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INTUITIONISTIC FUZZY αψ-CONNECTEDNESS BETWEEN INTUITIONISTIC FUZZY SETS

[1]Parimala Mani* & [2]Devi Ramasamy

[1]*Assistant Professor, Department of Mathematics, Bannari Amman Institute of Technology Sathyamangalam-638401, Tamil Nadu, India*

[2]*Principal, Kongunadu Arts and Science College, Coimbatore-641029, Tamil Nadu, India*

E-mail: [1]rishwanthpari@gmail.com, [2]rdevicbe@yahoo.com

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ABSTRACT

In this paper, we introduce the notion of intuitionistic fuzzy αψ-connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces and investigate some of their properties.

Keyword: *intuitionistic fuzzy topology, intuitionistic fuzzy closed set, intuitionistic fuzzy open set,intuitionistic fuzzy αψ-closed set, intuitionistic fuzzy αψ-connectedness between intuitionistic fuzzy sets.*

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1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [13] in 1965, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications.

Connectedness is one of the basic notions in topology. The concept of "connectedness between sets" was first introduced by Kuratowski [8] in general topology. A space X is said to connected between subset A and B if and only if there is no closed-open set F in X such that $A \subseteq F$ and $A \cap F = \varphi$ [Kuratowski 1968, p142]. Since then various weak and strong form of connectedness between sets such as s-connectedness between sets [5], p-connectedness between sets [9], GO-connectedness between sets [14] have been introduced and studied in general topology. In 1993 Thakur and Malviya [11] extended the notions of connectedness between sets in Fuzzy topology. Recently Thakur and Thakur [12] extended the concepts of connectedness between sets in intuitionistic fuzzy topology. In the present paper we introduced and study the concepts of intuitionistic fuzzy αψ-connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces

2. PRELIMINARIES

Throughout this paper, by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (briefly, IFTS). For a subset A of a space (X, τ), cl(A), int(A) and A denote the closure of A, the interior of A and the compliment of A respectively. Each intuitionistic fuzzy set (briefly, IFS) which belongs to (X, τ) is called an intuitionitic fuzzy open set (briefly, IFOS) in X. The complement A of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

We introduce some basic notions and results that are used in the sequel.

Definition 2.1: [1] Let X be a nonempty fixed set and I be the closed interval [0, 1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$$

where the mappings $\mu_A : X \rightarrow I$ and $\nu_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

***Corresponding author: [1]Parimala Mani*, *E-mail: [1]rishwanthpari@gmail.com**

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X\}.$$

Definition 2.2:[1] Let A and B are IFSs of the form A={ $\langle x, \mu_A(x),\nu_A(x) \rangle$: $x\in X$} and B={x, $\mu_B(x)$, $\nu_B(x)\rangle$:$x\in X$}. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$;

(ii) A = { $\langle x, \nu_A(x), \mu_A(x) \rangle$: $x \in X$};

(iii) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X\}$;

(iv) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X\}$.

We will use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(T1) $0_\sim, 1_\sim \in \tau$;
(T2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(T3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

Definition 2.4: [3] Let A be an IFS in IFTS X. Then
int(A) = $\cup\{G$: G is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;
cl(A) = $\cap\{G$: G is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A.

Definition 2.5: A subset A of an intuitionistic fuzzy space (X, τ) is called
1. an intuitionistic fuzzy pre-open set [7] if $A \subseteq$ int(cl(A)) and an intuitionistic fuzzy pre-closed set if cl(int(A)) $\subseteq$ A,
2. an intuitionistic fuzzy semi-open set [6] if $A \subseteq$ cl(int(A)) and an intuitionistic fuzzy semi-closed set if int(cl(A)) $\subseteq$ A,
3. an intuitionistic fuzzy α-open set [2] if $A \subseteq$ int(cl(int(A))) and an intuitionistic fuzzy α-closed set if cl(int(cl(A))) $\subseteq$ A,

The pre-closure (resp. semi-closure, α-closure) of a subset A of an intuitionistic fuzzy space (X, τ) is the intersection of all pre-closed (resp. semi-closed, α-closed) sets that contain A and is denoted by IF pcl(A) (resp. IF scl(A), IF αcl(A)).

Definition 2.6: A subset A of an intuitionistic fuzzy topological space (X, τ) is called
1. an intuitionistic fuzzy semi-generalized closed [10] (briefly, IF sg-closed) set if scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is semi-open in (X, τ);
2. an intuitionistic fuzzy ψ-closed set [4] if scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is IF sg-open in (X, τ).
3. an intuitionistic fuzzy αψ-closed [4] (briefly, IF αψCS) set if ψcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is IF α-open in (X, τ).

Definition 2.7: [4] Let A be an IFS in IFTS X. Then
IFαψint(A) = $\cup\{G$: G is an IFαψOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy αψ-interior of A;
IFαψcl(A)=$\cap\{$G:GisanIFαψCSinX and $G \supseteq A\}$is called an intuitionistic fuzzy αψ-closure of A.

Definition 2.8: [3] Two intuitionistic fuzzy sets A and B of X are said to be q-coincident (AqB for short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

Lemma 2.9: [3] For any two intuitionistic fuzzy sets A and B of X, $\neg$ (AqB) if and only if $A \subset B^c$.

Lemma 2.10: [3] For any intuitionistic fuzzy set A in (X, τ) we have:
1. A is an intuitionistic fuzzy closed set in X, cl(A) = A,
2. A is an intuitionistic fuzzy open set in X, int(A) = A,
3. $cl(A^c) = (int(A))^c$,
4. $int(A^c) = (cl(A))^c$.

Remark 2.11: [4] Every intuitionistic fuzzy closed (resp. intuitionistic fuzzy open) set is intuitionistic fuzzy αψ-closed (resp. intuitionistic fuzzy αψ-open) but the converse may not be true.

Definition 2.12:. An intuitionistic fuzzy topological space is said to be intuitionistic fuzzy αψ-connected if no non-empty intuitionistic fuzzy set is both intuitionistic fuzzy αψ-open and intuitionistic fuzzy αψ-closed.

Definition 2.13: [12] An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy closed open set F in X such that $A \subset F$ and $\neg$ (F q B).

3. ON INTUITIONISTIC FUZZY αψ-CONNECTEDNESS BETWEEN INTUITIONISTIC FUZZY SETS

Definition 3.1: An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy αψ-closed αψ-open set F in X such that $A \subset F$ and $\neg$(F q B).

Theorem 3.2: If an intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B, then it is intuitionistic fuzzy connected between A and B.

Proof: If (X, τ) is not intuitionistic fuzzy connected between A and B, then there exists an intuitionistic fuzzy closed open set F in X such that $A \subset F$ and $\neg$ (F qB). Then by Remark 2.11, F is an intuitionistic fuzzy αψ-closed αψ-open set in X such that $A \subset F$ and $\neg$ (F qB). Hence (X, τ) is not intuitionistic fuzzy αψ-connected between A and B, which contradicts our hypothesis.

Remark 3.3: The converse of Theorem 3.2 may be false, as the following example shows.

Example 3.4: Let X = {a, b} and U = {⟨ a, 0.5, 0.4 ⟩, ⟨ b, 0.6, 0.4 ⟩} , A = {⟨ a, 0.2, 0.7 ⟩, ⟨ b, 0.3, 0.6 ⟩} and B ={⟨ a, 0.5, 0.4 ⟩, ⟨ b, 0.4, 0. 5⟩} be intuitionistic fuzzy sets on X. Let $\tau = \{0_\sim, 1_\sim, U\}$ be an intuitionistic fuzzy topology on X. Then (X, τ) is intuitionistic fuzzy connected between A and B but it is not intuitionistic fuzzy αψ-connected between A and B.

Theorem 3.5: An intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B if and only if there is no intuitionistic fuzzy αψ-closed αψ-open set F in X such that $A \subset F \subset B^c$.

Proof:
Necessity: Let (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B. Suppose on the contrary, that F is an intuitionistic fuzzy αψ-closed αψ-open set in X such that $A \subset F \subset B^c$. Now $F \subset B^c$ which implies that $\neg$ (F q B). Therefore F is an intuitionistic fuzzy αψ-closed αψ-open set in X such that $A \subset F$ and $\neg$ (F q B). Hence (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B, which is a contradiction.

Sufficiency: Suppose on the contrary, that (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy αψ-closed αψ-open set F in X such that $A \subset F$ and $\neg$ (F q B). Now, $\neg$ (F q B) which implies that $F \subset B^c$. Therefore F is an intuitionistic fuzzy αψ-closed αψ-open set in X such that $A \subset F \subset B^c$, which contradicts our assumption.

Theorem 3.6: If an intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B, then A and B are non-empty.

Proof: If the intuitionistic fuzzy set A is empty, then A is an intuitionistic fuzzy αψ-closed αψ-open set in X and $A \subset B$. Now we claim that $\neg$ (A q B). If A q B, then there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

But $\mu_A(x) = 0$ and $\gamma_A(x) = 1$ for all $x \in X$. Therefore no point $x \in X$ for which $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$, which is a contradiction. Hence $\neg$ (A q B) and (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy set A and intuitionistic fuzzy set B.

Theorem 3.7: If an intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B and $A \subset A_1$ and $B \subset B_1$, then (X, τ) is intuitionistic fuzzy αψ-connected between A_1 and B_1.

Proof: Suppose (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A_1 and B_1. Then there exists an intuitionistic fuzzy αψ-closed αψ-open set F in X such that $A_1 \subset F$ and $\neg$ (F q B_1). Clearly, $A \subset F$. Now we claim that $\neg$ (F q B). If F q B, then there exists a point $x \in X$ such that $\mu_F(x) > \gamma_B(x)$ or $\gamma_F(x) < \mu_B(x)$. Without loss of generality suppose a point $x \in X$ such that $\mu_F(x) > \gamma_B(x)$. Now $B \subset B_1$, $\gamma_B(x) \geq \gamma_{B1}(x)$. And so $\mu_F(x) > \gamma_{B1}(x)$ and F q B_1, which is a contradiction. Consequently (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B.

Theorem 3.8: An intuitionistic fuzzy topological space (X, τ) is intuitionis-tic fuzzy αψ-connected between intuitionistic fuzzy sets A and B if and only if it is intuitionistic fuzzy αψ-connected between αψcl(A) and αψcl(B).

Proof:
Necessity: Follows from Theorem 3.7, because **A** $\subset$ αψcl(A) and B $\subset$ αψcl(B).

Sufficiency: Suppose (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B. Then there is an intuitionistic fuzzy αψ-closed αψ-open set F of X such that A $\subset$ F and ¬ (F q B). Since F is intuitionistic fuzzy αψ-closed and A $\subset$ F, αψcl(A) $\subset$ F . Now, ¬ (F q B) which implies that F $\subset$ B^c.

Therefore F = αψint F $\subset$ αψint(B^c) = $(\alpha\psi cl(B))^c$.

Hence (F q αψcl(B)) and X is not intuitionistic fuzzy αψ-connected between αψcl(A) and αψcl(B).

Theorem 3.9: Let (X, τ) be an intuitionistic fuzzy topological space and A and B be two intuitionistic fuzzy sets in X. If A q B then (X, τ) is intuitionistic fuzzy αψ-connected between A and B.

Proof: If F is any intuitionistic fuzzy αψ-closed αψ-open set of X such that A $\subset$ F, then A q B hence F q B.

Remark 3.10: The converse of Theorem 3.9 may not be true, as the following example shows.

Example 3.11: Let X = {a, b} and U ={⟨ a, 0.2, 0.6 ⟩, ⟨ b, 0.3, 0.5 ⟩} , A ={⟨ a, 0.4, 0.3⟩, ⟨ b, 0.3, 0.6 ⟩} and B={⟨ a, 0.2, 0.5 ⟩, ⟨ b, 0.5, 0. 4⟩}be intuitionistic fuzzy sets on X. Let τ = {0, 1, U} be an intuitionistic fuzzy topology on X. Then (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B but ¬ (A q B).

Theorem 3.12: An intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy αψ-connected if and only if it is intuitionistic fuzzy αψ-connected between every pair of its non-empty intuitionistic fuzzy sets.

Proof:
Necessity: Let A, B be any pair of intuitionistic fuzzy subsets of X. Suppose (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy αψ-closed αψ-open set F of X such that A $\subset$ F and ¬ (F q B). Since intuitionistic fuzzy sets A and B are non-empty, it follows that F is a non-empty proper intuitionistic fuzzy αψ-closed αψ-open set of X. Hence (X, τ) is not intuitionistic fuzzy αψ-connected.

Sufficiency: Suppose (X, τ) is not intuitionistic fuzzy αψ-connected. Then there exists a non-empty proper intuitionistic fuzzy αψ-closed αψ-open set F of X. Consequently X is not intuitionistic fuzzy αψ-connected between F and F^c, a contradiction.

Remark 3.13: If a fuzzy topological space (X, τ) is intuitionistic fuzzy αψ-connected between a pair of its intuitionistic fuzzy subsets it is not necessarily that (X, τ) is intuitionistic fuzzy αψ-connected between every pair of its intuitionistic fuzzy subsets and so is not necessarily intuitionistic fuzzy αψ-connected, as the following example shows.

Example 3.14: Let X = {a, b} and U ={⟨ a, 0.5, 0.4⟩, ⟨b, 0.6, 0.4⟩}, A ={⟨a, 0.4, 0.3⟩ , ⟨b, 0.6, 0.4⟩}, B = {⟨a, 0.5, 0.2⟩ , ⟨b, 0.4, 0.4⟩}, C = {⟨a, 0.2, 0.7⟩ , ⟨b, 0.3, 0.6⟩} and D = {⟨a, 0.5, 0.4⟩ , ⟨b, 0.4, 0.5⟩} be intuitionistic fuzzy sets on X.

Let τ = {$0_{\sim}$, $1_{\sim}$, U} be an intuitionistic fuzzy topology on X. Then (X, τ) is intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B but it is not intuitionistic fuzzy connected between intuitionistic fuzzy sets C and D. Also (X, τ) is not intuitionistic fuzzy αψ-connected.

Theorem 3.15: Let (Y, τ_Y) be a subspace of a intuitionistic fuzzy topological space (X, τ) and A, B be intuitionistic fuzzy subsets of Y. If (Y, τ_Y) is intuitionistic fuzzy αψ-connected between A and B then so is (X, τ).

Proof: Suppose, on the contrary, that (X, τ) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy αψ-closed αψ-open set F of X such that A $\subset$ F and ¬ (F q B).

Put F_Y = F ∩Y. Then F_Y is intuitionistic fuzzy αψ-closed αψ-open set in Y such that A $\subset$ F_Y and ¬ (F_Y q B).
Hence (Y, τ_Y) is not intuitionistic fuzzy αψ-connected between A and B, a contradiction.

Theorem 3.16 Let (Y, τ_Y) be an intuitionistic fuzzy closed open subspace of a intuitionistic fuzzy topological space (X, τ) and A, B be intuitionistic fuzzy subsets of Y . If (X, τ) is intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B, then so is (Y, τ_Y).

Proof: If (Y, τ_Y) is not intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B, then there exists an intuitionistic fuzzy αψ-closed αψ-open set F of Y such that $A \subset F$ and $\neg$ (F q B). Since Y is intuitionistic fuzzy closed open in X, F is an intuitionistic fuzzy αψ-closed αψ-open set in X. Hence X cannot be intuitionistic fuzzy αψ-connected between intuitionistic fuzzy sets A and B, a contradiction.

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