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INTUITIONISTIC FUZZY QUASI SEMI-GENERALIZED CLOSED MAPPINGS

R. SANTHI

Department of Mathematics, Nallamuthu Gounder, Mahalingam College, Pollachi - 642 001
Tamil Nadu – India
E-mail: santhifuzzy@yahoo.co.in

K. ARUN PRAKASH*

Department of Mathematics, Kongu Engineering College, Perundurai - 638 052. Tamil Nadu – India
**E-mail: arun.kannusamy@yahoo.co.in*

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ABSTRACT

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy quasi semi-generalized closed mappings and intuitionistic fuzzy quasi semi-generalized open mappings in intuitionistic fuzzy topological space. We investigate some of its fundamental properties and its characterizations.

Keywords and Phrases: *Intuitionistic fuzzy topology, Intuitionistic fuzzy semi-generalized closed set, Intuitionistic fuzzy semi-generalized open set, Intuitionistic fuzzy quasi semi-generalized closed mapping, Intuitionistic fuzzy quasi semi-generalized open mapping and Intuitionistic fuzzy semi-$T_{1/2}$ space.*

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1. INTRODUCTION:

Fuzzy set (FS), proposed by Zadeh [15] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. By adding the degree of non-membership to FS, Atanassov proposed intuitionistic fuzzy set (IFS) in 1983 [1] which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. Later on fuzzy topology was introduced by Chang in 1967.After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In last few years various concepts in fuzzy were extended to intuitionistic fuzzy sets. In 1997, Coker introduced the concept of intuitionistic fuzzy topological space. After this many concepts in fuzzy topological spaces were extended to intuitionistic fuzzy topological spaces. We introduce the concepts of intuitionistic fuzzy quasi semi-generalized closed mappings and intuitionistic fuzzy quasi semi-generalized open mappings as an extension of work done in the papers ([10], [11]). We have studied some of the basic properties regarding it. We also obtained some characterizations and preservation theorems with the help of intuitionistic fuzzy semi$T_{1/2}$ space.

2. PRELIMINARIES:

Definition: 2.1 [1] An *intuitionistic fuzzy set* (IFS, for short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \,/\, x \in X\}$$

where the functions $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non- membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Definition: 2.2 [1] Let A and B be IFS's of the forms
$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \,/\, x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle \,/\, x \in X\}$
Then,
(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$,

Corresponding author: K. ARUN PRAKASH*, *E-mail: arun.kannusamy@yahoo.co.in

(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle / x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \ \gamma_A(x) \vee \gamma_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \ \gamma_A(x) \wedge \gamma_B(x)\rangle / x \in X\}$
(f) $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$
(g) $\overline{\overline{A}} = A, \overline{1_\sim} = 0_\sim, \overline{0_\sim} = 1_\sim$.

Definition: 2.3 [1] Let $\alpha,\beta \in [0,1]$ with $\alpha+\beta \le 1$. An *intuitionistic fuzzy point* (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha, \beta), if \ x = p \\ (0,1), otherwise \end{cases}$$

Definition: 2.4 [3] An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFS's in X satisfying the following axioms:
(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $\bar{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition: 2.5 [3] Let X and Y are two non empty sets and f: X→ Y be a function. If

$$B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle / y \in Y\}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, \ f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle / x \in X\}$$

Definition: 2.6 [3] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* of A are defined by

$$\text{int}(A) = \cup \{G \mid G \text{ is an IFOS in X and } G \subseteq A\},$$
$$\text{cl}(A) = \cap \{K \mid K \text{ is an IFCS in X and } A \subseteq K\}.$$

Note that, for any IFS A in (X, τ), we have

$$\text{cl}(\bar{A}) = \overline{\text{int}(A)} \quad \text{and} \quad \text{int}(\bar{A}) = \overline{\text{cl}(A)}$$

Definition: 2.7 An IFS $A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\}$ in an IFTS (X, τ) is called an
(i) *intuitionistic fuzzy semiopen set* (IFSOS) if $A \subseteq \text{cl(int}(A))$ [5].
(ii) *intuitionistic fuzzy α-open set* (IFαOS) if $A \subseteq \text{int(cl(int}(A)))$ [5].
(iii) *intuitionistic fuzzy preopen set* (IFPOS) if $A \subseteq \text{int(cl}(A))$ [5].
(iv) *intuitionistic fuzzy regular open set* (IFROS) if $\text{int(cl}(A))=A$ [5].
(v) *intuitionistic fuzzy semi-pre open set* (IFSPOS) if there exists $B \in$ IFPO (X) such that $B \subseteq A \subseteq \text{cl}(B)$ [13].

An IFS A is called an intuitionistic fuzzy semiclosed set, intuitionistic fuzzy α-closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi-preclosed set, respectively (IFSCS, IFαCS, IFPCS, IFRCS and IFSPCS resp), if the complement $\bar{A}$ is an IFSOS, IFαOS, IFPOS, IFROS and IFSPOS respectively.

The family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy α-open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semi-preopen) sets of an IFTS (X,τ) is denoted by IFSO(X) (resp $IF_\alpha(X)$, IFPO(X), IFRO(X) and IFSPO(X)).

Definition: 2.8 [10] An IFS A of an IFTS (X, τ) is called an *intuitionistic fuzzy semi-generalized closed (intuitionistic fuzzy sg-closed) set* (IFSGCS) if scl (A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSOS..

The complement $\bar{A}$ of an intuitionistic fuzzy semi-generalized closed set A is called an *intuitionistic fuzzy semi-generalized open (intuitionistic fuzzy sg-open) set* (IFSGOS).

Definition: 2.9 [10] An IFTS (X,τ) is said to be an *intuitionistic fuzzy semi-$T_{1/2}$ space*, if every intuitionistic fuzzy sg-closed set in X is an intuitionistic fuzzy semiclosed in X.

Definition: 2.10 [7] Let $p_{(\alpha,\beta)}$ be an IFP of an IFTS (X, τ). An IFS A of X is called an *intuitionistic fuzzy neighborhood* (IFN) of $p_{(\alpha,\beta)}$, if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition: 2.11 [12] Let $p_{(\alpha,\beta)}$ be an IFP of an IFTS (X, τ). An IFS A of X is called an *intuitionistic fuzzy sg-neighborhood* of $p_{(\alpha,\beta)}$, if there exists an IFSOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition: 2.12 [7] Let (X,τ) be an IFTS and A = $\langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy semi-interior and intuitionistic fuzzy semi-closure of A are defined by

$$\text{sint}(A) = \cup \{G \mid G \text{ is an IFSOS in X and } G \subseteq A\},$$
$$\text{scl}(A) = \cap \{K \mid K \text{ is an IFSCS in X and } A \subseteq K\}.$$

Definition: 2.13 [11] Let (X,τ) be an IFTS and A = $\langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy semi-interior and intuitionistic fuzzy semi-closure of A are defined by

$$\text{sgint}(A) = \cup \{G \mid G \text{ is an IFSOS in X and } G \subseteq A\},$$
$$\text{sgcl}(A) = \cap \{K \mid K \text{ is an IFSCS in X and } A \subseteq K\}.$$

Definition: 2.14 [11] A mapping f: (X,τ) → (Y,κ) from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy semi-generalized continuous (intuitionistic fuzzy sg-continuous) mapping if $f^{-1}(A)$ is an IFSGCS in X, for every IFCS A in Y.

Definition: 2.15 [11] A mapping f: (X,τ) → (Y,κ) from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy semi-generalized irresolute (intuitionistic fuzzy sg-irresolute) mapping if $f^{-1}(A)$ is an IFSGCS in X, for every IFSGCS A in Y.

Definition: 2.16 A mapping f: (X,τ) → (Y,κ) from an IFTS (X,τ) into an IFTS (Y,κ) is said to be

(i) an intuitionistic fuzzy open mapping if f(A) is an IFOS in Y, for every IFOS A in X [6]
(ii) an intuitionistic fuzzy semi-open mapping if f(A) is an IFSOS in Y, for every IFOS A in X [6]
(iii) an intuitionistic fuzzy pre-open mapping if f(A) is an IFPOS in Y, for every IFOS A in X [6]
(iv) an intuitionistic fuzzy α- open mapping if if f(A) is an IFαOS in Y, for every IFOS A in X [6]
(v) an intuitionistic fuzzy semi-pre open mapping if if f(A) is an IFSPOS in Y, for every IFOS A in X [6]

Definition: 2.17 [12**]** A mapping f: (X,τ) → (Y,κ) from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy sg^*-closed if the image of every IFSGCS of X is an IFSGCS in Y.

3. INTUITIONISTIC FUZZY QUASI SEMI-GENERALIZED OPEN FUNCTIONS

Definition: 3.1 A function f: X→Y from an IFTS X into an IFTS Y is said to be an intuitionistic fuzzy quasi semi-generalized open (intuitionistic fuzzy quasi sg-open) mapping if f(B) is an IFOS in Y, for every IFSGOS B in X.

It is evident that, the concepts of intuitionistic fuzzy quasi sg-open mapping and intuitionistic fuzzy sg-continuous mapping coincide if the function is a bijection.

Theorem: 3.2 If (X, τ) is an intuitionistic fuzzy semi $T_{1/2}$ space, then a mapping f : (X, τ) → (Y, σ) is an intuitionistic fuzzy quasi sg-open mapping if and only if f(sint(B)) $\subseteq$ int(f(B)) for every IFS B of X.

Proof: Let f be an intuitionistic fuzzy quasi sg-open mapping. We have sint(B) $\subseteq$ B, f(sint(B)) $\subseteq$ f(B). Since sint(B) is an IFSOS, it is an IFSGOS. By our assumption f(sint(B)) is an IFOS in Y, f(sint(B)) $\subseteq$ int(f(B)).

Conversely assume that B is an IFSGOS in X. Since (X, τ) is an intuitionistic fuzzy semi $T_{1/2}$ space B is an IFSOS in X, then f(B) = f(sint(B)) $\subseteq$ int(f(B)), but int(f(B)) $\subseteq$ f(B). Consequently f(B) = int(f(B)), which implies f(B) is an IFOS in Y. Hence f is an intuitionistic fuzzy quasi sg-open mapping.

Lemma 3.3 Let (X, τ) is an intuitionistic fuzzy semi $T_{1/2}$ space. If f: (X, τ) $\rightarrow$ (Y, σ) is an intuitionistic fuzzy quasi sg-open mapping, then sint(f^{-1}(B)) $\subseteq$ f^{-1}(int(B)), for every IFS B of Y.

Proof: Let B be an IFS of Y. Then sint(f^{-1}(B)) is an IFSOS in X which implies sint(f^{-1}(B)) is an IFSGOS in X. Cleraly f(sint(f^{-1}(B))) $\subseteq$ f(f^{-1}(B)) and by hypothesis f(sint(f^{-1}(B))) is an IFOS in X.

Hence f(sint(f^{-1}(B))) $\subseteq$ int(f(f^{-1}(B))) $\subseteq$ int(B). Thus sint(f^{-1}(B)) $\subseteq$ f^{-1}(int(B)).

Theorem 3.4 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy open mapping.

Proof: Let f: X$\rightarrow$Y be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Then by our assumption f(B) is an IFOS in Y. Hence f is an intuitionistic fuzzy open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.5 Let X= {a, b}

$$\text{Let} \quad A = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right) \rangle$$

Then τ = {$0_\sim$, $1_\sim$, A} is an IFT on X.

Define a mapping f: (X,τ) $\rightarrow$ (X,τ) by f(a) = a and f(b) = b.

$$\text{IFSOS(X)} = \left\{0_\sim, 1_\sim, G_{a,b}^{(l_1,m_1),(l_2,m_2)};\ l_1 \epsilon\, [0.4,1], l_2 \epsilon\, [0.5,1], m_1 \epsilon\, [0,0.3], m_2 \in [0,0.4],\ l_i + m_i \leq 1,\ i = 1,2\right\}$$

$$\text{where} \quad G_{a,b}^{(l_1,m_1),(l_2,m_2)} = \langle x, \left(\frac{u}{l_1}, \frac{v}{l_2}\right), \left(\frac{u}{m_1}, \frac{v}{m_2}\right) \rangle$$

$$\text{IFSCS(X)} = \left\{0_\sim, 1_\sim, H_{a,b}^{(a_1,b_1),(a_2,b_2)};\ a_1 \epsilon\, [0,0.3], a_2 \epsilon\, [0,0.4], b_1 \epsilon\, [0.4,1], b_2 \in [0.5,1],\ a_i + b_i \leq 1,\ i = 1,2\right\}$$

$$\text{where} \quad H_{a,b}^{(a_1,b_1),(a_2,b_2)} = \langle x, \left(\frac{u}{a_1}, \frac{v}{a_2}\right), \left(\frac{u}{b_1}, \frac{v}{b_2}\right) \rangle$$

Clearly f is an intuitionistic fuzzy open mapping.

Let $B = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.7}\right), \left(\frac{a}{0.2}, \frac{b}{0.1}\right) \rangle$ be an IFSGOS in X.

But f (B) is not an IFOS in X. Therefore f is not an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.6 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy semi-open mapping.

Proof: Let f: X$\rightarrow$Y be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Then by our assumption f(B) is an IFOS in Y. Since every IFOS is an IFSOS, f(B) is an IFSOS in Y. Hence f is an intuitionistic fuzzy semi-open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.7 The mapping f in Example 3.5 is an intuitionistic fuzzy semi-open mapping which is not an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.8 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy α-open mapping.

Proof: Let f: X$\rightarrow$Y be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Then by our assumption f(B) is an IFOS in Y. Since every IFOS is an IFαOS, f(B) is an IFαOS in Y. Hence f is an intuitionistic fuzzy α-open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.9 The mapping f in Example 3.5 is an intuitionistic fuzzy α-open mapping which is not an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.10 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy semi-pre open mapping.

Proof: Let f: $X \to Y$ be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Then by our assumption f(B) is an IFOS in Y. Since every IFOS is an IFSPOS, f(B) is an IFSPOS in Y. Hence f is an intuitionistic fuzzy semi-pre open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.11 The mapping f in Example 3.5 is an intuitionistic fuzzy semi-pre open mapping which is not an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.12 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy preopen mapping.

Proof: Let f: $X \to Y$ be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Then by our assumption f(B) is an IFOS in Y. Since every IFOS is an IFPOS, f(B) is an IFPOS in Y. Hence f is an intuitionistic fuzzy preopen mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.13 The mapping f in Example 3.5 is an intuitionistic fuzzy preopen mapping which is not an intuitionistic fuzzy quasi sg-open mapping.

Theorem 3.14 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy sg-open mapping.

Proof: Let f: $X \to Y$ be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Then by our assumption f(B) is an IFOS in Y. Since every IFOS is an IFSGOS, f(B) is an IFSGOS in Y. Hence f is an intuitionistic fuzzy sg-open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.15 The mapping f in Example 3.5 is an intuitionistic fuzzy sg-open mapping which is not an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.16 Every intuitionistic fuzzy quasi sg-open mapping is an intuitionistic fuzzy sg^*-open mapping.

Proof: Let f: $X \to Y$ be an intuitionistic fuzzy quasi sg-open mapping and let B be an IFSGOS in X. By our assumption f(B) is an IFOS in Y. Since every IFOS is an IFSGOS, f(B) is an IFSGOS in Y. Hence f is an intuitionistic fuzzy sg^*-open mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example: 3.17 The mapping f in Example 3.5 is an intuitionistic fuzzy sg^*-open mapping which is not an intuitionistic fuzzy quasi sg-open mapping

Theorem: 3.18 A mapping f: $X \to Y$ is an intuitionistic fuzzy quasi sg-open mapping if and only if for any IFS B of Y and for any IFSGCS F of X containing $f^{-1}(B)$, there exists an IFCS G of Y containing B such that $f^{-1}(G) \subseteq F$.

Proof: Necessity: Assume that f is an intuitionistic fuzzy quasi sg-open mapping. Let B be an IFS of Y. Assume that F be an IFSGCS of X, $\overline{F}$ is an IFSGOS in X such that $f^{-1}(B) \subseteq F$. Since f is an intuitionistic fuzzy quasi sg-open mapping $f(\overline{F})$ is an IFOS in Y. Put G = $\overline{f(\overline{F})}$, $f^{-1}(B) \subseteq F$ implies $B \subseteq G$. We have G is an IFCS of Y such that $f^{-1}(G) \subseteq F$.

Sufficiency: Let A be an IFSGOS of X and put $B = \overline{f(A)}$. Then $\overline{A}$ is an IFSGCS in X containing $f^{-1}(B)$. By hypothesis, there exists an IFCS F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq \overline{A}$. Hence, we obtain $f(A) \subseteq \overline{F}$. On the other hand, it follows that $B \subseteq F$, $\overline{F} \subseteq \overline{B} = f(A)$. Thus, we obtain $f(A) = \overline{F}$ which is an IFOS in Y. Hence f is an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.19 A mapping f : $(X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy quasi sg-open mapping if and only if $f^{-1}(cl(B)) \subseteq scl(f^{-1}(B))$ for every IFS B of Y, where (X, τ) is an intuitionistic fuzzy semi $T_{1/2}$ space.

Proof: Assume that f is an intuitionistic fuzzy quasi sg-open mapping. For any IFS B of Y, $f^{-1}(B) \subseteq scl(f^{-1}(B))$. Since $scl(f^{-1}(B))$ is an IFSCS, it is an IFSGCS in X, by Theorem 3.18, there exists an IFCS F in Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq scl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(cl(B)) \subseteq f^{-1}(cl(F)) = f^{-1}(F) \subseteq scl(f^{-1}(B))$.

Converse: Let $B \subseteq Y$ and F be an IFSGCS of X containing $f^{-1}(B)$. Put G = cl(B), then we have $B \subseteq G$ and G is an IFCS and $f^{-1}(G) \subseteq sgcl(f^{-1}(B)) \subseteq F$. Then by Theorem 3.18, f is an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.20 Let f: X→Y be a mapping. Then the following statements are equivalent:

(i) f is an intuitionistic fuzzy quasi sg-open mapping;

(ii) for each IFS B of X, $f(sgint(B)) \subseteq int(f(B))$;

(iii) for each IFP $p_{(\alpha,\beta)}$ and each intuitionistic fuzzy sg-neighborhood B of $p_{(\alpha,\beta)}$ in X, there exists an IFN f(A) of $f(p_{(\alpha,\beta)})$ such that $A \subseteq B$.

Proof: (i) ⇒ (ii): Follows from the Theorem 3.2.

(ii) ⇒ (iii): Let $p_{(\alpha,\beta)} \in X$ and B be an arbitrary intuitionistic fuzzy sg-neighborhood of $p_{(\alpha,\beta)}$ in X. Then by definition there exists an IFSGOS C in X such that $p_{(\alpha,\beta)} \in C \subseteq B$. Then by (ii)

$f(C) = f(sgint(C)) \subseteq int(f(C))$ and hence $f(C) = int(f(C))$. Therefore f(C) is an IFOS in Y, such that $f(p_{(\alpha,\beta)}) \in f(C) \subseteq f(B)$. Taking f(C) = A, we have $A \subseteq f(B)$.

(iii)⇒ (i): Let B be an IFSGOS in X. Then for each $q_{(\alpha,\beta)} \in f(B)$, by (iii), there exists an IFN $A_{q_{(\alpha,\beta)}} \subseteq f(B)$. As $A_{q_{(\alpha,\beta)}}$ is an IFN of $q_{(\alpha,\beta)}$, there exists an IFOS $C_{q_{(\alpha,\beta)}}$ in Y such that $q_{(\alpha,\beta)} \in C_{q_{(\alpha,\beta)}} \subset A_{q_{(\alpha,\beta)}}$. Thus $f(B) = \cup \left\{C_{q_{(\alpha,\beta)}} : q_{(\alpha,\beta)} \in f(B)\right\}$ which is an IFOS in Y. This implies that f is an intuitionistic fuzzy quasi sg-open function.

Theorem: 3.21 Let f: X→Y and g: Y→Z be any two mappings and $g \circ f : X \to Z$ be an intuitionistic fuzzy quasi sg - open mapping. If g is an intuitionistic fuzzy continuous and injective mapping, then f is an intuitionistic fuzzy quasi sg-open mapping.

Proof: Let B be an IFSGOS in X. Since $g \circ f$ is an intui tionistic fuzzy quasi sg-open mapping, $(g \circ f)(B) = g(f(B))$ is an IFOS in Y. Also since g is an intuitionistic fuzzy continuous and injective mapping, $f(B) = g^{-1}((g \circ f)(B)) = g^{-1}(g(f(B)))$ is an IFOS in Y. Hence f is an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.22 Let f: X→Y be a mapping, where X is an intuitionistic fuzzy semi - $T_{1/2}$ space. Then the following are equivalent:

(i) f is an intuitionistic fuzzy quasi sg-open mapping;

(ii) $f(sint(B)) \subseteq int(f(B))$ for each IFS B of X.

Proof: (i) ⇒ (ii): Let B be an IFS in X. Then sint(B) is an IFSOS in X. Since every IFSOS is an IFSGOS, sint(B) is an IFSGOS in X. By our assumption, f(sint(B)) is an IFOS in Y. Then $f(sint(B)) = int(f(sint(B))) \subseteq int(f(B))$.

(ii)⇒ (i): Let B be an IFSGOS in X. Then by our assumption, $f(sint(B)) \subseteq int(f(B))$. Since (X,τ).is an intuitionistic fuzzy semi- $T_{1/2}$ space, B is an IFSOS in X, which implies $f(B) \subseteq int(f(B))$. But $int(f(B)) \subseteq f(B)$. Therefore f(B) = int(f(B)), which implies f(B) is an IFOS in Y. Hence f is an intuitionistic fuzzy quasi sg-open mapping.

Theorem: 3.23 Let f: X→Y and g: Y→Z be any two mappings, then the following statements hold:

(i) If f is an intuitionistic fuzzy quasi sg-open mapping and g is an intuitionistic fuzzy open mapping, then $g \circ f$ is an intuitionistic fuzzy quasi sg-open mapping.

(ii) If f and g are intuitionistic fuzzy quasi sg-open mapping, then $g \circ f$ is an intuitionistic fuzzy open mapping.

(iii) If f is an intuitionistic fuzzy open mapping and g is an intuitionistic fuzzy quasi sg-open mapping, then $g \circ f$ is an intuitionistic fuzzy open mapping.

(iv) If f is an intuitionistic fuzzy semi-open mapping and g is an intuitionistic fuzzy quasi sg-open mapping, then $g \circ f$ is an intuitionistic fuzzy open mapping.

(v) If f is an intuitionistic fuzzy α-open mapping and g is an intuitionistic fuzzy quasi sg-open mapping, then $g \circ f$ is an intuitionistic fuzzy open mapping.

(vi) If f is an intuitionistic fuzzy sg^*-open mapping and g is an intuitionistic fuzzy quasi sg-open mapping, then $g \circ f$ is an intuitionistic fuzzy quasi sg-open mapping.

(vii) If f is an intuitionistic fuzzy quasi sg-open mapping and g is an intuitionistic fuzzy quasi sg-closed mapping, into g is an intutitionistic fuzzy quasi sg-open mapping.

Proof:

(i) Let B be an IFSGOS in X. Since f is an intuitionistic fuzzy quasi sg-open mapping, f(B) is an IFOS in Y. Also since g is an intuitionistic fuzzy open mapping, g(f(B) is an IFOS in Z. Since (g∘f)(B) = g(f(B)), (g∘f)(B) is an IFOS in Z. Hence g∘f is an intuitionistic fuzzy quasi sg-open mapping.

(ii) Let B be an IFOS in X. Since every IFOS is an IFSGOS, B is an IFSGOS in X. Since f is an intuitionistic fuzzy quasi sg-open mapping, f(B) is an IFOS in Y. Since every IFOS is an IFSGOS, f(B) is an IFSGOS in Y. Also since g is an intuitionistic fuzzy quasi sg-open mapping, g(f(B) is an IFOS in Z. Since (g∘f)(B) = g(f(B)), (g∘f)(B) is an IFOS in Z. Hence g∘f is an intuitionistic fuzzy open mapping.

(iii) Let B be an IFOS in X. Since f is an intuitionistic fuzzy open mapping, f(B) is an IFOS in Y. Since every IFOS is an IFSGOS, f(B) is an IFSGOS in Y. Since g is an intuitionistic fuzzy quasi sg-open mapping, g(f(B) is an IFOS in Z. Since (g∘f)(B) = g(f(B)), (g∘f)(B) is an IFOS in Z. Hence g∘f is an intuitionistic fuzzy open mapping.

(iv) Let B be an IFOS in X. Since f is an intuitionistic fuzzy semi-open mapping, f(B) is an IFSOS in Y. Since every IFSOS is an IFSGOS, f(B) is an IFSGOS in Y. Since g is an intuitionistic fuzzy quasi sg-open mapping, g(f(B) is an IFOS in Z.
Since (g∘f)(B) = g(f(B)), (g∘f)(B) is an IFOS in Z. Hence g∘f is an intuitionistic fuzzy open mapping.

(v) Since every IFαOS is an IFSGOS, the proof follows immediately.

(vi) Proof is similar to (iv).

(vii) Obvious.

4. INTUITIONISTIC FUZZY QUASI SEMI-GENERALIZED CLOSED FUNCTIONS:

Definition: 4.1 A function f: X→Y from an IFTS X into an IFTS Y is said to be an intuitionistic fuzzy quasi semi-generalized closed (intuitionistic fuzzy quasi sg-closed) mapping if f(B) is an IFCS in Y, for every IFSGCS B in X.

It is evident that every intuitionistic fuzzy quasi sg-closed function is an intuitionistic fuzzy closed, intuitionistic fuzzy semi-closed, intuitionistic fuzzy α-closed and intuitionistic fuzzy sg-closed function. The converses are not true in general.

Lemma: 4.2 If f : X→Y is an intuitionistic fuzzy quasi sg-closed mapping, then $f^{-1}(int(B)) \subseteq sint(f^{-1}(B))$, for every IFS B of Y, where (X, τ) is an intuitionistic fuzzy semi $T_{1/2}$ space.

Proof: Let B be an IFS of Y. Then $sint(f^{-1}(B))$ is an IFSOS in X and by our assumption $sint(f^{-1}(B))$ is an IFSGOS in X which in turn implies that $scl(f^{-1}(\overline{B}))$ is an IFSCS in X. Since f is an intuitionistic fuzzy quasi sg-closed mapping, $f(scl(f^{-1}(\overline{B}))) \subseteq cl(f(f^{-1}(\overline{B}))) \subseteq cl(\overline{B})$. Hence $f^{-1}(int(B)) \subseteq sint(f^{-1}(B))$.

Theorem: 4.3 A bijective mapping f : X→Y is an intuitionistic fuzzy quasi sg-closed mapping if and only if for any IFS B of Y and for any IFSGOS G of X containing $f^{-1}(B)$, there exists an IFOS U of Y containing B such that $f^{-1}(U) \subseteq G$.

Proof: Similar to Theorem 3.18.

Theorem: 4.4 Let f: X→Y and g: Y→Z be any two functions such that g∘f: X→Z is an intuitionistic fuzzy quasi sg-closed mapping. Then:
(i) If f is an intuitionistic fuzzy sg-irresolute and surjective mapping, then g is an intuitionistic fuzzy closed mapping;
(ii) If g is an intuitionistic fuzzy sg-continuous and injective mapping, then f is an intuitionistic fuzzy sg^*-closed mapping.

Proof: (i) Suppose that B is an IFCS in Y. Since every IFCS is an IFSGCS, B is an IFSGCS in Y. Since f is an intuitionistic fuzzy sg-irresolute mapping, $f^{-1}(B)$ is an IFSGCS in X. Also since g∘f is an intuitionistic fuzzy quasi sg - closed mapping and f is surjective, $(g∘f)(f^{-1}(B))$ = g(B) is an IFCS in Z. Hence g is an intuitionistic fuzzy closed mapping.

(ii) Suppose that B is an IFSGCS in X. Since g∘f is an intuitionistic fuzzy quasi sg-closed mapping, (g∘f)(B) is an IFCS in Z. Also since g is an intuitionistic fuzzy sg-continuous and injective mapping, $g^{-1}(g∘f(B))$ = f(B), which is an IFSGCS in Y. Hence f is an intuitionistic fuzzy sg^*-closed mapping.

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