International Journal of Mathematical Archive-3(2), 2012, Page: 616-618
Available online through www.ijma.info ISSN 2229 – 5046

MAXIMUM INDEPENDENT SET COVER PEBBLING NUMBER OF A STAR

A. Lourdusamy*, C. Muthulakshmi@Sasikala

Department of Mathematics, St. xavier's College, Palayamkottai – 627002, India
Department of Mathematics, Sri Paramakalyani College, Alwarkurichi, India

E-mail: lourdusamy15@gmail.com

(Received on: 29-12-11; Accepted on: 14-01-12)

ABSTRACT

A pebbling move is defined by removing two pebbles from some vertex and placing one pebble on an adjacent vertex. A graph is said to be cover pebbled if every vertex has a pebble on it after a sequence of pebbling moves. The cover pebbling number γ (G) of a graph G is the minimum number of pebbles that must be placed on the vertices of G such that after a sequence of pebbling moves the graph can be cover pebbled no matter how the pebbles are initially placed on the vertices of the graph G. The maximum independent set cover pebbling number, ρ (G), of a graph G is the minimum number of pebbles that are placed on V(G) such that after a sequence of pebbling moves along a random walk, the set of vertices with pebbles forms a maximum independent set of G, regardless of their initial configuration. In this paper, we determine the maximum independent set cover pebbling number of a star.

Keywords: *Graph pebbling, cover pebbling, maximum independent set, maximum independent set cover pebbling, star*.

1. INTRODUCTION

Graphs considered here are simple, finite, undirected, and connected. Given a graph G, distribute k pebbles on its vertices in some configuration. A pebbling move is defined by removing two pebbles from some vertex and placing one pebble on an adjacent vertex in which each move takes place along a path. Let us consider the electrical network D. Here the trajectory of an electron is a random walk, which is a walk on D, whereby when it reaches node v, the wire along which to travel next is chosen at random, m wires incident with v being equiprobable. We imagine the same approach in the sequence of pebbling moves. Starting with a pebble distribution on the vertices of G, a pebbling move removes two pebbles from a vertex v, the edge along which to travel next is chosen at random, the d(v) edges incident with v being equiprobable and adds one pebble at a chosen adjacent vertex w. The *pebbling number*, π (G), of a graph G is the minimum number of pebbles that are placed on V(G), such that after a sequence of pebbling moves, a pebble can be moved to any root vertex v in G regardless of the initial configuration[2]. The *cover pebbling number*, γ (G) of a graph G is defined as the minimum number of pebbles needed to place a pebble on every vertex of a graph G using a sequence of pebbling moves, regardless of the initial configuration [3]. A set S of vertices in a graph G is said to be independent set (or an internally stable set) if no two vertices in the set S are adjacent. An independent set S is maximum if G has no independent set S' with |S'|>|S|. In this paper, we introduce the concept of maximum independent set cover pebbling number. We define the *maximum independent set cover pebbling number*, ρ (G) of a graph G, is the minimum number of pebbles that are placed on V(G) such that after a sequence of pebbling moves along random walks, the set of vertices with pebbles forms a maximum independent set of G, regardless of their initial configuration. Terms not defined here are used in the sense of Bondy and Murty.

1.1 Known Results

We find the following results with regard to the maximum independent set cover pebbling number of a complete graph and a path in [6].

Theorem 1.1: For K_n, $\rho(K_n)=n+1$ ($n\geq 2$).

Theorem 1.2: For a path P_3, $\rho(P_3)=6$.

Corresponding author: A. Lourdusamy*, *E-mail: lourdusamy15@gmail.com

1.2 Notations

Let us define f(G) to be the number of pebbles on G. For any vertex a of G, f(a) denotes the number of pebbles placed at the vertex a. For a, b $\in$ V(G), $a \xrightarrow{m} b$ refers to moving m pebbles to b from a if ab $\in$ E(G). Label the vertices of the star $K_{1,n}(n \geq 3)$ by $w, v_1, v_2, \ldots v_{n-1}, v_n$ such that deg(w)=n and deg(v_i)=1 (1$\leq$ i $\leq$ n). Clearly ρ ($K_{1,1}$)= ρ (K_2)=3, ρ ($K_{1,2}$)= ρ (P_3) =6. Let us compute the maximum independent set cover pebbling number of $K_{1,3}$ in the following theorem.

2. MAXIMUM INDEPENDENT SET COVER PEBBLING NUMBER OF A STAR $K_{1,n}$ (n≥3)

Theorem 2.1: The maximum independent set cover pebbling number of $K_{1,3}$ is ρ ($K_{1,3}$) =9.

Proof: Consider the following configuration: f(v_1)=8 and f(v)=0 for all v in V($K_{1,3}$)- {v_1}. Then we cannot cover the maximum independent set of $K_{1,3}$. Thus ρ ($K_{1,3}$) $\geq$ 9.

Now, consider the distribution of nine pebbles on the vertices of $K_{1,3}$.

Case 1: 1$\leq$ f (v_3) $\leq$ 3.

This implies that the path v_1wv_2 contains at least six pebbles and hence we are done since ρ (P_3) = 6.

Case 2: f (v_3) = 0.

We need at most four pebbles to place a pebble on v_3 from the vertices of V($K_{1,3}$) – {v_3}. If we use at most three pebbles to pebble v_3, then we are done since ρ (P_3) = 6 and v_1wv_2 contains at least six pebbles. Otherwise, we use exactly four pebbles to pebble v_3. Then f(v_1) =5 or f(v_2)=5 or f(v_1) $\geq$1 and f(v_2) $\geq$1 (after putting a pebble at v_3) and hence we are done.

Case 3: f (v_3) $\geq$ 4.

If either f (v_1) =0 or f (v_2) = 0 then apply case 2. Otherwise, consider the number of pebbles at w. If f (w) = 0, then clearly we are done. If f (w) = 2, then we move a pebble to v_1 and we are done. If f (w) = 1 or 3, then consider the following sequence of pebbling moves; $v_3 \xrightarrow{1} w \xrightarrow{1(or)2} v_3$ and hence we are done.

Thus ρ ($K_{1,3}$)$\leq$ 9.

Theorem 2.2: For $K_{1,n}$, ρ ($K_{1,n}$) = 4n-3 where n$\geq$3.

Proof: Let f(v_1)=4n-4 and f(v)=0 for all v in V($K_{1,n}$)-{v_1}. Then we cannot cover the maximum independent set of $K_{1,n}$. Thus ($K_{1,n}$) $\geq$ 4n-3.

Now, consider the distribution of 4n-3 pebbles on the vertices of $K_{1,n}$. By Theorem 2.1, the result is true for n=3. Assume n>3. Let f(v_i)=0 for some i. we can pebble the vertex v_i, using at most four pebbles from the vertices of V($K_{1,n}$)-{v_i} which is $K_{1,n-1}$ and hence we are done by induction. So assume that f(v_i)$\geq$1 for all i (1$\leq$i$\leq$n). Now consider the number of pebbles at w. If f(w) is even then we are done easily. Let f(w) be odd, say x. If f(v_i)$\geq$2, for some i (1$\leq$ i$\leq$ n), then consider the following sequence of pebbling moves : $v_i \xrightarrow{1} w \xrightarrow{\frac{x+1}{2}} v_i$ and hence we are done. So assume that f(v_i)=1 for all i(1$\leq$ i$\leq$ n). Then x=f(w)$\geq$ 3n-3$\geq$ 9 and we use the following sequence of pebbling moves: $w \xrightarrow{2} v_1 \xrightarrow{1} w \xrightarrow{\frac{x-3}{2}} v_1$ and hence we are done.

Thus ρ ($K_{1,n}$)$\leq$4n-3.

CONCLUSION:

In this paper, we have determined the maximum independent set cover pebbling number of a star. Finding the maximum independent set cover pebbling number of bipartite graphs is another interesting area of research.

REFERENCES:

[1] J. A. Bondy, U.S.R. Murty, Graph Theory 2008.

[2] F.R.K. Chung, Pebbling in hypercubes, SIAM J. Disc Math 2 (1989), 467-472.

[3] B. Crul, T. Cundiff, P. Feltman, G. H. Hurlbert, L. Pudwell, Z. Szanizlo, Z. Tuza, The cover pebbling number of graphs, Discrete Math 296 (2005), 15-23.

[4] J. Gardner, A. Teguia, A. Vuong, N. Watson, C. Yerger, Domination cover pebbling: Graph families, JCMCC 64 (2008), 255-271.

[5] G. Hurlbert, A survey of graph pebbling, Congressus Numerantium 139 (1999), 41-64.

[6] A. Lourdusamy, C. Muthulakshmi @ Sasikala, Maximum independent set cover pebbling number of complete graphs and paths, submitted for publication.

****

