interest with no attention paid to F* and to consider AQL as p*, .01 as α and n*- i as S. The value of n*is given by $a_0 + a_1$ i,where a_0 and a_1 could be determined from the graph of Murphy (1958). Values a_0 and a_1 are functions of α and K, where K is

$$K = (1-p^*)^i = \frac{f(1-F^*)}{(C+1-f)F^* - Cf} \tag{13}$$

It is observed that K values in (13) matches with Murphy (1958) when C=0.Tables IB and IIB are developed to give the S values for C=1and 2 of CSP-C plans indexed by AQL, f and sample frequency code letters.

Tables are arranged as table IA, IIA and IB, IIB so as to include CSP-C plans in subsequent revision of MIL-STD-1235C (1988).

SELECTION OF CSP –C PLANS:

The CSP-C plans with stopping rules can be selected from tables IA, IIA and IB,IIB with reference to C=1,C=2 respectively for a specified AOQC and f.

(i). For an AOQL of 0.53% and the specified sampling frequency, f=1/5, C=1, i = 184 and S = 588

start with the screening inspection and the screening inspection continues until the 588^{th} unit and if it exceeds 588 units without any decision the inspection has to be stopped at once and the reasons are to be explored

(ii). For an AOQL of 1.9% and the specified sampling frequency f =1/10 C=2, i=86 and S=508

The performance of CSP-C plan for various C values are observed to see whether the introduction of C is beneficial. The value of AFI at 0.1% AQL and the corresponding AOQL values are computed.

c	i	1/f	AOQL	AFI (at .1%) AQL
0	1790	96	0.001449	0.05941
1	2396	96	0.001292	0.05941
2	2770	95	0.001224	0.05998
3	3042	95	0.001186	0.05999
4	3255	96	0.001165	0.05943
5	3430	96	0.001141	0.05998
6	3580	96	0.001132	0.05941

Table values reveal that AOQL values decrease for the increase in C while the AFI remains constant. This establishes the advantage of CSP-C plan.

CONCLUSION:

For a given AOQL and f CSP –C plan requires a larger value of i than CSP-1. However, CSP-C is generally more economical than CSP-1 when quality is good, since the average fraction of the total number of units inspected in the long run is lesser for CSP-C plan.

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TABLE I-B VALUES OF S FOR CSP-C PLANS WHEN C=1

Samp Freq Code Ltr	f	AQL* IN %															
		.010	.015	.025	.040	.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
A	**1/2**	3079	1509	1190	737	498	442	333	199	123	87	55	34	23	12	10	6
B	**1/3**	6650	3197	2565	1589	1083	980	749	451	278	201	126	77	57	29	21	14
C	**1/4**	10097	4789	3886	2405	1648	1513	1166	701	432	315	199	124	88	48	38	19
D	**1/5**	13362	6275	5137	3179	2186	2031	1575	948	588	426	272	165	121	66	52	32
E	**1/7**	19423	8978	7452	4610	3186	3017	2362	1415	879	642	410	252	178	96	79	40
F	**1/10**	27560	12538	10556	6527	4535	4376	3457	2069	1281	944	609	376	263	145	114	61
G	**1/15**	39377	17597	15052	9303	6502	6416	5117	3062	1896	1423	914	550	406	223	178	104
H	**1/25**	59068	25789	22515	13907	9953	9798	8046	4827	2986	2273	1464	871	669	350	292	168
I	**1/50**	97051	41005	36861	22735	17166	16211	14141	8492	5236	4104	2605	1561	1202	621	544	297
J	**1/100**	152912	62384	57832	35639	28436	25716	23960	14381	8880	7082	4569	2697	2118	1048	964	502
K	**1/200**	233995	92064	88116	54266	46004	39660	39631	23809	14639	12063	7715	4485	3602	1791	1651	926
		.018	.033	.046	.074	.113	.143	.198	0.33	0.53	0.79	1.22	1.90	2.90	4.94	7.12	11.46
		AOQL IN %															

TABLE II-B VALUES OF S FOR CSP-C PLANS WHEN C=2

Samp Freq Code Ltr	f	AQL* IN %															
		.010	.015	0.025	.040	.065	0.100	0.150	0.250	0.400	0.650	1.0	1.5	2.5	4.0	6.5	10.0
A	**1/2**	4173	2032	1610	997	678	604	459	274	169	121	77	48	33	18	13	6
B	**1/3**	9001	4284	3467	2145	1467	1340	1033	617	382	277	177	109	77	43	32	19
C	**1/4**	13614	6385	5232	3239	2227	2070	1606	966	600	433	279	172	121	66	52	32
D	**1/5**	17959	8331	6898	4265	2944	2775	2163	1300	803	591	376	228	169	88	69	40
E	**1/7**	25940	11836	9940	6145	4267	4102	3229	1940	1202	888	565	355	251	134	102	61
F	**1/10**	36563	16401	13981	8644	6034	5925	4715	2828	1742	1319	840	508	373	194	160	88
G	**1/15**	51824	22802	19774	12217	8580	8633	6948	4171	2581	1962	1258	758	556	290	241	144
H	**1/25**	77020	33051	29295	18081	13302	12817	10873	6515	4034	3094	1982	1189	888	470	384	225
I	**1/50**	125047	51825	47365	29208	22728	20948	18936	11372	7004	5556	3559	2104	1655	854	758	388
J	**1/100**	194905	77921	73544	45293	37409	32923	31897	19124	11787	9606	6103	3620	2855	1409	1315	728
K	**1/200**	295999	113797	111063	68335	60073	52582	50339	31515	19390	16223	10385	6029	4923	2370	2392	1173
	AOQL IN %																

* AQL'S are provided as indices to simplify use of this table, but have no other meaning relative to the plans.

TABLE I-A VALUES OF i FOR CSP-1-C PLANS For C=1

Samp Freq Code Ltr	f	AQL* IN %															
		.010	.015	.025	.040	.065	0.10	0.150	0.250	0.400	0.650	1.0	1.5	2.5	4.0	6.5	10.0
A	**1/2**	2235	1218	874	543	355	281	202	121	75	50	32	20	13	7	5	3
B	**1/3**	3619	1974	1416	880	576	455	328	197	122	82	52	33	22	12	8	5
C	**1/4**	4633	2527	1812	1126	737	582	420	252	156	105	67	43	28	16	11	6
D	**1/5**	5435	2965	2126	1321	865	683	493	296	184	123	79	50	33	19	13	8
E	**1/7**	6673	3639	2610	1622	1062	839	606	363	226	151	97	62	40	23	16	9
F	**1/10**	8017	4372	3136	1949	1276	1008	728	436	271	181	117	75	48	28	19	11
G	**1/15**	9583	5227	3749	2330	1525	1205	870	521	324	217	140	89	58	34	23	14
H	**1/25**	11610	6333	4542	2823	1848	1460	1054	632	333	263	170	108	71	41	28	17
I	**1/50**	14452	7884	5655	3514	2301	1818	1312	787	489	328	211	135	88	51	35	21
J	**1/100**	17391	9486	6805	4229	2769	2187	1579	947	589	394	255	163	106	61	42	25
K	**1/200**	20413	11136	7988	4965	3250	2568	1854	1112	691	463	299	191	124	72	49	30
		.018	.033	.046	.074	.113	.143	.198	.330	.530	.790	1.22	1.9	2.9	4.94	7.12	11.46
		AOQL IN %															

TABLE II A VALUES OF i FOR CSP-1 (C) PLANS FOR C=2

Samp Freq Code Ltr	f	AQL* IN %															
		.010	.015	0.025	.040	.065	0.100	0.150	0.250	0.400	0.650	1.0	1.5	2.5	4.0	6.5	10.0
A	**1/2**	2714	1480	1061	659	432	341	246	147	91	61	39	25	16	9	6	3
B	**1/3**	4333	2363	1695	1053	689	544	393	235	146	98	63	40	26	15	10	6
C	**1/4**	5492	2995	2948	1335	874	690	498	299	186	124	80	51	33	19	13	8
D	**1/5**	6397	3489	2503	1555	1018	804	580	348	216	145	93	59	39	22	15	9
E	**1/7**	7771	4238	3040	1889	1237	977	705	423	263	176	113	73	47	27	18	11
F	**1/10**	9241	5040	3615	2247	1471	1162	839	503	312	210	135	86	56	32	22	13
G	**1/15**	10927	5960	4275	2657	1739	1374	992	595	370	248	160	102	66	38	26	16
H	**1/25**	13080	7134	5117	3180	2082	1645	1188	712	443	296	191	122	79	46	31	19
I	**1/50**	16052	8755	6280	3903	2555	2019	1457	874	543	364	235	150	98	57	39	23
J	**1/100**	19084	10410	7468	4641	3039	2401	1733	1039	646	433	279	179	116	67	46	28
K	**1/200**	22186	12098	8679	5394	3532	2790	2015	1208	751	503	325	208	135	78	54	32
		.018	.033	.046	.074	.113	.143	.198	.330	.530	.790	1.22	1.9	2.9	4.94	7.12	11.46
		AOQL IN %															

*AQL'S are provided as indices to simplify use of this table, but have no other meaning relative to the plans.