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(1, 2)*- $\ddot{g}$ -CLOSED AND (1, 2)*- $\ddot{g}$ -OPEN MAPS IN BITOPOLOGICAL SPACES

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ABSTRACT

A set A in a bitopological space X is said to be (1,2)- $\ddot{g}$ -closed set if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-sg-open in X. In this paper, we introduce (1,2)*- $\ddot{g}$ -closed map from a bitopological space (X, τ_1, τ_2) to a bitopological space (Y, σ_1, σ_2) as the image of every $\tau_{1,2}$-closed set is (1,2)*- $\ddot{g}$ -closed, and also we prove that the composition of two (1,2)*- $\ddot{g}$ -closed maps need not be a (1,2)*- $\ddot{g}$ -closed map. We also obtain some properties of (1, 2)*- $\ddot{g}$ -closed maps.*

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1. INTRODUCTION

Malghan [13] introduced the concept of generalized closed maps in topological spaces. Devi [5] introduced and studied sg-closed maps and gs-closed maps. Recently, Sheik John [22] defined ω-closed maps and studied some of their properties. In this paper, we introduce (1, 2)*- $\ddot{g}$ -closed maps, (1, 2)*- $\ddot{g}$ -open maps, (1, 2)*- $\ddot{g}^*$ -closed maps and (1, 2)*- $\ddot{g}^*$ -open maps in bitopological spaces and obtain certain characterizations of these class of maps.

2. PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (briefly, X, Y and Z) will denote bitopological spaces.

Definition 2.1: Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [15] if S = A $\cup$B where A $\in$ τ_1 and B $\in$ τ_2.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.2 [15]: Let S be a subset of a bitopological space X. Then
(1) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap$ {F : S $\subseteq$ F and F is $\tau_{1,2}$-closed }.
(2) the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup$ {F : F $\subseteq$ S and F is $\tau_{1,2}$-open }.

Definition 2.3: A subset A of a bitopological space X is called (1, 2)*-semi-open set [17] if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)).

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The complement of (1, 2)*-semi-open set is (1, 2)*-semi-closed.

The (1, 2)*-semi-closure [17] of a subset A of X, denoted by (1, 2)*-scl(A), is defined to be the intersection of all (1,2)*-semi-closed sets of X containing A. It is known that (1, 2)*-scl(A) is a (1,2)*-semi-closed set. For any subset A of an arbitrarily chosen bitopological space, the (1, 2)*-semi-interior [17] of A, denoted by (1, 2)*-sint(A), is defined to be the union of all (1,2)*-semi-open sets of X contained in A.

Definition 2.4: A subset A of a bitopological space X is called

(i) (1,2)*-generalized closed (briefly, (1,2)*-g-closed) set [19] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X. The complement of (1, 2)*-g-closed set is called (1, 2)*-g-open set;

(ii) (1, 2)*-ĝ-closed set [6] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X. The complement of (1, 2)*-ĝ-closed set is called (1,2)*-ĝ-open set;

(iii) (1,2)*-semi-generalized closed (briefly, (1,2)*-sg-closed) set [17] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X. The complement of (1, 2)*-sg-closed set is called (1,2)*-sg-open set;

(iv) (1,2)*-generalized semi-closed (briefly, (1,2)*-gs-closed) set [17] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X. The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;

(v) (1, 2)*- $\ddot{g}$ -closed set [8] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-sg-open in X. The complement of (1, 2)*- $\ddot{g}$ -closed set is called (1, 2)*- $\ddot{g}$ -open set.

(vi) (1,2)*-ψ -closed set [20] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-sg-open in X. The complement of (1, 2)*-ψ -closed set is called (1,2)*-ψ -open set;

The collection of all (1, 2)*- $\ddot{g}$ -closed sets is denoted by (1, 2)*- $\ddot{G}$ C(X).

Definition 2.5 [9]:
(i) For any A $\subseteq$ X, (1,2)*- $\ddot{g}$ -int(A) is defined as the union of all (1,2)*- $\ddot{g}$ -open sets contained in A. That is (1, 2)*- $\ddot{g}$ -int(A) = $\cup$ {G : G $\subseteq$ A and G is (1,2)*- $\ddot{g}$ -open}.

(ii) For every set A $\subseteq$ X, we define the (1, 2)*- $\ddot{g}$ -closure of A to be the intersection of all (1,2)*- $\ddot{g}$ -closed sets containing A.

In symbols, (1,2)*- $\ddot{g}$ -cl(A) = $\cap$ {F : A $\subseteq$ F $\in$ (1,2)*- $\ddot{G}$ C(X)}.

Definition 2.6 [7]: Let X be a bitopological space. Let x be a point of X and G be a subset of X. Then G is called an (1,2)*- $\ddot{g}$ -neighborhood of x (briefly, (1,2)*- $\ddot{g}$ -nbhd of x) in X if there exists an (1,2)*- $\ddot{g}$ -open set U of X such that x $\in$ U $\subset$ G.

Definition 2.7 [10]: A space X is called
(i) $T_{(1,2)*\text{-}\hat{g}}$-space if every (1, 2)*-ĝ-closed set in it is $\tau_{1,2}$-closed.
(ii) $T_{(1,2)*\text{-}\ddot{g}}$ -space if every (1, 2)*- $\ddot{g}$ -closed set in it is $\tau_{1,2}$-closed.

Definition 2.8: A map f: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called
(i) (1, 2)*-ĝ-continuous [23] if the inverse image of every $\sigma_{1,2}$-closed set in Y is (1,2)*-ĝ-closed set in X.
(ii) (1, 2)*- $\ddot{g}$ -continuous [7] if the inverse image of every $\sigma_{1,2}$-closed set in Y is (1,2)*- $\ddot{g}$ -closed set in X.
(iii) (1, 2)*-sg-irresolute [17] if f^{-1}(V) is (1, 2)*-sg-open in X, for every (1, 2)*-sg-open subset V in Y.
(iv) (1, 2)*-continuous [17] if f^{-1}(V) is $\tau_{1,2}$-open in X, for every $\sigma_{1,2}$-open subset V in Y.

Proposition 2.9 [9]: For any A $\subseteq$ X, the following hold:
(i) (1, 2)*- $\ddot{g}$ -int(A) is the largest (1,2)*- $\ddot{g}$ -open set contained in A.
(ii) A is (1, 2)*- $\ddot{g}$ -open if and only if (1, 2)*- $\ddot{g}$ -int(A) = A.

Proposition 2.10 [9] For any $A \subseteq X$, the following hold:

(i) (1, 2)*- $\ddot{g}$ -cl(A) is the smallest (1,2)*- $\ddot{g}$ -closed set containing A.
(ii) A is (1, 2)*- $\ddot{g}$ -closed if and only if (1, 2)*- $\ddot{g}$ -cl(A) = A.

Proposition 2.11 [9] For any two subsets A and B of X, the following hold:
(i) If $A \subseteq B$, then (1,2)*- $\ddot{g}$ -cl(A) $\subseteq$ (1,2)*- $\ddot{g}$ -cl(B).
(ii) (1,2)*- $\ddot{g}$ -cl($A \cap B$) $\subseteq$ (1,2)*- $\ddot{g}$ -cl(A) $\cap$ (1,2)*- $\ddot{g}$ -cl(B).

Theorem 2.12 [7]: A set A is (1, 2)*- $\ddot{g}$ -open if and only if $F \subseteq \tau_{1,2}$-int(A) whenever F is (1,2)*-sg-closed and $F \subseteq A$.

Definition 2.13: A map f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called
(i) (1, 2)*-g-closed [19] if f (V) is (1, 2)*-g-closed in Y, for every $\tau_{1,2}$-closed set V of X.
(ii) (1, 2)*-sg-closed [17] if f (V) is (1, 2)*-sg-closed in Y, for every $\tau_{1,2}$-closed set V of X.
(iii) (1, 2)*-gs-closed [17] if f (V) is (1, 2)*-gs-closed in Y, for every $\tau_{1,2}$-closed set V of X.
(iv) (1, 2)*- ψ -closed [20] if f (V) is (1, 2)*- ψ -closed in Y, for every $\tau_{1,2}$-closed set V of X.
(v) (1, 2)*-closed [17] if f (V) is $\sigma_{1,2}$-closed in Y, for every $\tau_{1,2}$-closed set V of X.

3. (1, 2)*- $\ddot{g}$ -CLOSED MAPS

We introduce the following definition:

Definition 3.1: A map f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (1, 2)*- $\ddot{g}$ -closed if the image of every $\tau_{1,2}$-closed set in X is (1,2)*- $\ddot{g}$ -closed in Y.

Example 3.2: Let X = Y = {a, b, c}, $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{b\}\}$. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Let $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, Y, \{a, b\}\}$. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open in Y and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed in Y. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is an (1, 2)*- $\ddot{g}$ -closed map.

Proposition 3.3: A map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*- $\ddot{g}$ -closed if and only if
(1, 2)*- $\ddot{g}$ -cl(f(A)) $\subseteq$ f($\tau_{1,2}$-cl(A)) for every subset A of X.

Proof: Suppose that f is (1,2)*- $\ddot{g}$ -closed and $A \subseteq X$. Then $\tau_{1,2}$-cl(A) is $\tau_{1,2}$-closed in X and so f($\tau_{1,2}$-cl(A)) is (1,2)*- $\ddot{g}$ -closed in Y. We have f(A) $\subseteq$ f($\tau_{1,2}$-cl(A)) and by Propositions 2.10 and 2.11,
(1,2)*- $\ddot{g}$ -cl(f(A)) $\subseteq$ (1,2)*- $\ddot{g}$ -cl(f($\tau_{1,2}$-cl(A))) = f($\tau_{1,2}$-cl(A)).

Conversely, let A be any $\tau_{1,2}$-closed set in X. Then A = $\tau_{1,2}$-cl(A) and so f(A) = f($\tau_{1,2}$-cl(A)) $\supseteq$ (1,2)*- $\ddot{g}$ -cl(f(A)), by hypothesis. We have f (A) $\subseteq$ (1, 2)*- $\ddot{g}$ -cl(f(A)). Therefore f (A) = (1, 2)*- $\ddot{g}$ -cl(f(A)). That is f (A) is (1, 2)*- $\ddot{g}$ -closed by Proposition 2.10 and hence f is (1, 2)*- $\ddot{g}$ -closed.

Proposition 3.4: Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a map such that (1,2)*- $\ddot{g}$ -cl(f(A)) $\subseteq$ f($\tau_{1,2}$-cl(A)) for every subset $A \subseteq X$. Then the image f (A) of a $\tau_{1,2}$-closed set A in X is $\ddot{g}$ -closed in Y.

Proof: Let A be a $\tau_{1,2}$-closed set in X. Then by hypothesis (1,2)*- $\ddot{g}$ -cl(f(A)) $\subseteq$ f($\tau_{1,2}$-cl(A)) = f(A) and so (1,2)*- $\ddot{g}$ -cl(f(A)) = f(A). Therefore f (A) is $\ddot{g}$ -closed in Y.

Theorem 3.5: A map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*- $\ddot{g}$ -closed if and only if for each subset S of Y and each $\tau_{1,2}$-open set U containing $f^{-1}(S)$ there is an (1,2)*- $\ddot{g}$ -open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Suppose f is (1, 2)*- $\ddot{g}$ -closed. Let $S \subseteq Y$ and U be an $\tau_{1,2}$-open set of X such that $f^{-1}(S) \subseteq U$. Then $V = (f(U^c))^c$ is an (1,2)*- $\ddot{g}$ -open set containing S such that $f^{-1}(V) \subseteq U$.

For the converse, let F be a $\tau_{1,2}$-closed set of X. Then $f^{-1}((f(F))^c) \subseteq F^c$ and F^c is $\tau_{1,2}$-open. By assumption, there exists an (1,2)*- $\ddot{g}$ -open set V in Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is (1, 2)*- $\ddot{g}$ -closed, f (F) is (1,2)*- $\ddot{g}$ -closed and therefore f is (1,2)*- $\ddot{g}$ -closed.

Proposition 3.6: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-sg-irresolute (1,2)*- $\ddot{g}$ -closed and A is an (1,2)*- $\ddot{g}$ -closed subset of X, then f(A) is (1,2)*- $\ddot{g}$ -closed in Y.

Proof: Let U be an (1, 2)*-sg-open set in Y such that $f(A) \subseteq U$. Since f is (1, 2)*-sg-irresolute, $f^{-1}(U)$ is an (1,2)*-sg-open set containing A. Hence $\tau_{1,2}\text{-cl}(A) \subseteq f^{-1}(U)$ as A is (1,2)*- $\ddot{g}$ -closed in X. Since f is (1,2)*- $\ddot{g}$ -closed, $f(\tau_{1,2}\text{-cl}(A))$ is an (1,2)*- $\ddot{g}$ -closed set contained in the (1,2)*-sg-open set U, which implies that $\tau_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) \subseteq U$ and hence $\tau_{1,2}\text{-cl}(f(A)) \subseteq U$. Therefore, f (A) is an (1,2)*- $\ddot{g}$ -closed set in Y.

The following example shows that the composition of two (1, 2)*- $\ddot{g}$ -closed maps need not be a (1, 2)*- $\ddot{g}$ -closed.

Example 3.7: Let X, Y and f be as in Example 3.2. Let Z = {a, b, c} and $\eta_1 = \{\phi, Z, \{c\}\}$ and $\eta_2 = \{\phi, Z, \{a, b\}\}$. Then the sets in {ϕ, Z, {c}, {a, b}} are called $\eta_{1,2}$-open and the sets in {ϕ, Z, {c}, {a, b}} are called $\eta_{1,2}$-closed. Let $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity map. Then both f and g are (1, 2)*- $\ddot{g}$ -closed maps but their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is not an (1,2)*- $\ddot{g}$ -closed map, since for the $\tau_{1,2}$-closed set {b ,c} in X, (g o f) ({b, c}) = {b, c}, which is not an (1, 2)*- $\ddot{g}$ -closed set in Z.

Corollary 3.8: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be (1,2)*- $\ddot{g}$ -closed and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be (1,2)*- $\ddot{g}$ -closed and (1,2)*-sg-irresolute, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- $\ddot{g}$ -closed.

Proof: Let A be a $\tau_{1,2}$-closed set of X. Then by hypothesis f (A) is an (1, 2)*- $\ddot{g}$ -closed set in Y. Since g is both (1,2)*- $\ddot{g}$ -closed and (1,2)*-sg-irresolute by Proposition 3.6, g(f(A)) = (g o f) (A) is (1,2)*- $\ddot{g}$ -closed in Z and therefore g o f is (1,2)*- $\ddot{g}$ -closed.

Proposition 3.9: Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be (1, 2)*- $\ddot{g}$ -closed maps where Y is a $T_{(1,2)^*\text{-}}$ $\ddot{g}$ -space. Then their composition $g \circ f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1, 2)*- $\ddot{g}$ -closed.

Proof: Let A be a $\tau_{1,2}$-closed set of X. Then by assumption f (A) is (1, 2)*- $\ddot{g}$ -closed in Y. Since Y is a $T_{(1,2)^*\text{-}}$ $\ddot{g}$ -space, f(A) is $\sigma_{1,2}$-closed in Y and again by assumption g(f(A)) is (1,2)*- $\ddot{g}$ -closed in Z. That is (g o f) (A) is (1, 2)*- $\ddot{g}$ -closed in Z and so g o f is (1, 2)*- $\ddot{g}$ -closed.

Proposition 3.10: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*- $\ddot{g}$ -closed, $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- $\ddot{g}$ -closed (resp. (1,2)*-g-closed, (1,2)*- ψ -closed, (1,2)*-sg-closed and (1,2)*-gs-closed) and Y is a $T_{(1,2)^*\text{-}}$ $\ddot{g}$ -space, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- $\ddot{g}$ -closed (resp. (1,2)*-g-closed, (1,2)*- ψ -closed, (1,2)*-sg-closed and (1,2)*-gs-closed).

Proof: Similar to Proposition 3.9.

Proposition 3.11: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a (1,2)*-closed map and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be an (1,2)*- $\ddot{g}$ -closed map, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- $\ddot{g}$ -closed.

Proof: Similar to Proposition 3.9.

Remark 3.12: If f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is an (1,2)*- $\ddot{g}$ -closed and g : $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*-closed, then their composition need not be an (1,2)*- $\ddot{g}$ -closed map as seen from the following example.

Example 3.13: Let X, Y and f be as in Example 3.2. Let Z = {a, b, c} and η_1 = {ϕ, Z, {a}} and η_2 = {ϕ, Z, {a, b}}. Then the sets in {ϕ, Z, {a}, {a, b}} are called $\eta_{1,2}$-open and the sets in {ϕ, Z, {c}, {b, c}} are called $\eta_{1,2}$-closed. Let g: $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity map. Then f is an (1, 2)*- $\ddot{g}$ -closed map and g is a (1, 2)*-closed map. But their composition g o f : $(X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is not an (1,2)*- $\ddot{g}$ -closed map, since for the $\tau_{1,2}$-closed set {a, c} in X, (g o f) ({a, c}) = {a, c}, which is not an (1,2)*- $\ddot{g}$ -closed set in Z.

Definition 3.14: A map f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) (1, 2)*- $\ddot{g}$ -irresolute if the inverse image of every (1,2)*- $\ddot{g}$ -closed set in Y is (1,2)*- $\ddot{g}$ -closed in X.
(ii) strongly (1,2)*- $\ddot{g}$ -continuous if the inverse image of every (1,2)*- $\ddot{g}$ -open set in Y is $\tau_{1,2}$-open in X.

Theorem 3.15: Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and g : $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two maps such that their composition g o f : $(X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is an (1,2)*- $\ddot{g}$ -closed map. Then the following statements are true.

(i) If f is (1, 2)*-continuous and surjective, then g is (1, 2)*- $\ddot{g}$ -closed.
(ii) If g is (1, 2)*- $\ddot{g}$ -irresolute and injective, then f is (1, 2)*- $\ddot{g}$ -closed.
(iii) If f is (1, 2)*-ĝ-continuous, surjective and (X,τ) is a $T_{(1,2)*}$- ĝ-space, then g is (1,2)*- $\ddot{g}$ -closed.
(iv) If g is strongly (1, 2)*- $\ddot{g}$ -continuous and injective, then f is (1, 2)*-closed.

Proof:
(i) Let A be a $\sigma_{1,2}$-closed set of Y. Since f is (1,2)*-continuous, $f^{-1}(A)$ is $\tau_{1,2}$-closed in X and since g o f is (1,2)*- $\ddot{g}$ -closed, (g o f)($f^{-1}(A)$) is (1,2)*- $\ddot{g}$ -closed in Z. That is g(A) is (1,2)*- $\ddot{g}$ -closed in Z, since f is surjective. Therefore g is an (1, 2)*- $\ddot{g}$ -closed map.

(ii) Let B be a $\tau_{1,2}$-closed set of X. Since g o f is (1, 2)*- $\ddot{g}$ -closed, (g o f) (B) is (1, 2)*- $\ddot{g}$ -closed in Z. Since g is (1, 2)*- $\ddot{g}$ -irresolute, g^{-1}((g o f)(B)) is (1,2)*- $\ddot{g}$ -closed set in Y. That is f(B) is (1,2)*- $\ddot{g}$ -closed in Y, since g is injective. Thus f is an (1, 2)*- $\ddot{g}$ -closed map.

(iii) Let C be a $\sigma_{1,2}$-closed set of Y. Since f is (1, 2)*-ĝ-continuous, $f^{-1}(C)$ is (1, 2)*-ĝ-closed in X. Since X is a T (1, 2)*-ĝ-space, $f^{-1}(C)$ is $\tau_{1,2}$-closed in X and so as in (i), g is an (1,2)*- $\ddot{g}$ -closed map.

(iv) Let D be a $\tau_{1,2}$-closed set of X. Since g o f is (1, 2)*- $\ddot{g}$ -closed, (g o f) (D) is (1, 2)*- $\ddot{g}$ -closed in Z. Since g is strongly (1, 2)*- $\ddot{g}$ -continuous, g^{-1}((g o f) (D)) is $\sigma_{1,2}$-closed in Y. That is f (D) is $\sigma_{1,2}$-closed set in Y, since g is injective. Therefore f is a (1, 2)*-closed map.

In the next theorem we show that (1, 2)*-normality is preserved under (1, 2)*-continuous (1, 2)*- $\ddot{g}$ -closed maps.

Theorem 3.16: If f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a (1,2)*-continuous, (1,2)*- $\ddot{g}$ -closed map from a (1,2)*-normal space X onto a space Y, then Y is (1,2)*-normal.

Proof: Let A and B be two disjoint $\sigma_{1,2}$-closed subsets of Y. Since f is (1, 2)*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2}$-closed sets of X. Since X is (1, 2)*-normal, there exist disjoint $\tau_{1,2}$-open sets U and V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is (1, 2)*- $\ddot{g}$ -closed, by Theorem 3.5, there exist disjoint (1, 2)*- $\ddot{g}$ -open sets G and H in Y such that $A \subseteq G$, $B \subseteq H$, $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$. Since U and V are disjoint, $\sigma_{1,2}$-int (G) and $\sigma_{1,2}$-int (H) are disjoint $\sigma_{1,2}$-open sets in Y. Since A is $\sigma_{1,2}$-closed, A is (1, 2)*-sg-closed and therefore we have by Theorem 2.12, $A \subseteq \sigma_{1,2}$-int(G). Similarly $B \subseteq \sigma_{1,2}$-int(H) and hence Y is (1,2)*-normal.

Analogous to an (1, 2)*- $\ddot{g}$ -closed map, we define an (1, 2)*- $\ddot{g}$ -open map as follows:

Definition 3.17: A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is said to be an (1,2)*- $\ddot{g}$ -open map if the image f(A) is (1,2)*- $\ddot{g}$ -open in Y for each $\tau_{1,2}$-open set A in X.

Proposition 3.18: For any bijection f: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2), the following statements are equivalent:

(i) $f^{-1:}$ (Y, σ_1, σ_2)$\rightarrow$ (X, τ_1, τ_2) is (1,2)*- $\ddot{g}$ -continuous.
(ii) f is (1,2)*- $\ddot{g}$ -open map.
(iii) f is (1,2)*- $\ddot{g}$ -closed map.

Proof:
(i) $\Rightarrow$ (ii). Let U be an $\tau_{1,2}$-open set of X. By assumption, $(f^{-1})^{-1}$(U) = f (U) is (1, 2)*- $\ddot{g}$ -open in Y and so f is (1, 2)*- $\ddot{g}$ -open.

(ii) $\Rightarrow$ (iii). Let F be a $\tau_{1,2}$-closed set of X. Then F^c is $\tau_{1,2}$-open set in X. By assumption, f (F^c) is (1,2)*- $\ddot{g}$ -open in Y. That is f (F^c) = $(f (F))^c$ is (1, 2)*- $\ddot{g}$ -open in Y and therefore f (F) is (1, 2)*- $\ddot{g}$ -closed in Y. Hence f is (1, 2)*- $\ddot{g}$ -closed.

(iii) $\Rightarrow$ (i). Let F be a $\tau_{1,2}$-closed set of X. By assumption, f (F) is (1, 2)*- $\ddot{g}$ -closed in Y. But f (F) = $(f^{-1})^{-1}$(F) and therefore f^{-1} is (1, 2)*- $\ddot{g}$ -continuous.

In the next two theorems, we obtain various characterizations of (1, 2)*- $\ddot{g}$ -open maps.

Theorem 3.19: Assume that the collection of all (1, 2)*- $\ddot{g}$ -open sets of Y is closed under arbitrary union. Let f: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be a map. Then the following statements are equivalent:

(i) f is an (1,2)*- $\ddot{g}$ -open map.
(ii) For a subset A of X, f($\tau_{1,2}$-int(A)) $\subseteq$ (1,2)*- $\ddot{g}$ -int(f(A)).
(iii) For each x $\in$ X and for each $\tau_{1,2}$-neighborhood U of x in X, there exists an (1,2)*- $\ddot{g}$ -neighborhood W of f(x) in Y such that W $\subset$ f(U).

Proof:
(i) $\Rightarrow$ (ii). Suppose f is (1,2)*- $\ddot{g}$ -open. Let A $\subseteq$ X. Then $\tau_{1,2}$-int(A) is $\tau_{1,2}$-open in X and so f($\tau_{1,2}$-int(A)) is (1,2)*- $\ddot{g}$ -open in Y. We have f($\tau_{1,2}$-int(A)) $\subseteq$ f(A). Therefore by Proposition 2.9, f ($\tau_{1,2}$-int(A)) $\subseteq$ (1,2)*- $\ddot{g}$ -int(f(A)).

(ii) $\Rightarrow$ (iii). Suppose (ii) holds. Let x $\in$ X and U be an arbitrary $\tau_{1,2}$-neighborhood of x in X. Then there exists an $\tau_{1,2}$-open set G such that x $\in$ G $\subseteq$ U. By assumption, f (G) = f ($\tau_{1,2}$-int (G)) $\subseteq$ (1,2)*- $\ddot{g}$ -int(f(G)). This implies f (G) = (1,2)*- $\ddot{g}$ -int(f(G)). By Proposition 2.9, we have f (G) is (1, 2)*- $\ddot{g}$ -open in Y. Further, f(x) $\in$ f (G) $\subseteq$ f (U) and so (iii) holds, by taking W = f (G).

(iii) $\Rightarrow$ (i). Suppose (iii) holds. Let U be any $\tau_{1,2}$-open set in X, x $\in$ U and f(x) = y. Then y $\in$ f(U) and for each y $\in$ f(U), by assumption there exists an (1,2)*- $\ddot{g}$ -neighborhood W_y of y in Y such that $W_y \subseteq$ f(U). Since W_y is an (1, 2)*- $\ddot{g}$ -neighborhood of y, there exists an (1, 2)*- $\ddot{g}$ -open set Vy in Y such that y $\in V_y \subseteq W_y$.

Therefore, f (U) = $\cup$ {V_y : y $\in$ f(U)} is an (1,2)*- $\ddot{g}$ -open set in Y. Thus f is an (1, 2)*- $\ddot{g}$ -open map.

Theorem 3.20: A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is (1,2)*- $\ddot{g}$ -open if and only if for any subset S of Y and for any $\tau_{1,2}$-closed set F containing f^{-1}(S), there exists an (1,2)*- $\ddot{g}$ -closed set K of Y containing S such that f^{-1}(K) $\subseteq$ F.

Proof: Similar to Theorem 3.5.

Corollary 3.21: A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*- $\ddot{g}$ -open if and only if

$f^{-1}((1,2)^*\text{-}\ddot{g}\text{-cl}(B)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$ for each subset B of Y.

Proof: Suppose that f is (1, 2)*- $\ddot{g}$ -open. Then for any $B \subseteq Y$, $f^{-1}(B) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$. By Theorem 3.20, there exists an (1, 2)*- $\ddot{g}$ -closed set K of Y such that $B \subseteq K$ and $f^{-1}(K) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$.

Therefore, $f^{-1}((1,2)^*\text{-}\ddot{g}\text{-cl}(B)) \subseteq (f^{-1}(K)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$, since K is an (1,2)*- $\ddot{g}$ -closed set in Y.

Conversely, let S be any subset of Y and F be any $\tau_{1,2}$-closed set containing $f^{-1}(S)$. Put K = (1, 2)*- $\ddot{g}$ -cl(S). Then K is an (1, 2)*- $\ddot{g}$ -closed set and $S \subseteq K$. By assumption, $f^{-1}(K) = f^{-1}((1,2)^*\text{-}\ddot{g}\text{-cl}(S)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(S)) \subseteq F$ and therefore by Theorem 3.20, f is (1,2)*- $\ddot{g}$ -open.

Finally in this section, we define another new class of maps called (1, 2)*- $\ddot{g}^*$ -closed maps which are stronger than (1, 2)*- $\ddot{g}$ -closed maps.

Definition 3.22: A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (1,2)*- $\ddot{g}^*$ -closed if the image f(A) is (1,2)*- $\ddot{g}$ -closed in Y for every (1,2)*- $\ddot{g}$ -closed set A in X.

For example the map f in Example 3.2 is an (1, 2)*- $\ddot{g}^*$ -closed map.

Remark 3.23: Since every $\tau_{1,2}$-closed set is an (1,2)*- $\ddot{g}$ -closed set we have (1,2)*- $\ddot{g}^*$ -closed map is an (1,2)*- $\ddot{g}$ -closed map. The converse is not true in general as seen from the following example.

Example 3.24: Let X = Y = {a, b, c} $\tau_1 = \{\phi, X, \{a, b\}\}$ and $\tau_2 = \{\phi, X\}$. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed. Let $\sigma_1 = \{\phi, Y, \{a\}\}$ and $\sigma_2 = \{\phi, Y, \{a, b\}\}$. Then the sets in {ϕ, Y, {a}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {b, c}} are called $\sigma_{1,2}$-closed.

Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is an (1, 2)*- $\ddot{g}$ -closed but not (1, 2)*- $\ddot{g}^*$ -closed map.

Since {a, c} is (1, 2)*- $\ddot{g}$ -closed set in X, but its image under f is {a, c} which is not (1,2)*- $\ddot{g}$ -closed set in Y.

Proposition 3.25: A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*- $\ddot{g}^*$ -closed if and only if $(1,2)^*\text{-}\ddot{g}\text{-cl}(f(A)) \subseteq f((1,2)^*\text{-}\ddot{g}\text{-cl}(A))$ for every subset A of X.

Proof: Similar to Proposition 3.3.

Analogous to (1, 2)*- $\ddot{g}^*$ -closed map we can also define (1, 2)*- $\ddot{g}^*$ -open map.

Proposition 3.26: For any bijection $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

(i) $f^{-1}: (Y, \sigma_1, \sigma_2) \to (X, \tau_1, \tau_2)$ is $\ddot{g}$ -irresolute.

(ii) f is (1,2)*- $\ddot{g}^*$ -open map.

(iii) f is (1,2)*- $\ddot{g}^*$ -closed map.

Proof: Similar to Proposition 3.18.

Proposition 3.27: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-sg-irresolute and (1,2)*- $\ddot{g}$ -closed, then it is an (1,2)*- $\ddot{g}^*$ -closed map.

Proof: The proof follows from Proposition 3.6.

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