INTUITIONISTIC FUZZY αψ-CONNECTEDNESS BETWEEN INTUITIONISTIC FUZZY SETS

¹Parimala Mani* & ²Devi Ramasamy

¹Assistant Professor, Department of Mathematics, Bannari Amman Institute of Technology Sathyamangalam-638401, Tamil Nadu, India

²Principal, Kongunadu Arts and Science College, Coimbatore-641029, Tamil Nadu, India

E-mail: ¹*rishwanthpari@gmail.com,* ²*rdevicbe@yahoo.com*

(Received on: 28-01-12; Accepted on: 24-02-12)

ABSTRACT

In this paper, we introduce the notion of intuitionistic fuzzy $\alpha\psi$ -connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces and investigate some of their properties.

Keyword: intuitionistic fuzzy topology, intuitionistic fuzzy closed set, intuitionistic fuzzy open set, intuitionistic fuzzy αψ-closed set, intuitionistic fuzzy αψ-connectedness between intuitionistic fuzzy sets.

Mathematical Subject Classification: 54 A 99, 03 E 99.

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [13] in 1965, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications.

Connectedness is one of the basic notions in topology. The concept of "connectedness between sets" was first introduced by Kuratowski [8] in general topology. A space X is said to connected between subset A and B if and only if there is no closed-open set F in X such that $A \subseteq F$ and $A \cap F = \phi$ [Kuratowski 1968, p142]. Since then various weak and strong form of connectedness between sets such as s-connectedness between sets [5], p-connectedness between sets [9], GO-connectedness between sets [14] have been introduced and studied in general topology. In 1993 Thakur and Malviya [11] extended the notions of connectedness between sets in Fuzzy topology. Recently Thakur and Thakur [12] extended the concepts of connectedness between sets in intuitionistic fuzzy topology. In the present paper we introduced and study the concepts of intuitionistic fuzzy $\alpha\psi$ -connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces

2. PRELIMINARIES

Throughout this paper, by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (briefly, IFTS). For a subset A of a space (X, τ) , cl(A), int(A) and A denote the closure of A, the interior of A and the compliment of A respectively. Each intuitionistic fuzzy set (briefly, IFS) which belongs to (X, τ) is called an intuitionitic fuzzy open set (briefly, IFOS) in X. The complement A of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

We introduce some basic notions and results that are used in the sequel.

Definition 2.1: [1] Let X be a nonempty fixed set and I be the closed interval [0, 1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

 $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \qquad \measuredangle(\mathbf{x}) + \nu_A(x) \leq 1$ for each $x \in X$.

Corresponding author: ¹Parimala Mani, *E-mail: ¹rishwanthpari@gmail.com International Journal of Mathematical Archive- 3 (2), Feb. – 2012

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}.$$

Definition 2.2:[1] Let A and B are IFSs of the form A={ $\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X$ } and B={ $x, \mu_B(x), \nu_B(x) \rangle : x \in X$ }. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$;

(ii) A = { $\langle x, v_A(x), \mu_A(x) \rangle : x \in X$ };

(iii) $A \cap B = \{ \langle x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x) \rangle : x \in X \};$

(iv)
$$A \cup B = \{ \langle x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x) \rangle : x \in X \}.$$

We will use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}.$

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

 $\begin{array}{l} (T1) \ 0_{\sim}, \ 1_{\sim} \in \tau; \\ (T2) \ G_1 \ \cap \ G_2 \in \tau \ \text{for any } G_1, \ G_2 \in \tau \ ; \\ (T3) \ \cup \ G_i \ \in \tau \ \text{for any arbitrary family } \{G_i : i \ \in \ J \ \} \subseteq \tau \ . \end{array}$

Definition 2.4: [3] Let A be an IFS in IFTS X. Then

int(A) = \cup {G : G is an IFOS in X and G \subseteq A} is called an intuitionistic fuzzy interior of A; cl(A) = \cap {G : G is an IFCS in X and G \supseteq A} is called an intuitionistic fuzzy closure of A.

Definition 2.5: A subset A of an intuitionistic fuzzy space (X, τ) is called

1. an intuitionistic fuzzy pre-open set [7] if $A \subseteq int(cl(A))$ and an intuitionistic fuzzy pre-closed set if $cl(int(A)) \subseteq A$,

2. an intuitionistic fuzzy semi-open set [6] if $A \subseteq cl(int(A))$ and an intuitionistic fuzzy semi-closed set if $int(cl(A)) \subseteq A$, 3. an intuitionistic fuzzy α -open set [2] if $A \subseteq int(cl(int(A)))$ and an intuitionistic fuzzy α -closed set if $cl(int(cl(A))) \subseteq A$,

The pre-closure (resp. semi-closure, α -closure) of a subset A of an intuitionistic fuzzy space (X, τ) is the intersection of all preclosed (resp. semi-closed, α -closed) sets that contain A and is denoted by IF pcl(A) (resp. IF scl(A), IF α cl(A)).

Definition 2.6: A subset A of an intuitionistic fuzzy topological space (X, τ) is called

- 1. an intuitionistic fuzzy semi-generalized closed [10] (briefly, IF sg-closed) set if scl(A) \subseteq U whenever A \subseteq U and U is semi-open in (X, τ);
- 2. an intuitionistic fuzzy ψ -closed set [4] if scl(A) \subseteq U whenever A \subseteq U and U is IF sg-open in (X, τ).
- 3. an intuitionistic fuzzy $\alpha\psi$ -closed [4] (briefly, IF $\alpha\psi$ CS) set if ψ cl(A) \subseteq U whenever A \subseteq U and U is IF α -open in (X, τ).

Definition 2.7: [4] Let A be an IFS in IFTS X. Then

IFa ψ int(A) = U{G : G is an IFa ψ OS in X and G \subseteq A} is called an intuitionistic fuzzy a ψ -interior of A; IFa ψ cl(A)= \cap {G:GisanIFa ψ CSinX and G \supseteq A} is called an intuitionistic fuzzy a ψ -closure of A.

Definition 2.8: [3] Two intuitionistic fuzzy sets A and B of X are said to be q-coincident (AqB for short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

Lemma 2.9: [3] For any two intuitionistic fuzzy sets A and B of X, \neg (AqB) if and only if A \subset B^c.

Lemma 2.10: [3] For any intuitionistic fuzzy set A in (X, τ) we have:

- 1. A is an intuitionistic fuzzy closed set in X, cl(A) = A,
- 2. A is an intuitionistic fuzzy open set in X, int(A) = A,
- 3. $cl(A^{c}) = (int(A))^{c}$,

4. $int(A^{c}) = (cl(A))^{c}$.

Remark 2.11: [4] Every intuitionistic fuzzy closed (resp. intuitionistic fuzzy open) set is intuitionistic fuzzy $\alpha\psi$ -closed (resp. intuitionistic fuzzy $\alpha\psi$ -open) but the converse may not be true.

Definition 2.12:. An intuitionistic fuzzy topological space is said to be intuitionistic fuzzy $\alpha\psi$ -connected if no nonempty intuitionistic fuzzy set is both intuitionistic fuzzy $\alpha\psi$ -open and intuitionistic fuzzy $\alpha\psi$ -closed.

Definition 2.13: [12] An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy closed open set F in X such that $A \subset F$ and \neg (F q B).

3. ON INTUITIONISTIC FUZZY $\alpha\psi\text{-}\text{CONNECTEDNESS}$ BETWEEN INTUITIONISTIC FUZZY SETS

Definition 3.1: An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F in X such that $A \subset F$ and \neg (F q B).

Theorem 3.2: If an intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B, then it is intuitionistic fuzzy connected between A and B.

Proof: If (X, τ) is not intuitionistic fuzzy connected between A and B, then there exists an intuitionistic fuzzy closed open set F in X such that $A \subset F$ and \neg (F qB). Then by Remark 2.11, F is an intuitionistic fuzzy $\alpha \psi$ -closed $\alpha \psi$ -open set in X such that $A \subset F$ and \neg (F qB). Hence (X, τ) is not intuitionistic fuzzy $\alpha \psi$ -connected between A and B, which contradicts our hypothesis.

Remark 3.3: The converse of Theorem 3.2 may be false, as the following example shows.

Example 3.4: Let $X = \{a, b\}$ and $U = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.6, 0.4 \rangle\}$, $A = \{\langle a, 0.2, 0.7 \rangle, \langle b, 0.3, 0.6 \rangle\}$ and $B = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.4, 0.5 \rangle\}$ be intuitionistic fuzzy sets on X. Let $\tau = \{0_{\sim}, 1_{\sim}, U\}$ be an intuitionistic fuzzy topology on X. Then (X, τ) is intuitionistic fuzzy connected between A and B but it is not intuitionistic fuzzy $\alpha\psi$ -connected between A and B.

Theorem 3.5: An intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B if and only if there is no intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F in X such that $A \subset F \subset B^c$.

Proof:

Necessity: Let (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B. Suppose on the contrary, that F is an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set in X such that $A \subset F \subset B^c$. Now $F \subset B^c$ which implies that \neg (F q B). Therefore F is an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set in X such that $A \subset F$ and \neg (F q B). Hence (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B, which is a contradiction.

Sufficiency: Suppose on the contrary, that (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F in X such that $A \subset F$ and \neg (F q B). Now, \neg (F q B) which implies that $F \subset B^c$. Therefore F is an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set in X such that $A \subset F$ and \neg (F q B). Now, \neg (F q B) which implies that $F \subset B^c$. Therefore F is an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set in X such that $A \subset F \subset B^c$, which contradicts our assumption.

Theorem 3.6: If an intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B, then A and B are non-empty.

Proof: If the intuitionistic fuzzy set A is empty, then A is an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set in X and A \subset B. Now we claim that \neg (A q B). If A q B, then there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

But $\mu_A(x) = 0$ and $\gamma_A(x) = 1$ for all $x \in X$. Therefore no point $x \in X$ for which $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$, which is a contradiction. Hence \neg (A q B) and (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy set A and intuitionistic fuzzy set B.

Theorem 3.7: If an intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B and A \subset A₁ and B \subset B₁, then (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between A₁ and B₁.

Proof: Suppose (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A_1 and B_1 . Then there exists an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F in X such that $A_1 \subset F$ and \neg (F q B₁). Clearly, $A \subset F$. Now we claim that \neg (F q B). If F q B, then there exists a point $x \in X$ such that $\mu_F(x) > \gamma_B(x)$ or $\gamma_F(x) < \mu_B(x)$. Without loss of generality suppose a point $x \in X$ such that $\mu_F(x) > \gamma_B(x)$. Now $B \subset B_1$, $\gamma_B(x) \ge \gamma_{B1}(x)$. And so $\mu_F(x) > \gamma_{B1}(x)$ and F q B₁, which is a contradiction. Consequently (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B.

Theorem 3.8: An intuitionistic fuzzy topological space (X, τ) is intuitionis-tic fuzzy $\alpha \psi$ -connected between intuitionistic fuzzy sets A and B if and only if it is intuitionistic fuzzy $\alpha \psi$ -connected between $\alpha \psi cl(A)$ and $\alpha \psi cl(B)$.

Proof:

Necessity: Follows from Theorem 3.7, because $\mathbf{A} \subset \alpha \psi cl(\mathbf{A})$ and $\mathbf{B} \subset \alpha \psi cl(\mathbf{B})$.

Sufficiency: Suppose (X, τ) is not intuitionistic fuzzy $\alpha \psi$ -connected between intuitionistic fuzzy sets A and B. Then there is an intuitionistic fuzzy $\alpha \psi$ -closed $\alpha \psi$ -open set F of X such that $A \subset F$ and \neg (F q B). Since F is intuitionistic fuzzy $\alpha \psi$ -closed and $A \subset F$, $\alpha \psi cl(A) \subset F$. Now, \neg (F q B) which implies that $F \subset B^c$.

Therefore $F = \alpha \psi int F \subset \alpha \psi int(B^c) = (\alpha \psi cl(B))^c$.

Hence (F q $\alpha \psi cl(B)$) and X is not intuitionistic fuzzy $\alpha \psi$ -connected between $\alpha \psi cl(A)$ and $\alpha \psi cl(B)$.

Theorem 3.9: Let (X, τ) be an intuitionistic fuzzy topological space and A and B be two intuitionistic fuzzy sets in X. If A q B then (X, τ) is intuitionistic fuzzy $\alpha \psi$ -connected between A and B.

Proof: If F is any intuitionistic fuzzy $\alpha \psi$ -closed $\alpha \psi$ -open set of X such that $A \subset F$, then A q B hence F q B.

Remark 3.10: The converse of Theorem 3.9 may not be true, as the following example shows.

Example 3.11: Let $X = \{a, b\}$ and $U = \{\langle a, 0.2, 0.6 \rangle, \langle b, 0.3, 0.5 \rangle\}$, $A = \{\langle a, 0.4, 0.3 \rangle, \langle b, 0.3, 0.6 \rangle\}$ and $B = \{\langle a, 0.2, 0.5 \rangle, \langle b, 0.5, 0.4 \rangle\}$ be intuitionistic fuzzy sets on X. Let $\tau = \{0, 1, U\}$ be an intuitionistic fuzzy topology on X. Then (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B but \neg (A q B).

Theorem 3.12: An intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected if and only if it is intuitionistic fuzzy $\alpha\psi$ -connected between every pair of its non-empty intuitionistic fuzzy sets.

Proof:

Necessity: Let A, B be any pair of intuitionistic fuzzy subsets of X. Suppose (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F of X such that $A \subset F$ and \neg (F q B). Since intuitionistic fuzzy sets A and B are non-empty, it follows that F is a non-empty proper intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set of X. Hence (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected.

Sufficiency: Suppose (X, τ) is not intuitionistic fuzzy $\alpha\psi$ -connected. Then there exists a non-empty proper intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F of X. Consequently X is not intuitionistic fuzzy $\alpha\psi$ -connected between F and F^c, a contradiction.

Remark 3.13: If a fuzzy topological space (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between a pair of its intuitionistic fuzzy subsets it is not necessarily that (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between every pair of its intuitionistic fuzzy subsets and so is not necessarily intuitionistic fuzzy $\alpha\psi$ -connected, as the following example shows.

Example 3.14: Let $X = \{a, b\}$ and $U = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.6, 0.4 \rangle\}$, $A = \{\langle a, 0.4, 0.3 \rangle, \langle b, 0.6, 0.4 \rangle\}$, $B = \{\langle a, 0.5, 0.2 \rangle, \langle b, 0.4, 0.4 \rangle\}$, $C = \{\langle a, 0.2, 0.7 \rangle, \langle b, 0.3, 0.6 \rangle\}$ and $D = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.4, 0.5 \rangle\}$ be intuitionistic fuzzy sets on X.

Theorem 3.15: Let (Y, τ_Y) be a subspace of a intuitionistic fuzzy topological space (X, τ) and A, B be intuitionistic fuzzy subsets of Y. If (Y, τ_Y) is intuitionistic fuzzy $\alpha\psi$ -connected between A and B then so is (X, τ) .

Proof: Suppose, on the contrary, that (X, τ) is not intuitionistic fuzzy $\alpha \psi$ -connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy $\alpha \psi$ -closed $\alpha \psi$ -open set F of X such that $A \subset F$ and \neg (F q B).

Put $F_Y = F \cap Y$. Then F_Y is intuitionistic fuzzy $\alpha \psi$ -closed $\alpha \psi$ -open set in Y such that $A \subset F_Y$ and $\neg (F_Y q B)$. Hence (Y, τ_Y) is not intuitionistic fuzzy $\alpha \psi$ -connected between A and B, a contradiction.

Theorem 3.16 Let (Y, τ_Y) be an intuitionistic fuzzy closed open subspace of a intuitionistic fuzzy topological space (X, τ) and A, B be intuitionistic fuzzy subsets of Y. If (X, τ) is intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B, then so is (Y, τ_Y) .

© 2012, IJMA. All Rights Reserved

Proof: If (Y, τ_Y) is not intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B, then there exists an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set F of Y such that $A \subset F$ and \neg (F q B). Since Y is intuitionistic fuzzy closed open in X, F is an intuitionistic fuzzy $\alpha\psi$ -closed $\alpha\psi$ -open set in X. Hence X cannot be intuitionistic fuzzy $\alpha\psi$ -connected between intuitionistic fuzzy sets A and B, a contradiction.

REFERENCES

- [1] K.T. Atanassov, intuitionstic fuzzy sets, Fuzzy sets and systems, 20 (1986), 87-96.
- [2] A.S. Bin Shahna, On fuzzy strong semi continuity and fuzzy pre continuity, Fuzzy sets and systems, 44 (1991), no.2, 303-308.
- [3] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88 (1997), 81-89
- [4] R. Devi and M. Parimala, Intuitionistic fuzzy αψ-closed sets in intuitionistic fuzzy topological spaces, (Submitted).
- [5] K.K. Dubey, and O.S. Panwar, Some properties of s-connectedness between sets and s-connected mapping. Indian J. pure Math.154 1984, 343-354.
- [6] H. Gurcay, D. Coker and A.H. Es, On fuzzy continuity in intuitionistic fuzzy topological spaces, J.Fuzzy Math., 5 (1997), no. 2, 365-378.
- [7] J.K. Joen, Y.B. Jun, J.H. Park, Intuitionistic fuzzy α-continuity and intuitionstic fuzzy pre continuity, IJMMS, 19 (2005), 3091-3101.
- [8] K.Kuratowski . Topology Vol. II (transl.), Academic Press New York (??).
- [9] R. Santhi and K. Arun Prakash, On Intuitionistic Fuzzy Semi-Generalized Closed Sets and its Applications, Int. J. Contemp. Math. Sciences, Vol. 5, 2010, no. 34, 1677 - 1688.
- [10] S.S.Thakur and P. Paik p-connectedness between sets, Jour of Sci. Res. B.H.U. 371 1987 59-63.
- [11] S.S.Thakur and R. Malviya. Fuzzy connectedness between fuzzy sets, The Journal of Fuzzy Mathematics Vol.11993 4. 757-759.
- [12] S.S. Thakur and Mahima Thakur, Intuitionistic fuzzy set Connected map-pings, The Journal of Fuzzy Mathematics (Accepted)
- [13] L.A. Zadeh, Fuzzy Sets, Information and Control, 181965,338-353.
- [14] Zorlutuna, M. Kucuk and Y. Kucuk, Slightly Generalized Continuous Functions, Kochi J. Math. 32008, 99-108.
