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INTUITIONISTIC FUZZY EXPONENTIAL MAP VIA GENERALIZED OPEN SET

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ABSTRACT

The concept of generalized intuitionistic fuzzy compact open topology is introduced.Some characterization of this topology are discussed.

Keywords:*generalized intuitionistic fuzzy locally Compact Hausdorff space;generalized intuitionistic fuzzy product topology; generalized intuitionistic fuzzy compact open topology;generalized intuitionistic fuzzy homeomorphism; generalized intuitionistic fuzzy evaluation map;generalized intuitionistic fuzzy Exponential map.*

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1 INTRODUCTION:

Ever since the introduction of fuzzy sets by Zadeh [19] and fuzzy topological space by Chang [7],several authors have tried successfully to generalize numerous pivot concepts of general topology to the fuzzy setting. The concept of intuitionistic fuzzy set was introduced and studied by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2,3,4]. The concept of fuzzy compact open topology was introduced by S.Dang and A .Behera[9].

In this paper the concept of generalized intuitionistic fuzzy compact open topology are introduced.Some interesting properties are discussed. In this paper the concepts of generalized intuitionistic fuzzy local compactness and generalized intuitionistic fuzzy product topology are developed. We have used the fuzzy locally compactness notion due to Wong[17],Christoph [8] and fuzzy product topology due to Wong [18].

Throughout this paper intuitionistic fuzzy topological spaces $(X,T),(Y,S)$ and (Z,R) will be replaced by X ,Y and Z respectively.

2 PRELIMINIARIES:

Definition: 2.1 *[10] Let* (X,T) *be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set* A *in* (X,T) *is said to be generalized intuitionistic fuzzy closed (in shortly GIF-closed) if* $IFcl(A)\subseteq G$ *whenever* $A\subseteq G$ *and G is intuitionistic fuzzy open. The complement of a GIF-closed set is GIF-open.*

Definition: 2.2 *[10]Let* (X,T) *be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in X.Then intuitionistic fuzzy generalized closure and intuitionistic fuzzy generalized interior of A are defined by*

- $IFGcl(A) = \bigcap$ {G: G is a GIF closed set in X and $A\subseteq G$ }.
- $IFGint(A) = \bigcup$ {G: G is a GIF open set in X and $A\supseteq G$ }.

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Definition: 2.3 *[10] Let* (X,T) *be an intuitionistic fuzzy topological space. If a family* $\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}$ *of GIF open sets in* (X,T) *satisfies the condition* $\bigcup\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}=1_{\sim}$*, then it is called a GIF open cover of* (X,T).

A finite subfamily of a GIF open cover $\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}$ of (X,T),which is also a GIF open cover of (X,T) is called a finite subcover of $\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}$.

Definition: 2.4 *[10] An intuitionistic fuzzy topological space* (X,T) *is called GIF compact iff every GIF open cover of* (X,T) *has a finite subcover.*

Definition: 2.5 *[10] Let* (X,T) *be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in* (X,T). *If a family* $\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}$ *of GIF open sets in* (X,T) *satisfies the condition* $A\subseteq\bigcup\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}=1_{\sim}$*, then it is called a GIF open cover of A.*

A finite subfamily of a GIF open cover $\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}$ of A,which is also a GIF open cover of A, is called a finite subcover of $\{\langle x,\mu_{G_i},\delta_{G_i}\rangle : i\in J\}$.

Definition: 2.6 *[10] An intuitionistic fuzzy set A is called GIF compact iff every GIF open cover of A has a finite subcover.*

Definition: 2.7 *[7] Let* X *and* Y *be fuzzy topological spaces.A mapping* $f: X\to Y$ *is said to be a fuzzy homeomorphism if* f *is bijective,fuzzy continuous and fuzzy open.*

Definition: 2.8 *[17] An fuzzy topological space* (X,T) *is called a fuzzy Hausdorff space or* T_2*-space if for any pair of distinct fuzzy points(i.e., fuzzy points with distinct supports)* x_t *and* y_r*,there exist fuzzy open sets* U *and* V *such that* $x_t\in U$, $y_r\in V$ *and* $U\wedge V=0_X$

Definition: 2.9 *[18] An fuzzy topological space* (X,T) *is said to be fuzzy locally compact if and only if for every fuzzy point* x_t *in* X *there exists a fuzzy open set* $U\in T$ *such that* $x_t\in U$ *and* U *is fuzzy compact,i.e., each fuzzy open cover of* U *has a finite subcover.*

Definition: 2.10 *[14] Let* $A=\langle x,\mu_A(x),\delta_A(x)\rangle$ *and* $B=\langle y,\mu_B(y),\delta_B(y)\rangle$ *be intuitionistic fuzzy sets of* X *and* Y *respectively.The product of two intuitionistic fuzzy sets* A *and* B *is defined as* $(A\times B)(x,y)=\langle (x,y), min(\mu_A(x),\mu_B(y)), max(\delta_A(x),\delta_B(y))\rangle$ *for all* $(x,y)\in X\times Y$.

Definition: 2.11 *[14] Let* $f_1: X_1\to Y_1$ *and* $f_2: X_2\to Y_2$. *The product* $f_1\times f_2: X_1\times X_2\to Y_1\times Y_2$ *is defined by:* $(f_1\times f_2)(x_1,x_2)=(f_1(x_1), f_2(x_2))\ \forall (x_1,x_2)\in X_1\times X_2$.

Lemma: 2.1 *[14] Let* $f_i: X_i\to Y_i\ (i=1,2)$ *be functions and* U, V *intuitionistic fuzzy sets of* Y_1, Y_2*,respectively, then* $(f_1\times f_2)^{-1}(U\times V)=f_1^{-1}(U)\times f_2^{-1}(V)\ \forall\ U\times V\in Y_1\times Y_2$

Remark: 2.1 Let X and Y be two fuzzy topological spaces with Y fuzzy compact.Let x_t be any fuzzy point in X.The fuzzy product space $X\times Y$ containing $\{x_t\}\times Y$.It is cleat that $\{x_t\}\times Y$ is fuzzy homeomorphic[7] to Y

Remark: 2.2 Let X and Y be two fuzzy topological spaces with Y fuzzy compact.Let x_t be any fuzzy point in X .The fuzzy product space $X \times Y$ containing $\{x_t\} \times Y$. $\{x_t\} \times Y$ is fuzzy compact[6].

Remark: 2.3 A fuzzy compact subspace of a fuzzy Hausdorff space is fuzzy closed[12].

3 INTUITIONISTIC FUZZY COMPACT OPEN TOPOLOGY:

Definition: 3.1 *Let X and Y be any two intuitionistic fuzzy topological spaces.A mapping $f : X \to Y$ is generalized intuitionistic fuzzy continuous iff every generalized intuitionistic fuzzy open set V in Y , there exists a generalized intuitionistic fuzzy open set U in X such that $f(U) \subseteq V$.*

Definition: 3.2 *A mapping $f : X \to Y$ is said to be generalized intuitionistic fuzzy homeomorphism if f is bijective ,generalized intuitionistic fuzzy continuous and generalized intuitionistic fuzzy open.*

Definition: 3.3 *Let X be an intuitionistic fuzzy topological space. X is said to be generalized intuitionistic fuzzy Hausdorff space or T_2 space if for any two intuitionistic fuzzy sets A and B with $A \cap B = 0_{\sim}$,there exist generalized intuitionistic fuzzy open sets U and V , such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = 0_{\sim}$.*

Definition: 3.4 *An intuitionistic fuzzy topological space X is said to be generalized intuitionistic fuzzy locally compact iff for any intuitionistic fuzzy set A ,there exists a generalized intuitionistic fuzzy open set G , such that $A \subseteq G$ and G is generalized intuitionistic fuzzy compact.That is each generalized intuitionistic fuzzy open cover of G has a finite subcover.*

Proposition: 3.1 A generalized intuitionistic fuzzy Hausdorff topological space X ,the following conditions are equivalent.
(a) X is generalized intuitionistic fuzzy locally compact
(b) for each intuitionistic fuzzy set A ,there exists a generalized intuitionistic fuzzy open set G in X such that $A \subseteq G$ and $GIFcl(G)$ is generalized intuitionistic fuzzy compact

Proof. $(a) \Rightarrow (b)$ By hypothesis for each intuitionistic fuzzy set A in X , there exists a generalized intuitionistic fuzzy open set G ,such that $A \subseteq G$ and G is generalized intuitionistic fuzzy compact.Since X is generalized intuitionistic fuzzy Hausdorff, by Remark 2.3(generalized intuitionistic fuzzy compact subspace of generalized intuitionistic fuzzy Hausdorff space is generalized intuitionistic fuzzy closed), G is generalized intuitionistic fuzzy closed,thus $G = GIFcl(G)$ .Hence $A \subseteq G = GIFcl(G)$ and $GIFcl(G)$ is generalized intuitionistic fuzzy compact.

$(b) \Rightarrow (a)$ Proof is simple.

Proposition: 3.2 Let X be a generalized intuitionistic fuzzy Hausdorff topological space.Then X is generalized intuitionistic fuzzy locally compact on an intuitionistic fuzzy set A in X iff for every generalized intuitionistic fuzzy open set G containing A , there exists a generalized intuitionistic fuzzy open set V ,such that $A \subseteq V$, $GIFcl(V)$ is generalized intuitionistic fuzzy compact and $GIFcl(V) \subseteq G$.

Proof. Suppose that X is generalized intuitionistic fuzzy locally compact on an intuitionistic fuzzy set A .By Definition 3.4,there exists a generalized intuitionistic fuzzy open set G ,such that $A \subseteq G$ and G is generalized intuitionistic fuzzy compact.Since X is generalized intuitionistic fuzzy Hausdorff space,by Remark 2.3(generalized intuitionistic fuzzy compact subspace of generalized intuitionistic fuzzy Hausdorff space is generalized intuitionistic fuzzy closed), G is generalized intuitionistic fuzzy closed,thus $G = GIFcl(G)$ .Consider an intuitionistic fuzzy set $A \subseteq \overline{G}$ .Since X is generalized intuitionistic fuzzy Hausdorff space,by Definition 3.3,for any two intuitionistic fuzzy sets A and B with $A \cap B = 0_{\sim}$, there exist a generalized intuitionistic fuzzy open sets C and D ,such that $A \subseteq C$, $B \subseteq D$ and $C \cap D = 0_{\sim}$ .Let $V = C \cap G$ .Hence $V \subseteq G$ implies $GIFcl(V) \subseteq GIFcl(G) = G$.

Since $GIFcl(V)$ is generalized intuitionistic fuzzy closed and G is generalized intuitionistic fuzzy compact,by Remark 2.3(every generalized intuitionistic fuzzy closed subset of a generalized intuitionistic fuzzy compact space is generalized intuitionistic fuzzy compact) it follows that $GIFcl(V)$ is intuitionistic fuzzy compact.Thus $A \subseteq GIFcl(V) \subseteq G$ and $GIFcl(G)$ is generalized intuitionistic fuzzy compact.

The converse follows from Proposition 3.1(b).

Definition: 3.5 *Let X and Y be two intuitionistic fuzzy topological spaces.The function $T : X \times Y \to Y \times X$ defined by $T(x, y) = (y, x)$ for each $(x, y) \in X \times Y$ is called an intuitionistic fuzzy switching map.*

Proposition: 3.3 The intuitionistic fuzzy switching map $T : X \times Y \to Y \times X$ defined as above is generalized intuitionistic fuzzy continuous.

We now introduce the concept of generalized intuitionistic fuzzy compact open topology in the set of all generalized intuitionitic fuzzy continuous functions from an intuitionistic fuzzy topological space X to an intuitionistic fuzzy topological space Y.

Definition: 3.6 *Let X and Y be two intuitionistic fuzzy topological spaces and let $Y^X = \{ f : X \to Y$ such that f is generalized intuitionistic fuzzy continuous $\}$.We give this class Y^X a topology called the generalized intuitionistic fuzzy compact open topology as follows:Let $\mathsf{K} = \{K \in I^X : K$ is generalized intuitionistic fuzzy compact $X\}$ and $\mathsf{V} = \{V \in I^X : V$ is generalized intuitionistic fuzzy open in $Y\}$.For any $K \in \mathsf{K}$ and $V \in \mathsf{V}$,let $S_{K,V} = \{ f \in Y^X : f(K) \subseteq V\}$.*

The collection of all such $\{S_{K,V} : K \in \mathsf{K}, V \in \mathsf{V}\}$ is generates an intuitionistic fuzzy structure on the class Y^X.

4 GENERALIZED INTUITIONISTIC FUZZY EVALUATION MAP AND GENERALIZED INTUITIONISTIC FUZZY EXPONENTIAL MAP:

We now consider the generalized intuitionistic fuzzy product topological space $Y^X \times X$ and define an generalized intuitionistic fuzzy continuous map from $Y^X \times X$ into Y.

Definition: 4.1 *The mapping $e : Y^X \times X \to Y$ defined by $e(f, A) = f(A)$ for each intuitionistic fuzzy set A in X and $f \in Y^X$ is called the generalized intuitionistic fuzzy evaluation map.*

Definition: 4.2 *Let X, Y and Z be three intuitionistic fuzzy topological spaces and $f : Z \times X \to Y$ be any function.Then the induced map $\hat{f} : X \to Y^Z$ is defined by $(\hat{f}(A_1))(A_2) = f(A_2, A_1)$ for intuitionistic fuzzy sets A_1 of X and A_2 of Z.*

Conversely,given a function $\hat{f} : X \to Y^Z$, a corresponding function f can be also be defined be the same rule.

Proposition: 4.1 Let X be a generalized intuitionistic fuzzy locally compact Hausdorff space.Then the generalized intuitionistic fuzzy evaluation map $e : Y^X \times X \to Y$ is generalized intuitionistic fuzzy continuous.

Proof. Consider $(f, A_1) \in Y^X \times X$,where $f \in Y^X$ and intuitionistic fuzzy set A_1 of X.Let V be a generalized intuitionistic fuzzy open set containing $f(A_1) = e(f, A_1)$ in Y.Since X is generalized intuitionistic fuzzy locally compact and f is generalized intuitionistic fuzzy continuous,by Proposition 3.2, there exists an generalized intuitionistic fuzzy open set U in X ,such that $A_1 \subseteq GIFcl(U)$ and $GIFcl(U)$ is generalized intuitionistic fuzzy compact and $f(GIFcl(U)) \subseteq V$.

Consider the generalized intuitionistic fuzzy open set $S_{GIFcl(U),V} \times U$ in $Y^X \times X$. (f, A_1) is such that $f \in S_{GIFcl(U),V}$ and $A_1 \subseteq U$ .Let (g, A_2) be such that $g \in S_{GIFcl(U),V}$ and $A_2 \subseteq U$ be arbitrary,thus $g(GIFcl(U)) \subseteq V$ .Since $A_2 \subseteq U$,we have $g(A_2) \subseteq V$ and $e(g, A_2) = g(A_2) \subseteq V$ .Thus $e(S_{GIFcl(U),V} \times U) \subseteq V$ .Hence e is generalized intuitionistic fuzzy continuous.

Proposition: 4.2 Let X and Y be two intuitionistic fuzzy topological spaces with Y is generalized intuitionistic fuzzy compact.Let A_1 be any intuitionistic fuzzy set in X and N be a generalized intuitionistic fuzzy open set in the generalized intuitionistic fuzzy product space $X \times Y$ containing $\{A_1\} \times Y$ .Then there exists some generalized intuitionistic fuzzy open W with $A_1 \subseteq W$ in X ,such that $\{A_1\} \times Y \subseteq W \times Y \subseteq N$.

Proof: It is clear that by Remark 2.1, $\{A_1\} \times Y$ is generalized intuitionistic fuzzy homeomorphism to Y and hence by Remark 2.2, $\{A_1\} \times Y$ is generalized intuitionistic fuzzy compact.We cover $\{A_1\} \times Y$ by the basis elements $\{U \times V\}$ (for the generalized intuitionistic fuzzy product topology) lying in N .Since $\{A_1\} \times Y$ is generalized intuitionistic fuzzy compact, $\{U \times V\}$ has a finite subcover,say a finite number of basis elements $U_1 \times V_1, ..., U_n \times V_n$ .Without loss of generality we assume that $\{A_1\} \subseteq U_i$ for each $i = 1,2,...,n$ .Since otherwise the basis elements would be superfluous.

Let $W = \bigcap_{i=1}^{n} U_i$ .Clearly W is generalized intuitionistic fuzzy open and $A_1 \subseteq W$ .We show that $W \times Y \subseteq \bigcup_{i=1}^{n} (U_i \times V_i)$ .Let (A_1, B) be an intuitionistic fuzzy set in $W \times Y$ .Now $(A_1, B) \subseteq U_i \times V_i$ for some i ,thus $B \subseteq V_i$ .But $A_1 \subseteq U_i$ for every $i = 1,2,...,n$ (because $A_1 \subseteq W$). Therefore, $(A_1, B) \subseteq U_i \times V_i$ as desired.But $U_i \times V_i \subseteq N$ for all $i = 1,2,...,n$ and $W \times Y \subseteq \bigcup_{i=1}^{n} (U_i \times V_i)$,therefore $W \times Y \subseteq N$.

Proposition: 4.3 Let Z be a generalized intuitionistic fuzzy locally compact Hausdorff space and X, Y be arbitrary intuitionistic fuzzy topological spaces.Then a map $f : Z \times X \to Y$ is generalized intuitionistic fuzzy continuous iff $\hat{f} : X \to Y^Z$ is generalized intuitionistic fuzzy continuous,where $\hat{f}$ is defined by the rule $(\hat{f}(A_1))(A_2) = f(A_2, A_1)$.

Proof. Suppose that $\hat{f}$ is generalized intuitionistic fuzzy continuous.Consider the functions $Z \times X \xrightarrow{i_Z \times \hat{f}} Z \times Y^Z \xrightarrow{t} Y^Z \times Z \xrightarrow{e} Y$,where i_Z denote the intuitionistic fuzzy identity function on Z , t denote the intuitionistic fuzzy switching map and e denote the generalized intuitionistic fuzzy evaluation map.Since $et(i_Z \times \hat{f})(A_2, A_1) = et(A_2, \hat{f}(A_1)) = e(\hat{f}(A_1), A_2) = (\hat{f}(A_1))(A_2) = f(A_2, A_1)$ it follows that $f = et(i_Z \times \hat{f})$ and f being the composition of generalized intuitionistic fuzzy continuous functions is itself generalized intuitionistic fuzzy continuous.

Conversely,suppose that f is generalized intuitionistic fuzzy continuous,let A_1 be any arbitrary intuitionistic fuzzy set in X .we have $\hat{f}(A_1) \in Y^Z$ .Consider $S_{K,U} = \{g \in Y^Z : g(K) \subseteq U, K \in I^Z$ is generalized intuitionistic fuzzy compact and $U \in I^Y$ is generalized intuitionistic fuzzy open},containing $\hat{f}(A_1)$ .We need to find a generalized intuitionistic fuzzy open W with $A_1 \subseteq W$,such that $\hat{f}(A_1) \subseteq S_{K,U}$;this will suffice to prove $\hat{f}$ to be a generalized intuitionistic fuzzy continuous map.

For any intuitionistic fuzzy set A_2 in K ,we have $(\hat{f}(A_1))(A_2) = f(A_2, A_1) \in U$ thus $f(K \times \{A_1\}) \subseteq U$, that is $K\times\{A_1\}\subseteq f^{-1}(U)$ .Since f is generalized intuitionistic fuzzy continuous, $f^{-1}(U)$ is a generalized intuitionistic fuzzy open set in $Z\times X$ .Thus $f^{-1}(U)$ is a generalized intuitionistic fuzzy open set $Z\times X$ containing $K\times\{A_1\}$ .Hence by Proposition 4.2,there exists a generalized intuitionistic fuzzy open W with $A_1\subseteq W$ in X ,such that $K\times\{A_1\}\subseteq K\times W\subseteq f^{-1}(U)$ .Therefore $f(K\times W)\subseteq U$ .Now for any $A_1\subseteq W$ and $A_2\subseteq K$, $f(A_2,A_1)=(\hat{f}(A_1))(A_2)\subseteq U$ .Therefore $\hat{f}(A_1)(K)\subseteq U$ for all $A_1\subseteq W$ .that is $\hat{f}(A_1)\in S_{K,U}$ for all $A_1\subseteq W$ .Hence $\hat{f}(W)\subseteq S_{K,U}$ as desired.

Proposition: 4.4 Let X and Z be two generalized intuitionistic fuzzy locally compact Hausdorff spaces.Then for any intuitionistic fuzzy topological space Y ,the function $E:Y^{Z\times X}\rightarrow(Y^Z)^X$ defined by $E(f)=\hat{f}$ (that is $E(f)(A_1)(A_2)=f(A_2,A_1)=(\hat{f}(A_1))(A_2))$ for all $f:Z\times X\rightarrow Y$ is a generalized intuitionistic fuzzy homeomorphism.

Proof:

(a) Clearly E is onto.

(b) For E to be injective.Let $E(f)=E(g)$ for $f,g:Z\times X\rightarrow Y$ .Thus $\hat{f}=\hat{g}$,where $\hat{f}$ and $\hat{g}$ are the induced maps of f and g respectively.Now for any intuitionistic fuzzy set A_1 in X and any intuitionistic fuzzy set A_2 in Z , $f(A_2,A_1)=(\hat{f}(A_1))(A_2)=(\hat{g}(A_1))(A_2)=g(A_2,A_1)$;thus $f=g$.

(c) For proving the generalized intuitionistic fuzzy continuity of E ,consider any generalized intuitionistic fuzzy subbasis neighbourhood V of $\hat{f}$ in $(Y^Z)^X$, that is V is of the form $S_{K,W}$ where K is a generalized intuitionistic fuzzy compact subset of X and W is generalized intuitionistic fuzzy open in Y^Z .Without loss of generality we may assume that $W=S_{L,U}$,where L is a generalized intuitionistic fuzzy compact subset of Z and U is a generalized intuitionistic fuzzy open set in Y .Then $\hat{f}(K)\subseteq S_{L,U}=W$ and this implies that $\hat{f}(K)(L)\subseteq U$ .Thus for any intuitionistic fuzzy set $A_1\subseteq K$ and for all intuitionistic fuzzy sets $A_2\subseteq L$ .We have $(\hat{f}(A_1))(A_2)\subseteq U$, that is $f(A_2,A_1)\subseteq U$ and therefore $f(L\times K)\subseteq U$ .Now since L is generalized intuitionistic fuzzy compact in Z and K is generalized intuitionistic fuzzy compact in X , $L\times K$ is also generalized intuitionistic fuzzy compact in $Z\times X$ [6] and since U is a generalized intuitionistic fuzzy open set in Y , we conclude that $f\in S_{L\times K,U}\subseteq Y^{Z\times X}$ .We assert that $E(S_{L\times K,U})\subseteq S_{K,W}$ .Let $g\in S_{L\times K,U}$ be arbitrary.Thus $g(L\times K)\subseteq U$, that is $g(A_2,A_1)=(\hat{g}(A_1))(A_2)\subseteq U$ for all intuitionistic fuzzy sets $A_2\subseteq L$ in Z and for all intuitionistic fuzzy sets $A_1\subseteq K$ in X .So $(\hat{g}(A_1))(L)\subseteq U$ for all intuitionistic fuzzy sets $A_1\subseteq K$ in X , that is $\hat{g}(A_1)\subseteq S_{L,U}=W$ for all intuitionistic fuzzy sets $A_1\subseteq K$ in U .Hence we have $\hat{g}(K)\subseteq W$,that is $\hat{g}=E(g)\in S_{K,W}$ for any $g\in S_{L\times K,U}$ .Thus $E(S_{L\times K,U})\subseteq S_{K,W}$ .This proves that E is generalized intuitionistic fuzzy continuous.

(d) For proving the generalized intuitionistic fuzzy continuity of E^{-1} ,we consider the following generalized intuitionistic fuzzy evaluation maps: $e_1:(Y^Z)^X\times X\rightarrow Y^Z$ defined by $e_1(\hat{f},A_1)=\hat{f}(A_1)$ where $\hat{f}\in(Y^Z)^X$ and A_1 is an intuitionistic fuzzy set in X and $e_2:Y^Z\times Z\rightarrow Y$ defined by $e_2(g,A_2)=g(A_2)$ where $g\in Y^Z$ and A_2 is a intuitionistic fuzzy set in Z .Let ψ denote the composition of the following generalized intuitionistic fuzzy continuous functions $\psi:(Z\times X)\times(Y^Z)^X \xrightarrow{T} (Y^Z)^X \times (Z\times X) \xrightarrow{i\times t}$ $(Y^Z)^X \times (X\times Z) \xrightarrow{=} ((Y^Z)^X \times X)\times Z \xrightarrow{e_1\times i_Z} (Y^Z)\times Z \xrightarrow{e_2} Y$, where i, i_Z denote the intuitionistic fuzzy identity maps on $(Y^Z)^X$ and Z respectively and T, t denote the intuitionistic fuzzy switching maps.Thus $\psi:(Z\times X)\times(Y^Z)^X \to Y$ that is $\psi \in Y^{(Z\times X)\times(Y^Z)^X}$.

We consider the map $\hat{E}: Y^{(Z\times X)\times(Y^Z)^X} \to (Y^{(Z\times X)})^{(Y^Z)^X}$ (as defined in the statement of the proposition in fact it is E).So $\hat{E}(\psi):(Y^Z)^X \to Y^{(Z\times X)}$.Now for any intuitionistic fuzzy sets A_2 in Z, A_1 in X and $f \in Y^{(Z\times X)}$, again to check that $(\hat{E}(\psi)\circ E)(f)(A_2, A_1) = f(A_2, A_1)$; hence $\hat{E}(\psi)\circ E$ = identity. Similarly for any $\hat{g} \in (Y^Z)^X$ and intuitionistic fuzzy sets A_1 in X, A_2 in Z, again to check that

$(E\circ\hat{E}(\psi))(\hat{g})(A_1, A_2) = (\hat{g}(A_1))(A_2)$; hence $E\circ\hat{E}(\psi)$ =identity.Thus E is a generalized intuitionistic fuzzy homeomorphism.

Definition: 4.3 *The map E in Proposition 4.4 is called the generalized intuitionistic fuzzy exponential map.*As easy consequence of Proposition 4.4 is as follows.

Proposition: 4.5 Let X, Y and Z be three generalized intuitionistic fuzzy locally compact Hausdorff spaces. Then the map $N: Y^X \times Z^Y \to Z^X$ defined by $N(f, g) = g\circ f$ is generalized intuitionistic fuzzy continuous.

Proof: Consider the following compositions:

$$X\times Y^X\times Z^Y \xrightarrow{T} Y^X\times Z^Y\times X \xrightarrow{t\times i_X} Z^Y\times Y^X\times X \xrightarrow{=} Z^Y\times(Y^X\times X) \xrightarrow{i\times e_2} Z^Y\times Y \xrightarrow{e_2} Z$$

where T, t denote the intuitionistic fuzzy switching maps, i_X, i denote the intuitionistic fuzzy identity functions on X and Z^Y respectively and e_2 denote the generalized intuitionistic fuzzy evaluation maps.

Let $\varphi = e_2\circ(i\times e_2)\circ(t\times i_X)\circ T$ .By proposition 4.4,we have an exponential map. $E: Z^{X\times Y^X\times Z^Y} \to (Z^X)^{Y^X\times Z^Y}$.

Since $\varphi \in Z^{X\times Y^X\times Z^Y}$, $E(\varphi) \in (Z^X)^{Y^X\times Z^Y}$ .Let $N = E(\varphi)$,that is $N: Y^X\times Z^Y \to Z^X$ is an generalized intuitionistic fuzzy continuous.For $f\in Y^X, g\in Z^Y$ and for any intuitionistic fuzzy set A_1 in X ,it is easy to see that $N(f, g)(A_1) = g(f(A_1))$.

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