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# DISPERSION OF A SOLUTE IN A FLUID FLOW THROUGH A PIPE SUBJECT TO AN EXTERNAL ACCELERATION

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***ABSTRACT***

*In this paper the dispersion of a solute in an oscillatory flow of a Newtonian fluid in circular pipe in the presence of a uniform transverse magnetic field is studied. Periodic body acceleration is also applied. Exact solution of the equation governing the flow is obtained using the finite Hankel and Laplace transform technique. Generalized dispersion method is employed to solve the diffusion equation and obtained an exact solution. It is observed that the dispersion coefficient assumes positive and negative values as the solute moves forward and backward due to the oscillating nature of the flow. The effect of magnetic field on dispersion coefficient is found to decrease the dispersion coefficient. The presence of body acceleration is observed to enhance dispersion coefficient. The effects of magnetic field and body acceleration on mean concentration are also discussed.*

***Key Words:*** *Generalized dispersion model, Newtonian fluid, dispersion coefficient*

## 1. INTRODUCTION:

Human beings often undergo accelerative disturbances in many situations. For example, while landing and taking off air crafts, travelling in a tractor, operating a jack hammer and due to sudden movements of body during sports activities, the human body experiences external accelerations. It is observed that prolonged exposure to such acceleration resulted in several disorders like headache, loss of vision, increase in pulse rate, abdominal pain, venous pooling of blood in neck, lungs, brain etc. To protect the body from these ill effects several protective devices have been designed [6]. On the other hand, if accelerations are given to the body, by properly timing with respect to heart beat it is found that they have therapeutic effects on systemic circulation [2]. Experiments conducted on pigs showed that in the case of a cardiogenic shock, blood pressure and cardiac output increased when acceleration was given to the body in synchrony with heart beat. This method was suggested to be a salient feature for assisting circulation, particularly in the case of patients who are to be treated for cardiogenic shock.

The study of dispersion of a soluble matter in a solvent flowing in channels or pipes has importance in many chemical and biological systems. It is observed that oscillatory flow effects the entire dispersion process when the amplitude of the pressure gradient is larger than the mean pressure gradient. Using the method of moments, Aris [1] investigated the dispersion of a solute in an oscillatory flow. It is observed that the effective molecular diffusivity contained terms proportional to the sequence of the amplitude of the pressure pulse. Chatwin [4] analyzed the dispersion of a passive contaminant along the axis of a tube in which the flow is driven by a longitudinal pressure gradient varying harmonically with time. He showed that strong oscillatory effects dominate the character of contaminant cloud over time intervals of many periods. Smith [14], in his study showed that the sensitivity to time of release of contaminant during a cycle and the importance of the location of a discharge source on the flow. Jaeger and Kurzweg [7] obtained the dispersion coefficient in an oscillatory flow and observed that it is proportional to the sequence of the amplitude of the oscillation and the first power of frequency in the Womersley number to vary from 3 to 15. Watson [16] studied the exact analysis for the diffusion in an oscillatory flow in a pipe of arbitrary cross section. The resultant flux of the diffusing substance has been analytically evaluated for the cases of a circular pipe and channel. The general behaviour of the flux for an arbitrary cross – section in the limiting cases of slow and fast oscillations were discussed. Sarkar and Jayaraman [13] analysed the dispersion of a solute in an annulus in the presence of an oscillatory flow field. Ramana and Sarojamma [11] investigated the phenomenon of dispersion of a solute in blood vessels when the body is subjected to a periodic body acceleration using the generalized dispersion model. They observed that the body acceleration enhances the dispersion coefficient in large and small arteries.

In this paper, a mathematical model is developed to understand the effects of magnetic field and external accelerations on the phenomenon of dispersion of a solute in blood flows.

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## 2 MATHEMATICAL FORMULATION:

Consider the flow of blood modelling in an artery assuming it to be a circular tube and blood is modeled as a Newtonian fluid. A uniform transverse magnetic field of strength $B_0$ is applied. For $t^* > 0$, the flow is subjected to a periodic body acceleration G in the axial direction. Let $a_0$ be the amplitude, $f_b$ the frequency in Hz and $\phi$ the lead angle of G with respect to the heart action, $\omega_b = 2\pi f_b$ is the circular frequency. The body acceleration G is given by

$$G = a_0 \cos(\omega_b t^* + \phi) \tag{1}$$

We represent the pumping action of heart by the pressure gradient $\frac{\partial p^*}{\partial z^*}$ produced by it. In human beings $\frac{\partial p^*}{\partial z^*}$ takes the approximate form

$$\frac{\partial p^*}{\partial z^*} = A_0 + A_1 \cos \omega_p t^* \tag{2}$$

where $A_0$ is the constant component of pressure gradient, $A_1$ is the amplitude of the fluctuating component, $\omega_p = 2\pi f_p$ and $f_p$ is the pulse frequency.

We assume that the flow is laminar and axi-symmetric and frequency of body acceleration $f_b$ is so small that the wave effects can be neglected.

Under the above mentioned assumptions, the equation of motion for flow, following [12, 15] in cylindrical polar co-ordinates (r, θ, z) can be written in the non-dimensional form as

$$\frac{1}{Sc}\frac{\partial w}{\partial t} = D_1 \cos(\omega_1 t + \phi) + D_2 + D_3 \cos(\omega_2 t) + \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - M^2 w \tag{3}$$

with non dimensional variables

$$w = \frac{w^*}{w_0}, \quad r = \frac{r^*}{R}, \quad t = \frac{D_m t^*}{R^2}, \quad C = \frac{C^*}{C_0}, \quad z = \frac{D_m \bar{z}}{R^2 w_0} \tag{4}$$

where $\rho$ is the density of blood, $\mu$ is the coefficient of viscosity of blood, $\sigma$ is the electrical conductivity of the blood, $w$ represents the non dimensional velocity, $w_0$ is velocity in plane Poiseuille flow , R is the radius of the tube

$D_1 = \frac{\rho a_0 R^2}{\mu u_0}$; $D_2 = \frac{A_0 R^2}{\mu u_0}$; $D_3 = \frac{A_1 R^2}{\mu u_0}$; $\omega_1 = \frac{\omega_b R^2}{D_m} = \frac{\omega_b R^2}{\upsilon}\frac{\upsilon}{D_m}$; $M = B_0 R \sqrt{\sigma/\mu}$ is the Hartmann number $\omega_2 = \frac{\omega_p R^2}{D_m} = \frac{\omega_p R^2}{\upsilon}\frac{\upsilon}{D_m}$; $A_r = \frac{a_0}{A_0}$; $F_r = \frac{\omega_1}{\omega_2}$, $Sc = \frac{\upsilon}{D_m}$ is the Schmidts number ,

The initial and boundary conditions in the non dimensional form are given by

$$w(r,0) = 2\sum_{k=1}^{\infty} \frac{J_0(r\lambda_k)}{\lambda_k J_1(\lambda_k)} \frac{D_2 + D_3 + D_1 \cos\phi}{\lambda_k^2 + M^2} \tag{5a}$$

$w$ and $\nabla^2 w$ are all finite at r = 0 (5b)

$w = 0$ and $\nabla^2 w = 0$ at r = 1 (5c)

where $\nabla^2 = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)$

where $\lambda_k$ are the roots of the equation $J_0(\lambda_k) = 0$

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By applying Laplace transform and Hankel transform to equation (3) we obtain the required solution as

$$w(r, t) = 2\sum_{k=1}^{\infty} w^*(\lambda_k, t)\ \frac{J_0(\lambda_k r)}{J_1^2(\lambda_k)} = 2\sum_{k=1}^{\infty} \frac{J_0(\lambda_k r)}{\lambda_k J_1(\lambda_k)} \left\{ \frac{D_2}{\beta} + \frac{D_3(\beta \cos(\omega_2 t) + m \sin(\omega_2 t))}{m^2 + \beta^2} \right.$$

$$\left. + D_1 \left( \frac{\beta \cos(\omega_1 t + \phi) + \omega_1 m \sin(\omega_1 t + \phi)}{\beta^2 + m^2 \omega_1^2} \right) - e^{-ht} \left( \frac{D_2}{\beta} + \frac{D_3 \beta}{m^2 + \beta^2} + \frac{D_1(\beta \cos\phi + \omega_1 m \sin(\phi))}{m^2 \omega_1^2 + \beta^2} - \frac{\gamma}{\beta} \right) \right\} \quad (6)$$

We consider the dispersion of a bolus of a solute which is initially of $z_s$ units in length and of uniform concentration $C_0$. For a fully developed, laminar flow in a tube, the unsteady convective diffusion equation which describes the local concentration *C* of the solute as a function of longitudinal (axial) coordinate *z*, transverse (radial) co-ordinate *r* and time *t* can be written in non-dimensional form as

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{Pe^2} \frac{\partial^2}{\partial z^2} \right) C \quad (7)$$

where $C = \frac{C^*}{C_0}$, $z = \frac{D_m z^*}{R^2 w_0}$, $D_m$ is the molecular diffusivity, and Pe = $\frac{R u_0}{D_m}$ is the Peclet number. (8)

The initial and boundary conditions are

$$C(0, r, z) = 1 \text{ if } |z| \le z_s / 2 \quad (9a)$$

$$C(0, r, z) = 0 \text{ if } |z| > z_s / 2 \quad (9b)$$

$$C(t, r, \infty) = 0 \quad (9c)$$

$$\frac{\partial C}{\partial r}(t, 0, z) = 0 = \frac{\partial C}{\partial r}(t, 1, z) \quad (9d)$$

**3. METHOD OF SOLUTION:**

Following the solution procedure of [5] we assume the concentration C (t, r, z) as a series expansion in $\partial^n C_m / \partial z^n$ given by

$$C(t, r, z) = \sum_{n=0}^{\infty} f_n(t, r)\ \frac{\partial^n C_m}{\partial z^n} \quad (10)$$

where $C_m = 2 \int_0^1 C\ r\ dr$ (11)

is the mean concentration over a cross section

Multiply equation (7) by 2r and integrating with respect to r from 0 to 1, we get

$$\frac{\partial C_m}{\partial t} = \frac{1}{Pe^2} \frac{\partial^2 C_m}{\partial z^2} - 2 \frac{\partial}{\partial z} \int_0^1 u(t, r)\ C(t, z, r,) r\ dr \quad (12)$$

If we introduce (10) into (12), the following dispersion model for $C_m$ is obtained as

$$\frac{\partial C_m}{\partial t} = \sum_{i=1}^{\infty} K_i(t)\ \frac{\partial^i C_m}{\partial z^i} \quad (13)$$

where the $K_i$'s are given by

$$K_i(t) = \frac{\delta_{i2}}{Pe^2} - 2 \int_0^1 u(t, r)\ f_{i-1}(t, r)\ r\ dr, \quad i = 1, 2, 3\ldots \quad (14)$$

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and $\delta_{i,2}$ is the Kronecker delta. The first two terms on the right hand side of (13) describe the transport of $C_m$ in the axial direction z through convection and diffusion respectively, and therefore the coefficients $K_1$ and $K_2$ are termed as the convection and diffusion coefficients for $C_m$. For a steady flow in the absence of the magnetic field, $K_1 = 0$ and $K_2 = 1/Pe^2 + 1/192$ [5]. But, both the dispersion coefficients $K_1$ and $K_2$ are harmonic functions of time when dispersion is considered in an oscillatory flow field [8].

Substituting (10) in (7), using equation (13) and equating the coefficients of $\frac{\partial^n C_m}{\partial z^n}$, gives the partial differential equation for $f_n$ as

$$\frac{\partial f_n}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_n}{\partial r}\right) - u(t,r)\, f_{n-1} + \frac{1}{Pe^2} f_{n-2} - \sum_{i=1}^{n} K_i f_{n-i}, \quad n = 1, 2\ldots \tag{15}$$

where $f_{-1} = 0$ *and* $f_0 = 1$.

From equation (9), the initial and boundary conditions on $f_n$ are obtained as

$$f_n(0, r) = 0 \tag{16a}$$

$$\frac{\partial f_n}{\partial r}(t, 0) = 0 = \frac{\partial f_n}{\partial r}(t,1), \; n = 1, 2\ldots \tag{16b}$$

Equations (14) and (15) lead to coupled system of partial equations with boundary and initial conditions described by equations (16)

Thus, from (6) and using (14) and $f_0 = 1$, we get

$$\begin{aligned} K_1(t) = -4\sum_{k=1}^{\infty}\frac{1}{\lambda_k^2}\Bigg\{&\frac{D_2}{\beta} + D_3\frac{[\beta\cos\omega_2 t + m\sin\omega_2 t]}{(m^2+\beta^2)} + D_1\frac{[\beta\cos(\omega_1 t+\phi)+\omega_1 m\sin(\omega_1 t+\phi)]}{m^2{\omega_1}^2+\beta^2} \\ &- e^{-ht}\left(\frac{D_2}{\beta} + D_3\frac{\beta}{m^2+\beta^2} + D_1\frac{[\beta\cos\phi+\omega_1 m\sin\phi]}{m^2{\omega_1}^2+\beta^2} - \frac{\gamma}{\beta}\right)\Bigg\} \end{aligned} \tag{17}$$

Using equation (15) and $f_0 = 1$, the partial differential equation in $f_n$ for n = 1 can be expressed as,

$$\frac{\partial f_1}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_1}{\partial r}\right) - w(t,r) - K_1(t) \tag{18}$$

From equation (16), we get initial and boundary conditions for $f_1$ as

$$f_1(0, r) = 0 \tag{19a}$$

$$\frac{\partial f_1}{\partial r}(t, 0) = 0 = \frac{\partial f_1}{\partial r}(t,1) \tag{19b}$$

Solving the non-homogeneous partial differential equation (18) satisfying the conditions (19a - b), the expression for $f_1$ is obtained as

$$f_1(t, r) = \sqrt{2}\sum_{n=1}^{\infty} B_n(t)\,\frac{J_0(\mu_n r)}{J_0(\mu_n)} \tag{20}$$

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$$B_n\ (t) = -2\sqrt{2}\ e^{-\mu_n^2 t}\sum_{m=1}^{\infty} E_1\{E_2\left(e^{\mu_n^2 t}-1\right)+E_3\left[e^{\mu_n^2 t}\left(\mu_n^2\cos\omega_2 t+\sin\omega_2 t\right)-\mu_n^2\right]$$
$$+E_4\left[e^{\mu_n^2 t}\left(\mu_n^2\sin\omega_2 t-\cos\omega_2 t\right)+1\right]+E_5\left[e^{\mu_n^2 t}\left(\omega_1\sin\omega_1 t+\mu_n^2\cos\omega_1 t\right)-\mu_n^2\right] \quad (21)$$
$$-E_6\left[e^{\mu_n^2 t}\left(\mu_n^2\sin\omega_1 t-\omega_1\cos\omega_1 t\right)+\omega_1\right]+E_7\left[e^{\mu_n^2 t}\left(\mu_n^2\sin\omega_1 t-\omega_1\cos\omega_1 t\right)+\omega_1\right]$$
$$+E_8\left[e^{\mu_n^2 t}\left(\omega_1\sin\omega_1 t+\mu_n^2\cos\omega_1 t\right)-\mu_n^2\right]-E_9\left(\left(e^{\left(\mu_n^2-h\right)t}-1\right)/\mu_n^2-h\right)\}$$

where $E_1=\dfrac{1}{\lambda_k^2-\mu_n^2}$, $E_2=\dfrac{D_2}{\beta\,\mu_n^2}$, $E_3=\dfrac{D_3\,\beta}{\left(m^2+\beta^2\right)\left(\mu_n^2+\omega_2^2\right)}$, $E_4=\dfrac{D_3\,P}{\left(P^2+C_1^2\right)\left(\mu_n^2+\omega_2^2\right)}$,

$E_6=\dfrac{D_1\,\beta\cos\phi}{\left(\beta^2+m^2\omega_1^2\right)\left(\mu_n^2+\omega_1^2\right)}$ $E_6=\dfrac{D_1\,\beta\sin\phi}{\left(\beta^2+m^2\omega_1^2\right)\left(\mu_n^2+\omega_1^2\right)}$, $E_7=\dfrac{D_1\,\omega_1 m\cos\phi}{\left(\beta^2+m^2\omega_1^2\right)\left(\mu_n^2+\omega_1^2\right)}$

$E_8=\dfrac{D_1\,\omega_1 m\sin\phi}{\mu_n^2+\omega_1^2}$, $E_9=\dfrac{D_2}{\beta}+\dfrac{D_3\beta}{m^2+\beta^2}+\dfrac{D_1\left(\beta\cos\phi+\omega_1\,m\sin\phi\right)}{\beta^2+\omega_1^2 m^2}-\dfrac{\gamma}{\beta}$

where $\mu_n$'s are zeros of the Bessel's function $J_1$.

Using (20) and (14) we get the expression for $K_2$ as

$$K_2(t)-\frac{1}{Pe^2}=2\sqrt{2}\sum_{n=1}^{\infty}B_n(t)\,H_n(t) \quad (22)$$

where $H_n(t)=\sum_{k=1}^{\infty}\dfrac{1}{\lambda_k^2-\mu_n^2}\left\{\dfrac{D_2}{\beta}+\dfrac{D_3\left[\beta\cos(\omega_2 t)+m\sin(\omega_2 t)\right]}{\left(m^2+\beta^2\right)}-e^{-ht}\left(\dfrac{D_2}{\beta}+\dfrac{D_3\,\beta}{m^2+\beta^2}+\dfrac{D_1\left(\beta\cos\phi+\omega_1\,m\sin\phi\right)}{\beta^2+\omega_1^2\,m^2}-\dfrac{\gamma}{\beta}\right\}$ (23)

**Solution for Mean Concentration $C_m$:**

Neglecting $K_3(t)$ and higher order coefficients, the generalized dispersion model leads to

$$\frac{\partial C_m}{\partial t}=K_1(t)\frac{\partial C_m}{\partial z}+K_2(t)\frac{\partial^2 C_m}{\partial z^2} \quad (24)$$

The initial and boundary conditions for $C_m$ are given by

$$C_m(0,z)=1,\ \text{if}\ |z|\le z_s/2 \quad (25a)$$

$$C_m(0,z)=0,\ \text{if}\ |z|>z_s/2 \quad (25b)$$

$$C_m(t,\infty)=0 \quad (25c)$$

The solution of the mean concentration for equation (24) with the help of the condition (25) is given by

$$C_m=\frac{1}{2}\left[\ erf\left(\frac{z_s/2-z_1}{2\sqrt{\xi}}\right)+\ erf\left(\frac{z_s/2-z_1}{2\sqrt{\xi}}\right)\right] \quad (26)$$

where $z_1=z+\int_0^t K_1(\eta)d\eta$, $\xi=\int_0^t K_2(\eta)d\eta$ (27)

**4. RESULTS AND DISCUSSION:**

The objective of the present investigation is to understand the combined effect of body acceleration and magnetic field on the dispersion of solutes in blood flows in human beings modelling blood as a Newtonian fluid. To obtain a quantitative idea of the effects of body acceleration on the phenomenon of dispersion of solutes in blood flow, the results are discussed in large and small arteries. The relevant data for various arteries is compiled from published literature [3, 9, 10, 17] and is presented in Table 1

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**Table: 1 Data for different arteries**

| Sl. No. | Artery | Radius ($\times 10^{-2}$m) | Average Velocity ($\times 10^{-2}$m s$^{-1}$) | $A_0$ ($\times$10 Kg m$^{-2}$ s$^{-1}$) | $A_1$ ($\times$10 Kg m$^{-2}$ s$^{-1}$) |
|---|---|---|---|---|---|
| 1 | Aorta | 1.0 | 45.6 | 7.3 | 1.46 |
| 2 | Femoral | 0.5 | 50.0 | 32.0 | 6.4 |
| 3 | Carotid | 0.4 | 50.0 | 50.0 | 10.0 |
| 4 | Coronary | 0.15 | 98.25 | 698.65 | 139.74 |

The dispersion process in oscillatory flows varies from that of steady flow case. In an oscillator flow it is possible that the flow might have changed direction before the dispersion process had time to become fully effective. Due to fluctuations in velocity the dispersion coefficient assumes both positive and negative values. In a period of oscillation owing to the reversal flow the solute would be carried backward along with flow and thus negative values for dispersion are induced. Therefore, in a period of oscillation the dispersion of solutes contracts at each flow reversal. In addition to the fluctuating character of the dispersion of solute, the dispersion is further influenced by the body acceleration and magnetic field.

The results have been discussed in aorta, femoral, carotid and coronary arteries for different values of amplitudes of body acceleration, Hartmann number M, Schmidt's number Sc and the slug input length $z_s$ .

Fig. 1 (a-d) describes the distribution of mean concentration along the axial direction at different times ($T_n = nT_r/4$ for $0 \le n \le 7$ and $T_r = 2\pi/\alpha^2 Sc$), when the slug input length $z_S = 0.02$, $a_0 = 0.98$, Sc = 100, M=1 and the phase angle $\phi = \pi/3$. It is observed that in all the arteries the peak value of the mean concentration $C_m$ decreases as the dispersion time decreases and subsequently the profile becomes flattened. The peak value for $T_1$ is attained at origin in all arteries and the peak value drifts to the right of origin for the subsequent time values. The peak value in aorta is attained in the vicinity of origin. However, as the radius of the artery decreases the peak is drifted very much away from origin. In femoral and carotid the peak value of $C_m$ is attained in the intervals (0, 0.5) while in coronary it is attained in $0 < z < 18$.

Fig 2 (a-d) depicts the variation of mean concentration $C_m$ versus axial distance for different values of the amplitude of the body acceleration when Sc = 100, M=1 and the phase angle $\phi = \pi/3$.. It is seen that in aorta, when the amplitude of the body acceleration is 0.98 there is no much variation when compared to the corresponding case in the absence of body acceleration. But when $a_0$ =1.96 there is a significant decrease in the peak of the mean concentration. The peak value of the mean concentration is drifted towards the right of the origin. However, in femoral and carotid arteries the mean concentration is decreased significantly by the presence of body acceleration and further decreased with increase in amplitude of body acceleration. In femoral artery the peak value of mean concentration in the presence of body acceleration is reduced from 0.999 to 0.644 ($a_0 = 0.98$). When $a_0 = 1.96$ it is half of the peak value of that in the absence of the body acceleration. It is observed that the effect of body acceleration in coronary on $C_m$ is not appreciable.

Fig 3(a-d) presents the variation of mean concentration versus the axial distance when Sc = 100 and $a_0 = 0.98$ for different values of Hartmann number. It is observed that the presence of magnetic field increases the mean concentration. As the intensity of the magnetic field (i.e., Hartmann number) increases $C_m$ is also increased. In aorta there is a two fold increase in the peak value of $C_m$ when M = 2 to that of the value in the absence of the magnetic field. The points of the peak values are drifted towards the origin as M increases. A similar behavior is noticed in femoral and carotid arteries. But in coronary artery the influence of magnetic field is relatively less. The peak value of mean concentration increases from 0.5062 to 0.7885 when M takes the value from 0 to 2 in coronary artery.

Fig 4 (a-d) shows the variation of dispersion coefficient $K_2$ versus time when Sc = 1, M = 7 for different values of amplitude of body acceleration. Due to the oscillatory nature of the flow, $K_2$ shows an oscillatory behavior. It assumes positive and negative values due to the forward and backward movement of the solute. It is also observed that it is harmonic. The presence of body acceleration increases the magnitude of the dispersion coefficient. When M = 7 and $a_0$ =1.96 the maximum value of $K_2$ in aorta is twice that of the corresponding case in the absence of body acceleration. In femoral and carotid arteries the impact of body acceleration is very meager and in coronary artery the effect of body acceleration is negligible.

Fig 5(a-d) describes the effect of magnetic field on $K_2$ in one cycle of time. It is noticed that the presence of the magnetic field decreases the dispersion coefficient in all arteries. The negative values of $K_2$ are not symmetrical with reference to origin. In aorta in the presence of magnetic field (M = 1) the maximum value of $K_2$ is reduced to 3.025 from 4.2357 when M = 0. When M = 2, the maximum value of $K_2$ is three times less than that of the non-magnetic

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case. When M = 3, 4 the maximum and minimum values are symmetrical about the origin. For M = 4 the maximum value of $K_2$ is reduced by 12 times of that value corresponding to M = 1.When M = 7, $K_2$ is almost uniformly zero. A similar trend is noticed in the rest of the three arteries qualitatively. However, the magnitude of the dispersion coefficient increases with decrease in the size of the artery.

Fig 6 (a-d) describes the mean concentration $C_m$ along the axial direction for different values of $A_r$, the ratio of amplitudes of the body acceleration to that of the pressure gradient. When the amplitude of body acceleration is half of the amplitude of the pressure gradient, the peak value of the mean concentration in aorta occurs at z =1 and when it is equal the point of peak value drifts to the right of z =1 and it decreases from 0.0173 to 0.0154. With an increase in $A_r$ the point of peak value of $C_m$ drifts further and the magnitude is also decreased in aorta. In the remaining arteries also $C_m$ shows a similar behavior qualitatively.

The variation of ratio of frequencies of body acceleration to that of pressure gradient ($F_r$) on dispersion coefficient is shown in Fig 7 (a-d). In aorta, when frequency of body acceleration is half of the pressure gradient, $K_2$ assumes negative values attaining a minimum and starts increasing and assumes positive values again attaining a maximum value and shows a periodical behavior in the remaining cycle. When the frequencies are equal, $K_2$ increases, assuming positive values and then decreases assuming negative values and shows the same behavior in the rest of the cycle as in the case $F_r = 0.5$ when the frequency of body acceleration is twice that of the pressure gradient, it is qualitatively same as in the case $F_r = 0.5$. In rest of all arteries a similar behavior is noticed.

Fig 8 (a-d) illustrates the mean concentration for different values of Hartmann number in the four arteries when the observation point is inside the slug input. It is observed that the presence of magnetic field enhances the $C_m$ and it reduces with increase in time. A similar behavior is noticed in the remaining arteries also. Fig 9 (a-d) shows the distribution of mean concentration when the point of observation is outside the slug. In aorta it is noticed that there is a sudden rise in $C_m$ and attains its peak value and drops significantly in course of time. The presence of magnetic field and increase in magnetic field reduces the value of $C_m$ and the drop in its value after attaining its peak value is also controlled. In femoral the peak values are observed to be the same. In carotid qualitatively a similar behavior as in aorta is noticed. But the peak values are observed to be attained almost at the same time for all of M. In coronary the effect of magnetic field as $C_m$ versus time is negligible.

## 5 CONCLUSIONS:

The dispersion of a solute in blood flow under the influence of periodic body acceleration and a uniform transverse magnetic field is studied. The governing equations of flow and dispersion are solved employing finite Hankel and Laplace transformation and generalized dispersion model. The study brings out the development of the mass transport due to the introduction of a solute in terms of the convection and dispersion coefficients. The results are discussed in large and small arteries. Due to the fluctuations in the velocity owing to the oscillatory flow the dispersion coefficient assumes positive and negative values. The effect of magnetic field is found to decrease the dispersion coefficient and is significant in aorta. In aorta $K_2$ is reduces by 12 times of the corresponding value when M = 1. While in the other arteries the effect of magnetic field on $K_2$ is not as prominent as in aorta. The effect of body acceleration on the dispersion coefficient $K_2$ in aorta is found to be significant where as in femoral and carotid arteries its effect is meager, while in coronary artery it is negligible. The mean concentration $C_m$ is found to increase in aorta, femoral and carotid arteries and in coronary artery it is relatively less.

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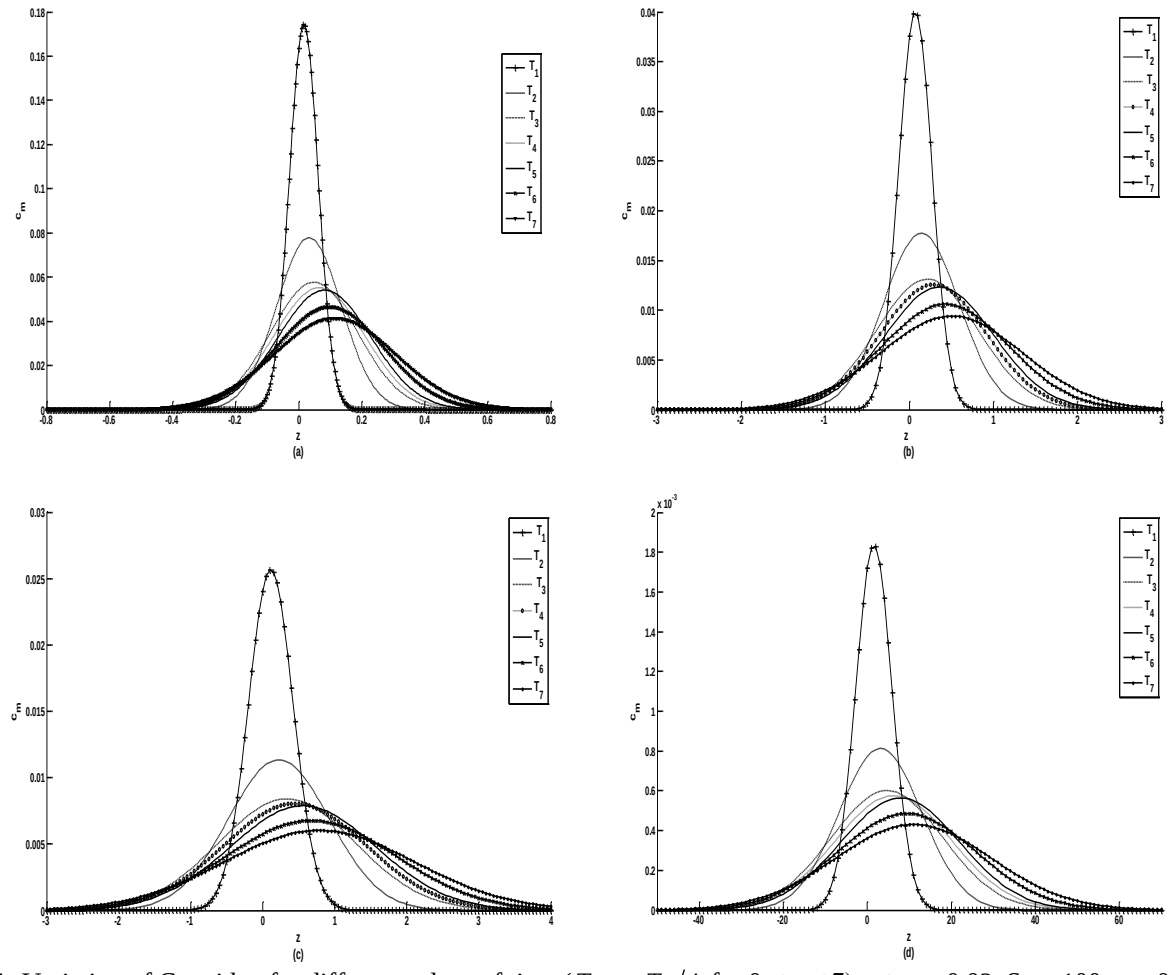


**Fig 1:** Variation of $C_m$ with z for different values of time ( $T_n = nT_r/4$ for $0 \le n \le 7$) at $z_s = 0.02$, Sc = 100, $a_0$ =0.98, M=1 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

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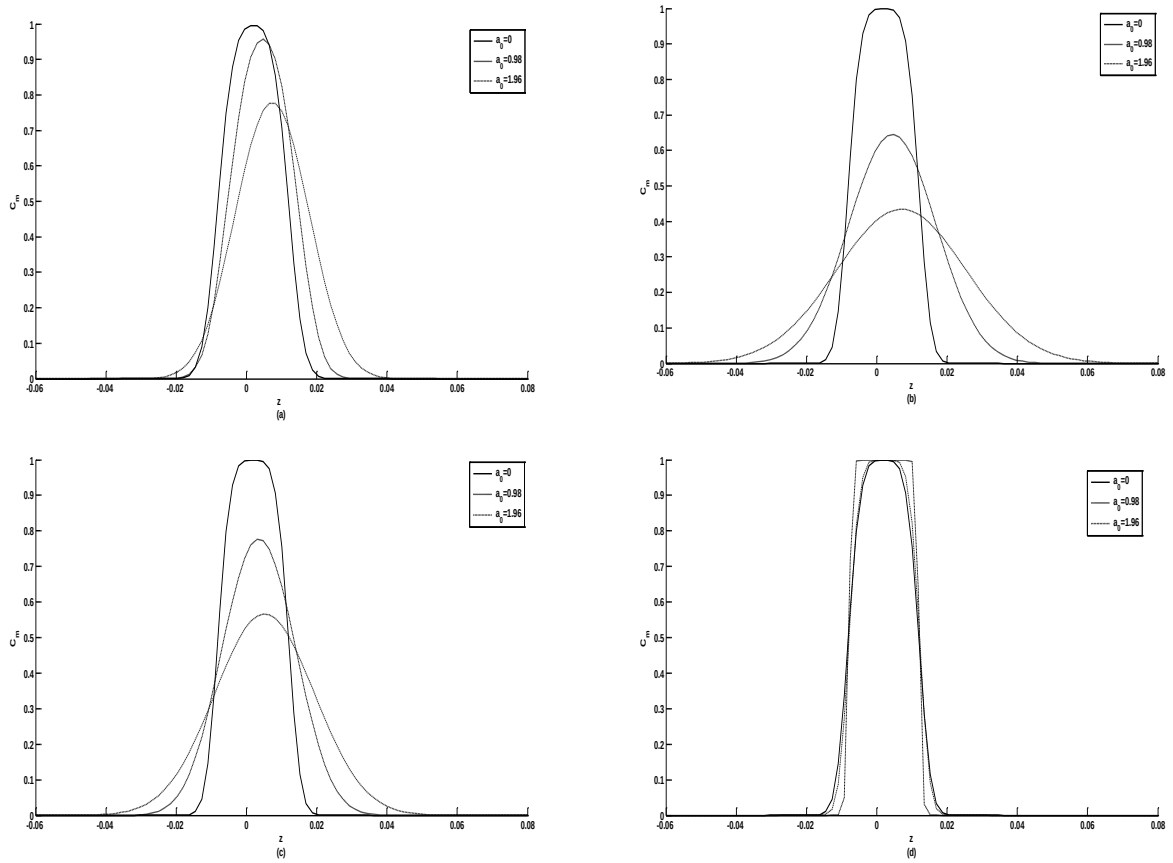


**Fig 2:** Variation of $C_m$ with z for different values of $a_0$ at $z_s$ = 0.02, Sc = 100, M=1 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

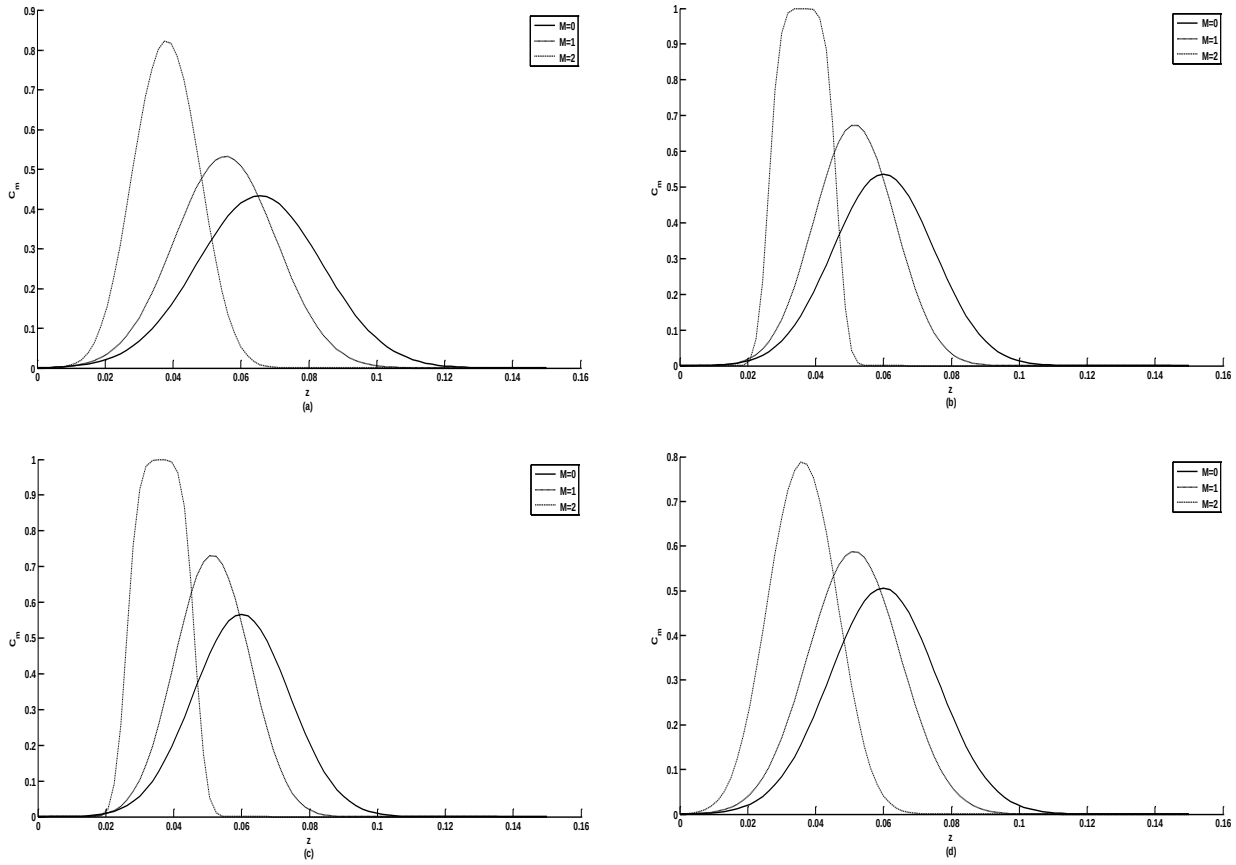


**Fig 3:** Variation of $C_m$ with z for different values of M at $z_s$ = 0.02, Sc = 100 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

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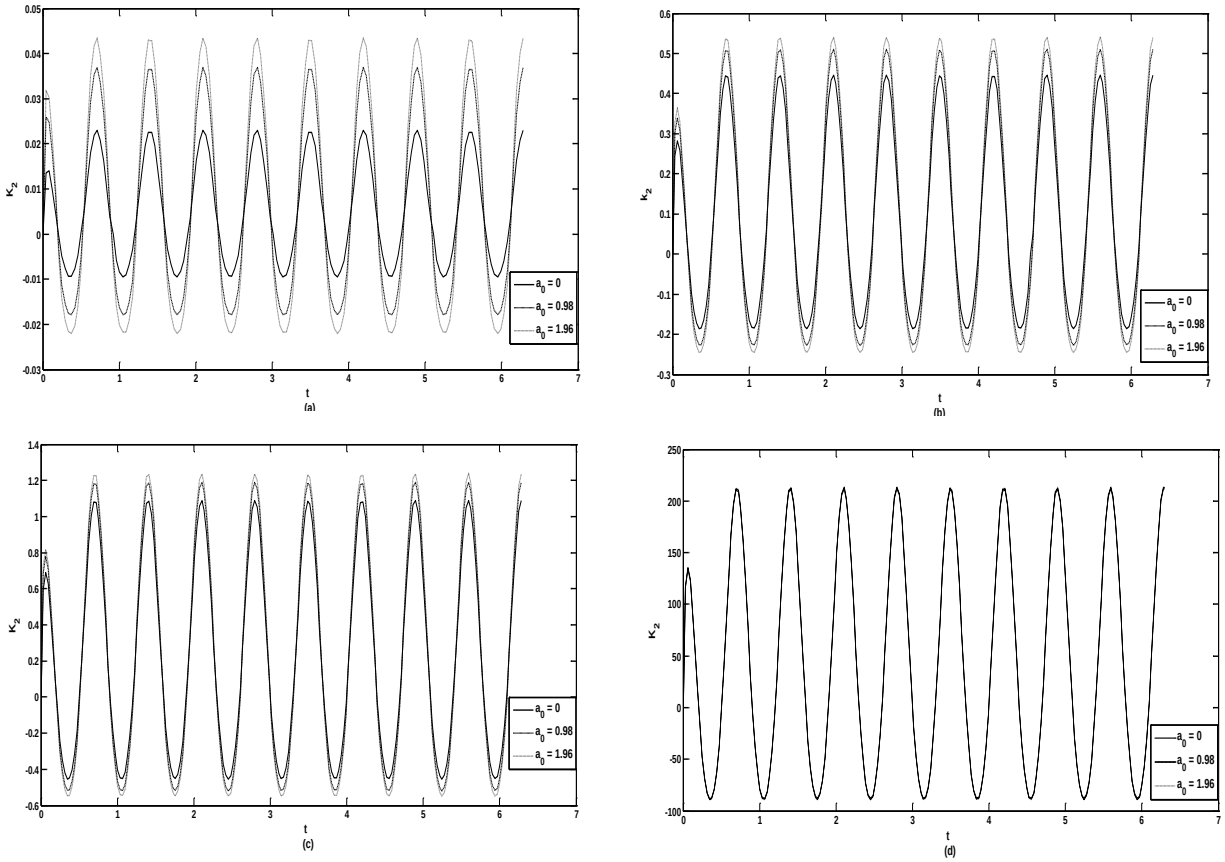


**Fig 4:** Variation of $K_2$ with t for different values of $a_0$ at Sc = 1, M = 7 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

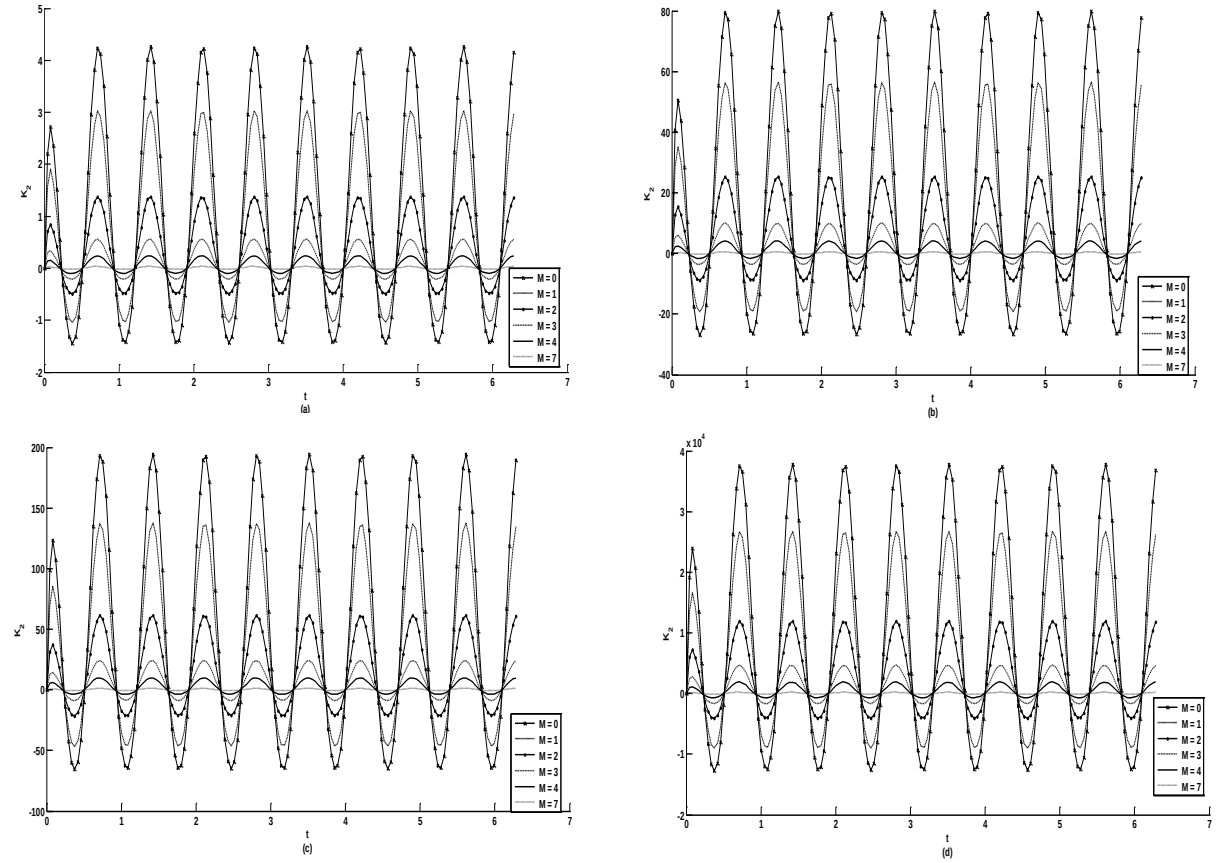


**Fig 5:** Variation of $K_2$ with t for different values of M at Sc = 1, $a_0$ = 0.98 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

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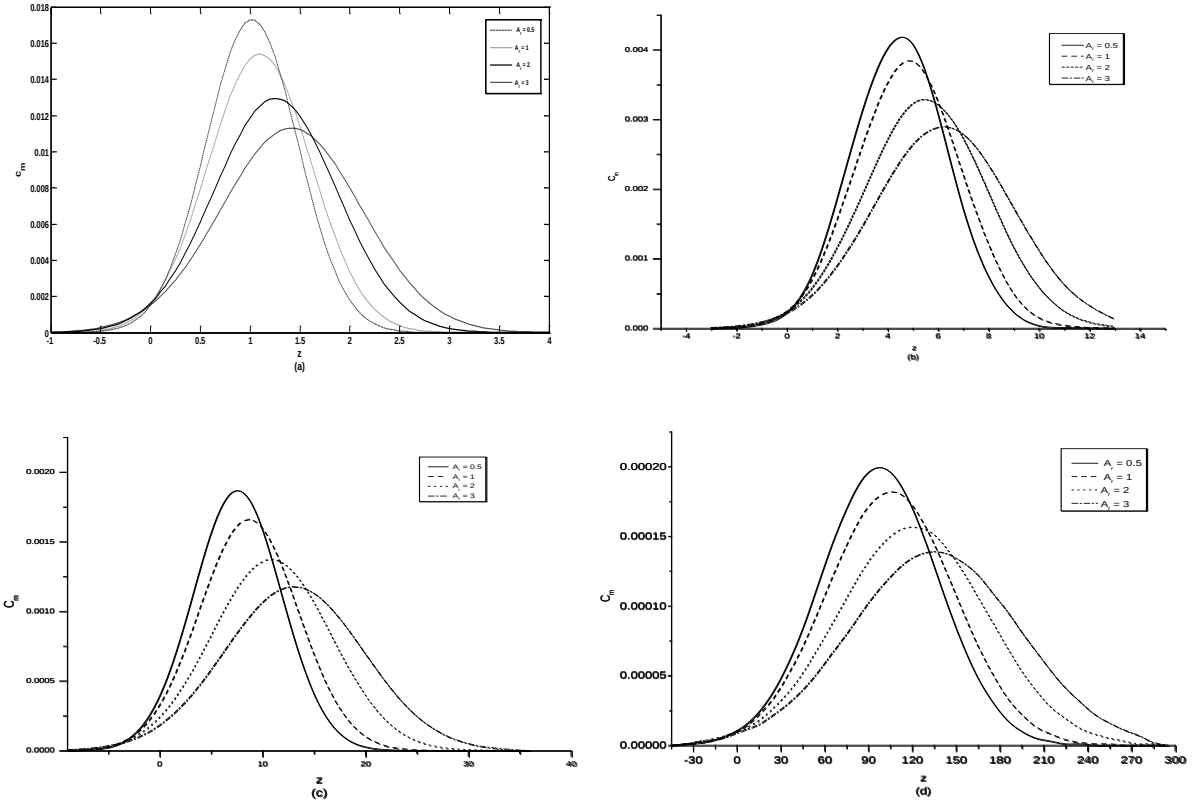


**Fig 6:** Variation of $C_m$ with z for different values of $A_r$ when Sc = 1, M=1 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

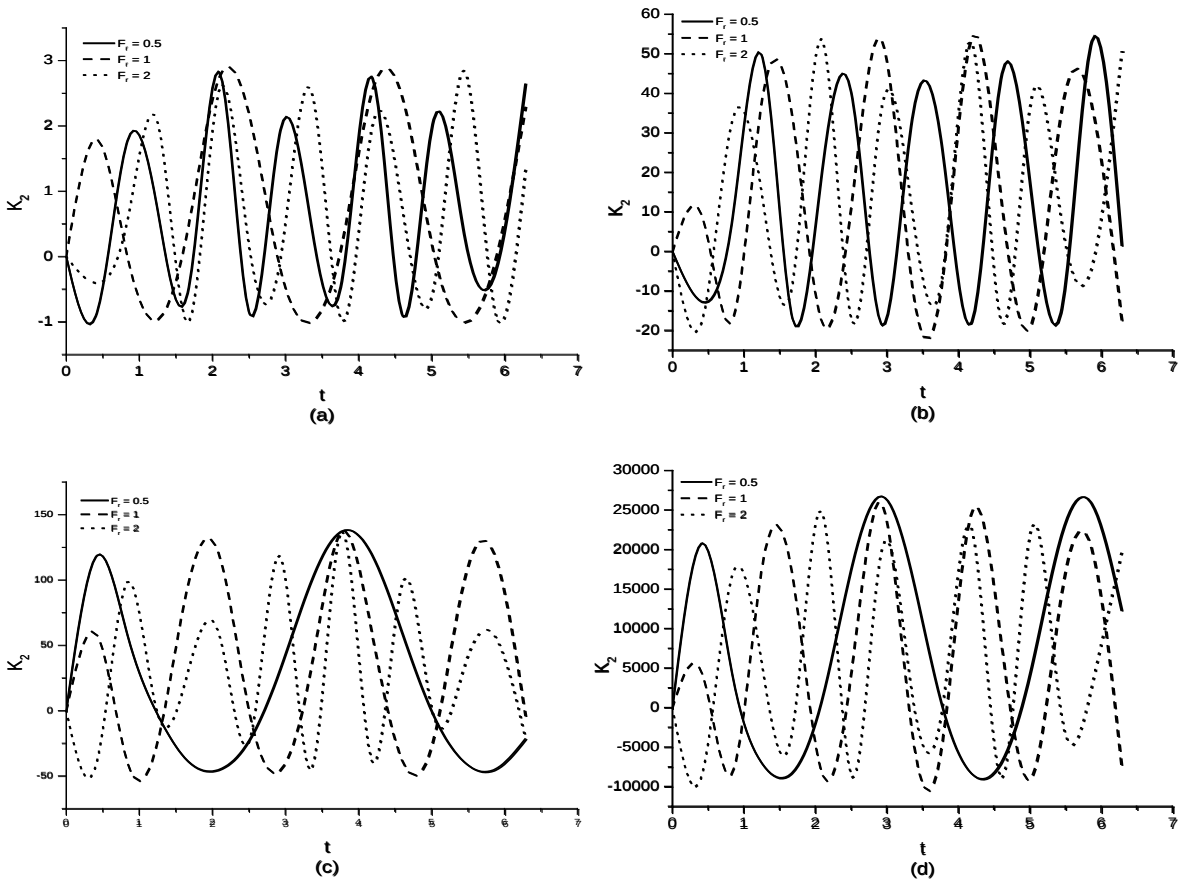


**Fig 7:** Variation of $K_2$ with t for different values of $F_r$ when Sc = 1 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

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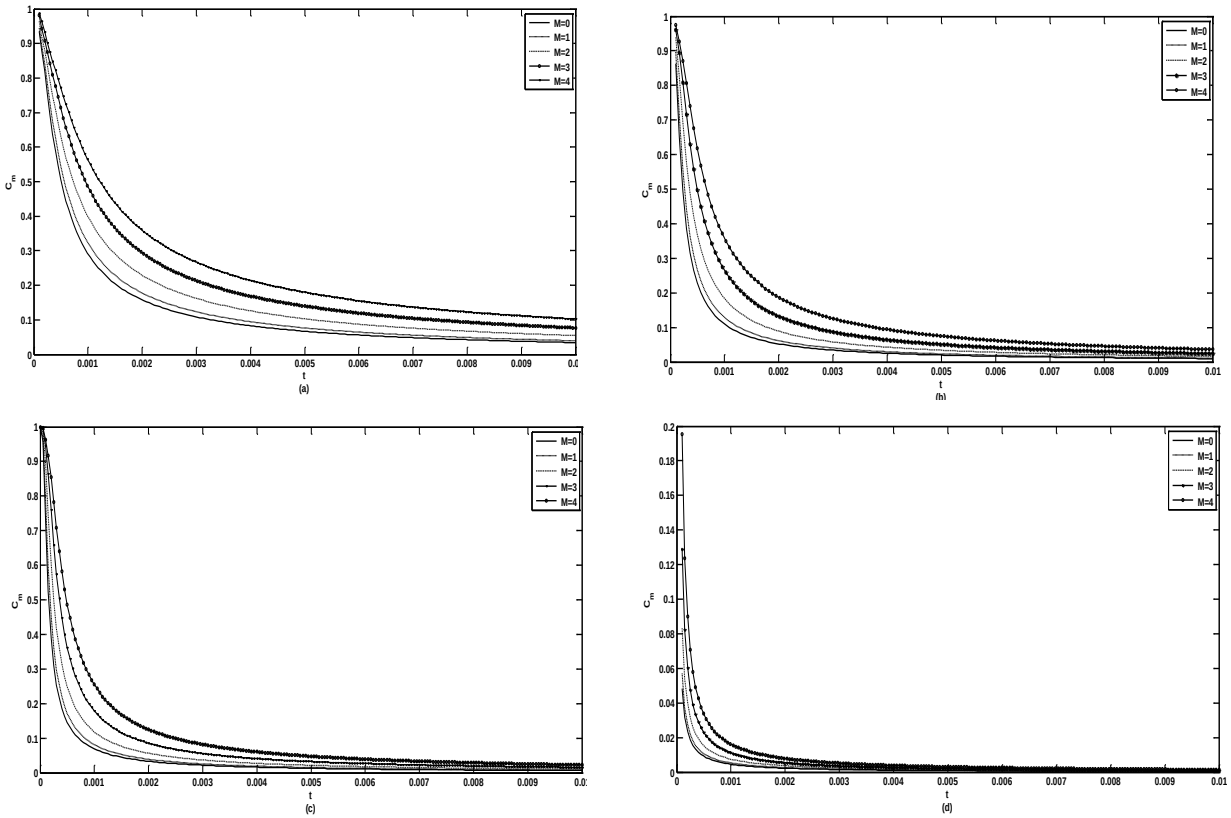


**Fig 8:** Variation of $C_m$ with t for different values of M at $z_s = 0.02$, Sc = 1, z = 0.005 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

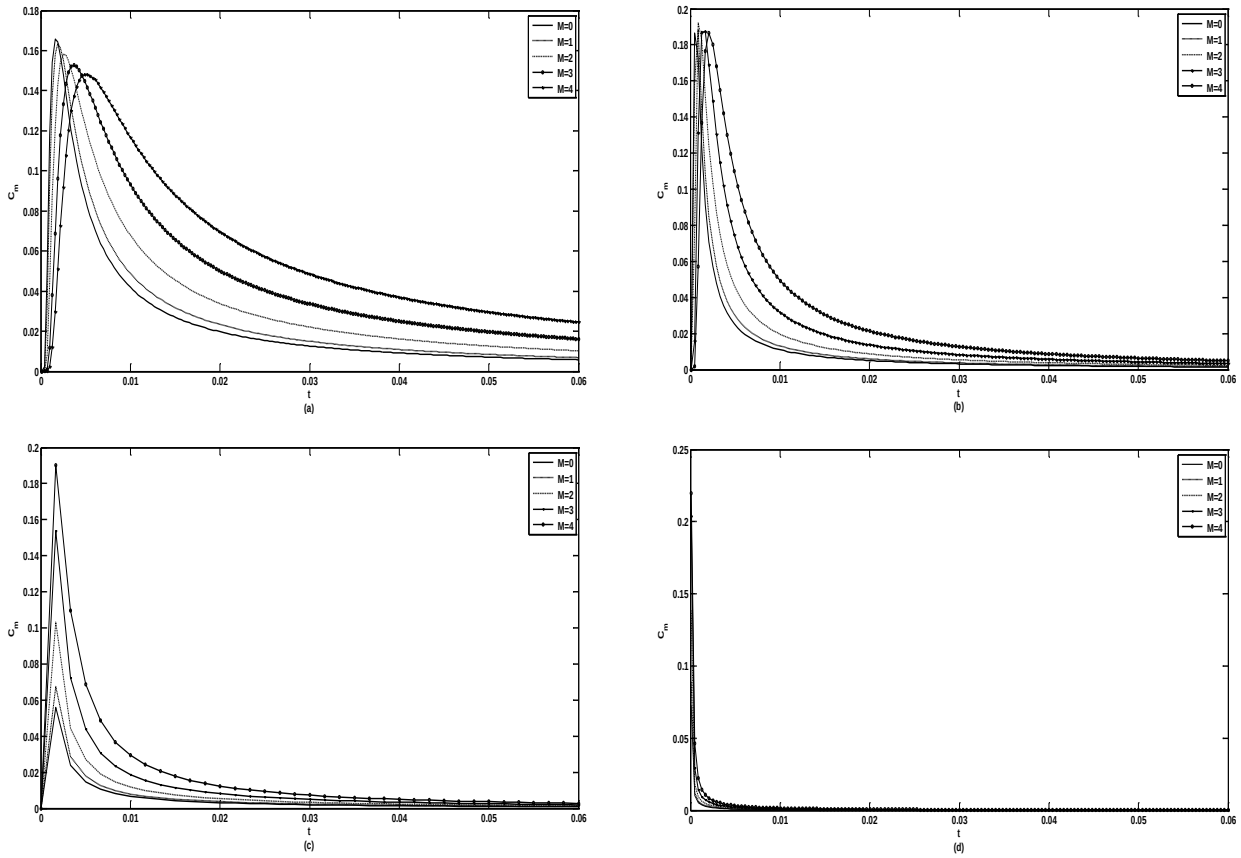


**Fig 9:** Variation of $C_m$ with t for different values of M at $z_s = 0.02$, Sc = 1,z = 0.05 in (a) Aorta (b) Femoral (c) Carotid (d) Coronary

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