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M-CONTINUITY AND ITS DECOMPOSITIONS

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ABSTRACT

The aim of this paper is to introduce the notions of R-locally m-closed sets and π-locally m-closed sets and some new subsets of minimal spaces and to obtain decompositions of M-continuity.

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1. INTRODUCTION

In [4] Maki introduced the notions of minimal structures and minimal spaces. Popa and Noiri [6] introduced a new notion of M-continuous functions as a function defined between sets satisfying some minimal conditions. In 1970, the notion of generalized closed (briefly, g-closed) sets were introduced and investigated by Levine [3]. Recently, many modifications of g-closed sets have defined and investigated. One among them is mg-closed sets which were introduced by Noiri and studied in [5]. In [5], he also introduced locally m-closed sets in minimal spaces.

In this paper, we introduce the notions of R-locally m-closed sets and π-locally m-closed sets, some new subsets of minimal spaces and obtain decompositions of M-continuity. Also we investigate some properties and characterizations of these sets with some theorems, examples and counter examples.

2. PRELIMINARIES

Definition 2.1[4]: A subfamily $m_x \subset P(X)$ is said to be a minimal structure on X if $\phi, X \in m_x$. The pair (X, m_x) is called a minimal space (or an m-space). A subset A of X is said to be m-open if A $\in m_x$. The complement of an m-open set is called m-closed set. We set m-Int(A)= $\bigcup\{U : U \subset A, U \in m_x\}$ and m-Cl(A) $=\cap\{F : A \subset F, X-F \in m_x\}$.

Lemma 2.2 [6]: Let (X, m_x) be an m-space and $A \subset X$. Then $x \in$ m-Cl(A) if and only if $U \bigcap A \neq \phi$ for every $U \in m_x$ containing x.

A minimal space (X, m_x) has the property [в] if the union of any family of subsets belonging to m_x belongs to m_x.

Proposition 2.3 [6]: Let (X, m_x) be a minimal space.

(i) For any two subsets A, B of X, the following properties hold:
  (a) $A \subset$ m-Cl(A) and A = m-Cl(A) if A is a m-closed set.

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(b) m-Int(A) $\subset$ A and A = m-Int(A) if A is an m-open set.
(c) A $\subset$ B $\Rightarrow$ m-Cl(A) $\subset$ m-Cl(B) and A $\subset$ B $\Rightarrow$ m-Int(A) $\subset$ m-Int(B).
(d) m-Cl(m-Cl(A)) = m-Cl(A).
(e) $(\text{m-Cl}(A))^c$ = m-Int(A^c) and $(\text{m-Int}(A))^c$ = m-Cl(A^c).
(f) m-Cl(ϕ) = ϕ; m-Cl(X) = X; m-Int(ϕ) = ϕ; m-Int(X) = X.

(ii) The following are equivalent.
(a) m_x has the property [ʙ].
(b) If m-Int(A) = A, then A $\in m_x$.
(c) If m-Cl(B) = B, then X – B $\in m_x$.

Definition 2.4 [8]: A subset A of a minimal space (X, m_x) is said to be
(a) regular m-open if A = m-Int(m-Cl(A)),
(b) m-semi open if A $\subset$ m-Cl(m-Int(A)),
(c) m-π-open if it is the finite union of regular m-open sets of A.

Definition 2.5 [5]: A subset A of a minimal space (X, m_x) is said to be mg-closed if m-Cl(A) $\subset$ U whenever A $\subset$ U and U is m-open in X.

Definition 2.6[5]: A subset A of an m-space (X, m_x) is said to be locally m-closed if A = U $\cap$ V where U is m-open and V is m-closed.

Lemma 2.7 [8]: For the subsets of a minimal space (X, m_x) satisfying property [ʙ], every m-π-open set is an m-open set but not conversely.

Example 2.8: Let (X, m_x) be a minimal space satisfying property [ʙ], such that X = {a, b, c} and m_x = {φ, {c}, {a, b}, {a, c}, X}. Then A = {a, c} is an m-open set but not an m-π-open set.

Remark 2.9 [8]: The implication in Lemma 2.7 will not hold if m_x does not have property [ʙ] as shown in the following Example 2.10.

Example 2.10: Let (X, m_x) be a minimal space such that X = {a, b, c} and m_x = {ϕ, X, {a}, {b}}. Then A = {a, b} is an m-π-open set but not an m-open set.

Lemma 2.11 [8]: For the subsets of a minimal space (X, m_x), every regular m-open set is an m-π-open set but not conversely.

Example 2.12: Let (X, m_x) be a minimal space such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then A = {a, b} is an m-π-open set but not a regular m-open set.

Remark 2.13 [1]: For the subsets of a minimal space (X, m_x), every m-open set is m-semi open set but not conversely.

Definition 2.14 [6]: A function f : (X, m_x) $\rightarrow$ (Y, m_y) is said to be M-continuous if for each x $\in$ X and each V $\in m_y$ containing f(x), there exists U $\in m_x$ containing x such that f(U) $\subset$ V.

Lemma 2.15 [6]: For a function f : (X, m_x) $\rightarrow$ (Y, m_y) where m_x satisfies property [ʙ], the following are equivalent.
1. f is M-continuous;
2. $f^{-1}(V)$ is m_x -open for every m_y -open set V of Y;
3. $f^{-1}(K)$ is m_x -closed for every m_y -closed set K of X.

3. STRONGER FORMS OF LOCALLY m-CLOSED SETS

Definition 3.1: A subset A of an m-space (X, m_x) is said to be
(a) R-locally m-closed if A = U $\cap$ V where U is regular m-open and V is m-closed,
(b) π-locally m-closed if A = U $\cap$ V where U is m-π-open and V is m-closed.

Definition 3.2: A subset A of an m-space (X, m_x) is said to be
(a) m-rg-closed if m-Cl(A) $\subset$ U whenever U is regular m-open in X and A $\subset$ U,
(b) m-πg-closed if m-Cl(A) $\subset$ U whenever U is m-π-open in X and A $\subset$ U,
(c) mω-closed if m-Cl(A) $\subset$ U whenever U is m-semi open in X and A $\subset$ U.

Lemma 3.3 [9]: For the subsets of an m-space (X, m_x), the following implications hold.
m-closed $\Rightarrow$ mω-closed $\Rightarrow$ mg-closed.

Lemma 3.4: For the subsets of an m-space (X, m_x) satisfying property [в], we have the following implications.
mg-closed $\Rightarrow$ m- πg-closed $\Rightarrow$ m-rg-closed.

Lemma 3.5: Let (X, m_x) be an m-space and $A \subset X$. If A is m-closed, then
(i) A is locally m-closed set but not conversely.
(ii) A is R-locally m-closed set but not conversely.
(iii) A is π-locally m-closed set but not conversely.

Remark 3.6: None of the implications in Lemmas 3.3, 3.4 and 3.5 is reversible as seen in the following Examples.

Example 3.7: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = { ϕ , X, {b}, {a, b}, {a, c}}. Then A = {b, c} is an mω-closed set but not an m-closed.

Example 3.8: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = { ϕ , X, {c}}. Then A = {a} is an mg-closed set but not an mω-closed.

Example 3.9: Let (X, m_x) be an m-space satisfying property [в] such that X = {a, b, c} and m_x = { ϕ , X, {c}, {a, b}, {a, c}}. Then A = {a} is an m-πg-closed set but not an mg-closed.

Example 3.10: Let (X, m_x) be an m-space satisfying property [в] such that X = {a, b, c} and m_x = { ϕ , X, {a}, {b}, {a, b}}. Then A = {a, b} is an m-rg-closed set but not an m-πg-closed.

Example 3.11: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = { ϕ , X, {a}, {b}}. Then
(i) A = {b} is locally m-closed set but not a m-closed set.
(ii) A = {a} is both R-locally m-closed set and π-locally m-closed set but not a m-closed.

Proposition 3.12: Let (X, m_x) be an m-space and A a subset of X.
1. If A is m-π-open, then A is π-locally m-closed set.

2. If A is R-locally m-closed set, then A is π-locally m-closed set.

Proposition 3.13: Let (X, m_x) be an m-space satisfying property [в] and A a subset of X. Then the following holds.

If A is π-locally m-closed set, then A is locally m-closed set.

Remark 3.14: The converses of the above Propositions 3.12 and 3.13 need not be true as shown in the following examples.

Example 3.15: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then A = {a, c} is π-locally m-closed set but not m-π-open.

Example 3.16: Let (X, m_x) be an m-space such that X = {a, b, c, d} and m_x = {φ, {a}, {b}, {a, b}, {a, c}, {b, c}, {a, b, c}, X}. Then A = {a, b, c} is π-locally m-closed set but it is not R-locally m-closed set.

Example 3.17: Let (X, m_x) be an m-space satisfying property [в] such that X = {a, b, c} and m_x = { ϕ , X, {c}, {a, b}, {a, c}}. Then A = {a} is locally m-closed set but not π-locally m-closed set.

Remark 3.18: For the subsets of an m-space (X, m_x) satisfying property [в], by Propositions 3.12 and 3.13, we have the following implications.

R-locally m-closed set ⇒ π-locally m-closed set ⇒ locally m-closed set.

Theorem 3.19: A subset A of an m-space (X, m_x)satisfying property [в] is m-closed if and only if it is
(i) locally m-closed and mg-closed. [4]
(ii) R-locally m-closed and m-rg-closed.
(iii) π-locally m-closed and m-πg-closed.

Proof: (i) Necessity is trivial. We prove only sufficiency. Let A be locally m-closed set and mg-closed set. Since A is locally m-closed, A = U ∩ V, where U is m-open and V is m-closed. So, we have A = U∩V ⊂ U. Since A is mg-closed, m-Cl(A)⊂ U. Also A = U ∩ V ⊂ V and V is m-closed, then m-Cl(A) ⊂ V. Consequently, we have m-Cl(A) ⊂ U ∩ V = A and hence A is m-closed.

(ii) and (iii) It is similar to that of (i).

Theorem 3.20: For a subset A of an m-space (X, m_x) satisfying property [ʙ], the following are equivalent.
(i) A is m-closed.
(ii) A is R-locally m-closed and mg-closed.
(iii) A is R-locally m-closed and m-rg-closed.

Theorem 3.21: For a subset A of an m-space (X, m_x) satisfying property [ʙ], the following are equivalent.
(i) A is m-closed.
(ii) A is π-locally m-closed and mω-closed.
(iii) A is locally m-closed and mg-closed.

Theorem 3.22: For a subset A of an m-space (X, m_x) satisfying property [ʙ], the following are equivalent.
(i) A is m-closed.
(ii) A is locally m-closed and mω-closed.
(iii) A is locally m-closed and mg-closed.

Theorem 3.23: For a subset A of an m-space (X, m_x) satisfying property [ʙ], the following are equivalent.
(i) A is m-closed.
(ii) A is R-locally m-closed and mω-closed.
(iii) A is π-locally m-closed and mg-closed.
(iv) A is π-locally m-closed and m-πg-closed.

Theorem 3.24
For a subset A of an m-space (X, m_x) satisfying property [ʙ], the following are equivalent.
(i) A is m-closed.
(ii) A is R-locally m-closed and mg-closed.
(iii) A is R-locally m-closed and m-πg-closed.
(iv) A is R-locally m-closed and m-rg-closed.

Remark 3.25
1. The notions of locally m-closed sets and mg-closed sets (resp. mω-closed sets) are independent.

2. The notions of π-locally m-closed sets and mg-closed sets (resp. mω-closed sets, m-πg-closed sets) are independent.

3. The notions of R-locally m-closed sets and mg-closed sets (resp. mω-closed sets, m-rg-closed sets, m-πg-closed sets) are independent.

Example 3.26

(i) Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {c}, {a, b}, {a, c}, X}. Then A = {b, c} is both mg-closed set and mω-closed set but it is not locally m-closed set.

(ii) Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {a}, {c}, X}. Then A = {c} is locally m-closed set but it is not mg-closed set.

(iii) Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then A = {a} is locally m-closed set but it is not mω-closed.

Example 3.27

(i) Let (X, m_x) be an m-space satisfying property [ʙ] such that X = {a, b, c} and m_x = {φ, {c}, {a, b}, {a, c}, X}. Then A = {b, c} is both mg-closed set and mω-closed set but it is neither R-locally m-closed set nor π-locally m-closed set. Moreover it is both m-rg-closed set and m-πg-closed set.

(ii) Let (X, m_x) be an m-space satisfying property [в] such that X = {a, b, c, d} and m_x = {φ, {a}, {b}, {a, b}, {a, c}, {b, c}, {a, b, c}, X}. Then A = {a} is both R-locally m-closed set and π-locally m-closed set but it is neither mg-closed set nor mω-closed set. Moreover it is neither m-rg-closed set nor m-πg-closed set.

4. ON NEW SUBSETS OF MINIMAL SPACES

Definition 4.1 [2]: Let A be a subset of a minimal space (X, m_x). Then the m-kernel of the set A, is denoted by $\Lambda_m(A)$, is the intersection of all m-open supersets of A.

Definition 4.2[2]: A subset A of a minimal space (X, m_x) is called Λ_m-set if A= $\Lambda_m(A)$.

Definition 4.3 [2]: A subset A of an m-space (X, m_x) is called (Λ, m)-closed if A = U ∩ V where U is Λ_m-set and V is m-closed.

Lemma 4.4:
(i) Every locally m-closed set is (Λ, m)-closed.
(ii) Every m-closed set is (Λ, m)-closed but not conversely.[2]

Example 4.5: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then A = {a} is (Λ, m)-closed set but not m-closed.

Lemma 4.6 [2]: For a subset A of an m-space (X, m_x) satisfying property [в], the following conditions are equivalent.
(i) A is (Λ, m)-closed.
(ii) A = L ∩ m-Cl(A) where L is Λ_m-set.
(iii) A = $\Lambda_m(A)$ ∩ m-Cl(A).

Lemma 4.7: A subset A ⊂ (X, m_x) is mg-closed if and only if m-Cl(A) ⊂ $\Lambda_m(A)$.

Proof: Suppose that A ⊂ X is mg-closed set. Let x ∈ m-Cl(A). Suppose x ∉ $\Lambda_m(A)$.Then there exists an m-open set U containing A such that x∉ U. Since A is mg-closed set, A ⊂ U and U is m-open implies that m-Cl(A) ⊂ U and so x ∉ m-Cl(A), a contradiction. Therefore m-Cl(A) ⊂ $\Lambda_m(A)$. Conversely, suppose m-Cl(A) ⊂ $\Lambda_m(A)$ ⊂ U. Therefore A is mg-closed.

Theorem 4.8: For a subset A of an m-space (X, m_x) satisfying property [в], the following conditions are equivalent.
(i) A is m-closed.
(ii) A is mg-closed and locally m-closed.
(iii) A is mg-closed and (Λ, m)-closed.

Proof: (i) ⇒ (ii) ⇒ (iii) Obvious.

(iii) ⇒ (i) Since A is mg-closed, so by Lemma 4.7, m-Cl(A)⊂ $\Lambda_m(A)$. Since A is (Λ, m)-closed, so by Lemma 4.6,

A = $\Lambda_m(A)$ ∩ m-Cl(A) = m-Cl(A). Hence A is m-closed.

The following two examples show that the concepts of mg-closed sets and (Λ, m)-closed sets are independent.

Example 4.9: Let (X, m_x) be an m-space such that X = {a, b, c, d} and m_x = {φ, {a}, {b}, {a, b}, {a, c}, {b, c}, {a, b, c}, X}. Then A = {a, c} is (Λ, m)-closed set but not mg-closed.

Example 4.10: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {b}, {a, b}, {a, c}, X}. Then A= {b, c} is mg-closed set but not (Λ, m)-closed.

Definition 4.11: Let A be a subset of a minimal space (X, m_x). Then
(i) The m-r-kernel of the set A, denoted by m-r-ker(A), is the intersection of all regular m-open supersets of A.
(ii) The m-π-kernel of the set A, denoted by m-π-ker(A), is the intersection of all m-π-open supersets of A.

Definition 4.12: A subset A of a minimal space (X, m_x) is called
(i) Λ_{mr}-set if A = m-r-ker(A).
(ii) $\Lambda_{m\pi}$-set if A = m-π-ker(A).

Definition 4.13: A subset A of an m-space (X, m_x) is called
(i) (Λ, mr)-closed if A = L ∩ F where L is Λ_{mr}-set and F is m-closed.
(ii) (Λ, mπ)-closed if A = L ∩ F where L is $\Lambda_{m\pi}$-set and F is m-closed.

Lemma 4.14: Every m-closed set is (Λ, mr)-closed but not conversely.
(i) Every π-locally m-closed set is (Λ, mπ)-closed.
(ii) Every m-closed set is (Λ, mπ)-closed but not conversely.
(iii) Every R-locally m-closed set is (Λ, mr)-closed.

Example 4.15: Let (X, m_x) be an m-space such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then
1. A = {a} is (Λ, mr)-closed set but not m-closed.
2. A = {a} is (Λ, mπ)-closed set but not m-closed.

Lemma 4.16: For a subset A of an m-space (X, m_x) satisfying property [в], the following are equivalent.

(a) 1. A is (Λ, mr)-closed .
2. A = L ∩ m-Cl(A) where L is Λ_{mr}-set.
3. A = m-r-ker(A) ∩ m-Cl(A).

(b) 1. A is (Λ, mπ)-closed.
2. A = L ∩ m-Cl(A) where L is $\Lambda_{m\pi}$-set.
3. A = m-π-ker(A) ∩ m-Cl(A).

Lemma 4.17
(i) A subset A ⊂ (X, m_x) is m-πg-closed if and only if m-Cl(A) ⊂ m-π-ker(A).
(ii) A subset A ⊂(X, m_x) is m-rg-closed if and only if m-Cl(A) ⊂ m-r-ker(A).

Theorem 4.18: For a subset A of an m-space (X, m_x) satisfying property [в], the following are equivalent.
(a) 1. A is m-closed.
2. A is m-π-closed and π-locally m-closed.
3. A is m-πg-closed and (Λ, mπ)-closed.
(b) 1. A is m-closed.
2. A is m-rg-closed and R-locally m-closed.
3. A is m-rg-closed and (Λ, mr)-closed.

Remark 4.19: By Examples 4.20 and 4.21, we realize that the following concepts are independent.
1. (Λ, mπ)-closed sets and m-πg-closed sets.
2. (Λ, mr)-closed sets and m-rg-closed sets.

Example 4.20: Let (X, m_x) be an m-space satisfying property [в], such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then
(i) A = {a} is (Λ, mπ)-closed but not m-πg-closed.
(ii) A = {c} is m-πg-closed but not (Λ, mπ)-closed.

Example 4.21
(i) Let (X, m_x) be an m-space satisfying property [в], such that X = {a, b, c} and m_x = {φ, {b}, {a, b}, {a, c}, X}. Then A = {b, c} is m-rg-closed but not (Λ, mr)-closed.
(ii) Let (X, m_x) be an m-space satisfying property [в], such that X = {a, b, c} and m_x = {φ, {a}, {b}, X}. Then A = {a} is (Λ, mr)-closed but not m-rg-closed.

5. DECOMPOSITIONS OF M-CONTINUITY

Definition 5.1: A function f : (X, m_x) → (Y, m_y) where m_x satisfies property [B] is said to be M-g-continuous(resp. M-rg-continuous, M-ω-continuous, M-πg-continuous) if $f^{-1}(A)$ is mg-closed (resp. m-rg-closed, mω-closed, m-πg-closed) in (X, m_x) for every m-closed set A of (Y, m_y).

Definition 5.2: A function f : (X, m_x) → (Y, m_y) where m_x satisfies property [B] is called
(i) locally M-continuous if $f^{-1}(A)$ is locally m-closed in (X, m_x) for every m-closed set A of (Y, m_y).
(ii) R-locally M-continuous if $f^{-1}(A)$ is R-locally m-closed in (X, m_x) for every m-closed set A of (Y, m_y).
(iii) π-locally M-continuous if $f^{-1}(A)$ is π-locally m-closed in (X, m_x) for every m-closed set A of (Y, m_y).

Theorem 5.3: A function f : (X, m_x) → (Y, m_y) where m_x satisfies property [B] is M-continuous if and only if it is
(i) locally M-continuous and M-g-continuous.
(ii) R-locally M-continuous and M-rg-continuous
(iii) π-locally M-continuous and M-πg-continuous.

Proof: It is an immediate consequence of Theorem 3.19.

Theorem 5.4: Let (X, m_x) be an m-space satisfying property [B]. For a function f: (X, m_x) → (Y, m_y), the following are equivalent.
(1) f is M-continuous.
(2) f is R-locally M-continuous and M-g-continuous.
(3) f is R-locally M-continuous and M-rg-continuous.

Proof: It is an immediate consequence of Theorem 3.20.

Theorem 5.5: Let (X, m_x) be an m-space satisfying property [B]. For a function f : (X, m_x) → (Y, m_y), the following are equivalent.
(1) f is M-continuous.
(2) f is π-locally M-continuous and M-ω-continuous.
(3) f is locally M-continuous and M-g-continuous.

Proof: It is an immediate consequence of Theorem 3.21.

Theorem 5.6: For a function f : (X, m_x) → (Y, m_y) where m_x satisfies property [B], the following are equivalent.
(1) f is M-continuous.
(2) f is locally M-continuous and M-ω-continuous.
(3) f is locally M-continuous and M-g-continuous.

Proof: It is an immediate consequence of Theorem 3.22.

Theorem 5.7
Let (X, m_x) be an m-space satisfying property [B]. For a function f : (X, m_x) → (Y, m_y), the following are equivalent.
(1) f is M-continuous.
(2) f is R-locally M-continuous and M-ω-continuous.
(3) f is π-locally M-continuous and M-g-continuous.
(4) f is π-locally M-continuous and M-πg-continuous.

Proof: It is an immediate consequence of Theorem 3.23.

Theorem 5.8: Let (X, m_x) be an m-space satisfying property [B]. For a function f : (X, m_x) → (Y, m_y), the following are equivalent.
(1) f is M-continuous.
(2) f is R-locally M-continuous and M-g-continuous.
(3) f is R-locally M-continuous and M-πg-continuous.
(4) f is R-locally M-continuous and M-rg-continuous.

Proof: It is an immediate consequence of Theorem 3.24.

Definition 5.9: A function f : (X, m_x) → (Y, m_y) where m_x satisfies property [B] is said to be (Λ, M)-continuous (resp. (Λ, Mπ)-continuous, (Λ, Mr)-continuous) if $f^{-1}(A)$ is (Λ, m)-closed (resp. (Λ, mπ)-closed, (Λ, mr)-closed) in (X, m_x) for every m-closed set A of (Y, m_y)

Theorem 5.10: For a function f : (X, m_x) → (Y, m_y), satisfying property [B], the following are equivalent.
(1) f is M-continuous.
(2) f is M-g-continuous and locally M-continuous.
(3) f is M-g-continuous and (Λ, M)-continuous.

Proof: It is an immediate consequence of Theorem 4.8.

Theorem 5.11: For a function f : (X, m_x) → (Y, m_y) satisfying property [B], the following are equivalent.
(a) 1. f is M-continuous.
2. f is M-πg-continuous and π-locally M-continuous.
3. f is M-πg-continuous and (Λ, Mπ)-continuous.

(b) 1. f is M-continuous.
2. f is M-rg-continuous and R-locally M-continuous.
3. f is M-rg-continuous and (Λ, Mr)-continuous.

Proof: It is an immediate consequence of Theorem 4.18.

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