



(1, 2)*- \ddot{g} -CLOSED AND (1, 2)*- \ddot{g} -OPEN MAPS IN BITOPOLOGICAL SPACES

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ABSTRACT

A set A in a bitopological space X is said to be $(1,2)^*\text{-}\ddot{g}$ -closed set if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*\text{-sg}$ -open in X . In this paper, we introduce $(1,2)^*\text{-}\ddot{g}$ -closed map from a bitopological space (X, τ_1, τ_2) to a bitopological space (Y, σ_1, σ_2) as the image of every $\tau_{1,2}$ -closed set is $(1,2)^*\text{-}\ddot{g}$ -closed, and also we prove that the composition of two $(1,2)^*\text{-}\ddot{g}$ -closed maps need not be a $(1,2)^*\text{-}\ddot{g}$ -closed map. We also obtain some properties of $(1, 2)^*\text{-}\ddot{g}$ -closed maps.

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1. INTRODUCTION

Malghan [13] introduced the concept of generalized closed maps in topological spaces. Devi [5] introduced and studied sg-closed maps and gs-closed maps. Recently, Sheik John [22] defined ω -closed maps and studied some of their properties. In this paper, we introduce $(1, 2)^*\text{-}\ddot{g}$ -closed maps, $(1, 2)^*\text{-}\ddot{g}$ -open maps, $(1, 2)^*\text{-}\ddot{g}^*$ -closed maps and $(1, 2)^*\text{-}\ddot{g}^*$ -open maps in bitopological spaces and obtain certain characterizations of these class of maps.

2. PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2) , (Y, σ_1, σ_2) and (Z, η_1, η_2) (briefly, X , Y and Z) will denote bitopological spaces.

Definition 2.1: Let S be a subset of X . Then S is said to be $\tau_{1,2}$ -open [15] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$ -open set is called $\tau_{1,2}$ -closed.

Notice that $\tau_{1,2}$ -open sets need not necessarily form a topology.

Definition 2.2 [15]: Let S be a subset of a bitopological space X . Then

- (1) the $\tau_{1,2}$ -closure of S , denoted by $\tau_{1,2}\text{-cl}(S)$, is defined as $\cap \{F : S \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed} \}$.
- (2) the $\tau_{1,2}$ -interior of S , denoted by $\tau_{1,2}\text{-int}(S)$, is defined as $\cup \{F : F \subseteq S \text{ and } F \text{ is } \tau_{1,2}\text{-open} \}$.

Definition 2.3: A subset A of a bitopological space X is called $(1, 2)^*\text{-semi-open}$ set [17] if $A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$.

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The complement of (1, 2)*-semi-open set is (1, 2)*-semi-closed.

The (1, 2)*-semi-closure [17] of a subset A of X, denoted by (1, 2)*-scl(A), is defined to be the intersection of all (1,2)*-semi-closed sets of X containing A. It is known that (1, 2)*-scl(A) is a (1,2)*-semi-closed set. For any subset A of an arbitrarily chosen bitopological space, the (1, 2)*-semi-interior [17] of A, denoted by (1, 2)*-sint(A), is defined to be the union of all (1,2)*-semi-open sets of X contained in A.

Definition 2.4: A subset A of a bitopological space X is called

(i) (1,2)*-generalized closed (briefly, (1,2)*-g-closed) set [19] if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X. The complement of (1, 2)*-g-closed set is called (1, 2)*-g-open set;

(ii) (1, 2)*- \hat{g} -closed set [6] if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-semi-open in X. The complement of (1, 2)*- \hat{g} -closed set is called (1,2)*- \hat{g} -open set;

(iii) (1,2)*-semi-generalized closed (briefly, (1,2)*-sg-closed) set [17] if (1,2)*-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-semi-open in X. The complement of (1, 2)*-sg-closed set is called (1,2)*-sg-open set;

(iv) (1,2)*-generalized semi-closed (briefly, (1,2)*-gs-closed) set [17] if (1,2)*-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X. The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;

(v) (1, 2)*- \ddot{g} -closed set [8] if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-sg-open in X. The complement of (1, 2)*- \ddot{g} -closed set is called (1, 2)*- \ddot{g} -open set.

(vi) (1,2)*- ψ -closed set [20] if (1,2)*-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-sg-open in X. The complement of (1, 2)*- ψ -closed set is called (1,2)*- ψ -open set;

The collection of all (1, 2)*- \ddot{g} -closed sets is denoted by (1, 2)*- $\ddot{G}C(X)$.

Definition 2.5 [9]:

(i) For any $A \subseteq X$, (1,2)*- \ddot{g} -int(A) is defined as the union of all (1,2)*- \ddot{g} -open sets contained in A. That is (1, 2)*- \ddot{g} -int(A) = $\cup \{G : G \subseteq A \text{ and } G \text{ is } (1,2)^*\text{-}\ddot{g}\text{-open}\}$.

(ii) For every set $A \subseteq X$, we define the (1, 2)*- \ddot{g} -closure of A to be the intersection of all (1,2)*- \ddot{g} -closed sets containing A.

In symbols, (1,2)*- \ddot{g} -cl(A) = $\cap \{F : A \subseteq F \in (1,2)^*\text{-}\ddot{G}C(X)\}$.

Definition 2.6 [7]: Let X be a bitopological space. Let x be a point of X and G be a subset of X. Then G is called an (1,2)*- \ddot{g} -neighborhood of x (briefly, (1,2)*- \ddot{g} -nbhd of x) in X if there exists an (1,2)*- \ddot{g} -open set U of X such that $x \in U \subseteq G$.

Definition 2.7 [10]: A space X is called

(i) $T_{(1,2)^*\text{-}\hat{g}}$ -space if every (1, 2)*- \hat{g} -closed set in it is $\tau_{1,2}$ -closed.

(ii) $T_{(1,2)^*\text{-}\ddot{g}}$ -space if every (1, 2)*- \ddot{g} -closed set in it is $\tau_{1,2}$ -closed.

Definition 2.8: A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) (1, 2)*- \hat{g} -continuous [23] if the inverse image of every $\sigma_{1,2}$ -closed set in Y is (1,2)*- \hat{g} -closed set in X.

(ii) (1, 2)*- \ddot{g} -continuous [7] if the inverse image of every $\sigma_{1,2}$ -closed set in Y is (1,2)*- \ddot{g} -closed set in X.

(iii) (1, 2)*-sg-irresolute [17] if $f^{-1}(V)$ is (1, 2)*-sg-open in X, for every (1, 2)*-sg-open subset V in Y.

(iv) (1, 2)*-continuous [17] if $f^{-1}(V)$ is $\tau_{1,2}$ -open in X, for every $\sigma_{1,2}$ -open subset V in Y.

Proposition 2.9 [9]: For any $A \subseteq X$, the following hold:

(i) (1, 2)*- \ddot{g} -int(A) is the largest (1,2)*- \ddot{g} -open set contained in A.

(ii) A is (1, 2)*- \ddot{g} -open if and only if (1, 2)*- \ddot{g} -int(A) = A.

Proposition 2.10 [9] For any $A \subseteq X$, the following hold:

- (i) $(1, 2)^*-\ddot{g}\text{-cl}(A)$ is the smallest $(1, 2)^*-\ddot{g}$ -closed set containing A .
- (ii) A is $(1, 2)^*-\ddot{g}$ -closed if and only if $(1, 2)^*-\ddot{g}\text{-cl}(A) = A$.

Proposition 2.11 [9] For any two subsets A and B of X , the following hold:

- (i) If $A \subseteq B$, then $(1, 2)^*-\ddot{g}\text{-cl}(A) \subseteq (1, 2)^*-\ddot{g}\text{-cl}(B)$.
- (ii) $(1, 2)^*-\ddot{g}\text{-cl}(A \cap B) \subseteq (1, 2)^*-\ddot{g}\text{-cl}(A) \cap (1, 2)^*-\ddot{g}\text{-cl}(B)$.

Theorem 2.12 [7]: A set A is $(1, 2)^*-\ddot{g}$ -open if and only if $F \subseteq \tau_{1,2}\text{-int}(A)$ whenever F is $(1, 2)^*\text{-sg-closed}$ and $F \subseteq A$.

Definition 2.13: A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (i) $(1, 2)^*\text{-g-closed}$ [19] if $f(V)$ is $(1, 2)^*\text{-g-closed}$ in Y , for every $\tau_{1,2}$ -closed set V of X .
- (ii) $(1, 2)^*\text{-sg-closed}$ [17] if $f(V)$ is $(1, 2)^*\text{-sg-closed}$ in Y , for every $\tau_{1,2}$ -closed set V of X .
- (iii) $(1, 2)^*\text{-gs-closed}$ [17] if $f(V)$ is $(1, 2)^*\text{-gs-closed}$ in Y , for every $\tau_{1,2}$ -closed set V of X .
- (iv) $(1, 2)^*\text{-}\psi\text{-closed}$ [20] if $f(V)$ is $(1, 2)^*\text{-}\psi\text{-closed}$ in Y , for every $\tau_{1,2}$ -closed set V of X .
- (v) $(1, 2)^*\text{-closed}$ [17] if $f(V)$ is $\sigma_{1,2}$ -closed in Y , for every $\tau_{1,2}$ -closed set V of X .

3. $(1, 2)^*-\ddot{g}$ -CLOSED MAPS

We introduce the following definition:

Definition 3.1: A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be $(1, 2)^*-\ddot{g}$ -closed if the image of every $\tau_{1,2}$ -closed set in X is $(1, 2)^*-\ddot{g}$ -closed in Y .

Example 3.2: Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y\}$ and $\sigma_2 = \{\emptyset, Y, \{a, b\}\}$. Then the sets in $\{\emptyset, Y, \{a, b\}\}$ are called $\sigma_{1,2}$ -open in Y and the sets in $\{\emptyset, Y, \{c\}\}$ are called $\sigma_{1,2}$ -closed in Y . Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is an $(1, 2)^*-\ddot{g}$ -closed map.

Proposition 3.3: A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^*-\ddot{g}$ -closed if and only if

$$(1, 2)^*-\ddot{g}\text{-cl}(f(A)) \subseteq f(\tau_{1,2}\text{-cl}(A)) \text{ for every subset } A \text{ of } X.$$

Proof: Suppose that f is $(1, 2)^*-\ddot{g}$ -closed and $A \subseteq X$. Then $\tau_{1,2}\text{-cl}(A)$ is $\tau_{1,2}$ -closed in X and so $f(\tau_{1,2}\text{-cl}(A))$ is $(1, 2)^*-\ddot{g}$ -closed in Y . We have $f(A) \subseteq f(\tau_{1,2}\text{-cl}(A))$ and by Propositions 2.10 and 2.11, $(1, 2)^*-\ddot{g}\text{-cl}(f(A)) \subseteq (1, 2)^*-\ddot{g}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) = f(\tau_{1,2}\text{-cl}(A))$.

Conversely, let A be any $\tau_{1,2}$ -closed set in X . Then $A = \tau_{1,2}\text{-cl}(A)$ and so $f(A) = f(\tau_{1,2}\text{-cl}(A)) \supseteq (1, 2)^*-\ddot{g}\text{-cl}(f(A))$, by hypothesis. We have $f(A) \subseteq (1, 2)^*-\ddot{g}\text{-cl}(f(A))$. Therefore $f(A) = (1, 2)^*-\ddot{g}\text{-cl}(f(A))$. That is $f(A)$ is $(1, 2)^*-\ddot{g}$ -closed by Proposition 2.10 and hence f is $(1, 2)^*-\ddot{g}$ -closed.

Proposition 3.4: Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a map such that $(1, 2)^*-\ddot{g}\text{-cl}(f(A)) \subseteq f(\tau_{1,2}\text{-cl}(A))$ for every subset $A \subseteq X$. Then the image $f(A)$ of a $\tau_{1,2}$ -closed set A in X is \ddot{g} -closed in Y .

Proof: Let A be a $\tau_{1,2}$ -closed set in X . Then by hypothesis $(1, 2)^*-\ddot{g}\text{-cl}(f(A)) \subseteq f(\tau_{1,2}\text{-cl}(A)) = f(A)$ and so $(1, 2)^*-\ddot{g}\text{-cl}(f(A)) = f(A)$. Therefore $f(A)$ is \ddot{g} -closed in Y .

Theorem 3.5: A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^*-\ddot{g}$ -closed if and only if for each subset S of Y and each $\tau_{1,2}$ -open set U containing $f^{-1}(S)$ there is an $(1, 2)^*-\ddot{g}$ -open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Suppose f is (1, 2)*- \ddot{g} -closed. Let $S \subseteq Y$ and U be an $\tau_{1,2}$ -open set of X such that $f^{-1}(S) \subseteq U$. Then $V = (f(U^c))^c$ is an (1,2)*- \ddot{g} -open set containing S such that $f^{-1}(V) \subseteq U$.

For the converse, let F be a $\tau_{1,2}$ -closed set of X . Then $f^{-1}((f(F))^c) \subseteq F^c$ and F^c is $\tau_{1,2}$ -open. By assumption, there exists an (1,2)*- \ddot{g} -open set V in Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is (1, 2)*- \ddot{g} -closed, $f(F)$ is (1,2)*- \ddot{g} -closed and therefore f is (1,2)*- \ddot{g} -closed.

Proposition 3.6: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-sg-irresolute (1,2)*- \ddot{g} -closed and A is an (1,2)*- \ddot{g} -closed subset of X , then $f(A)$ is (1,2)*- \ddot{g} -closed in Y .

Proof: Let U be an (1, 2)*-sg-open set in Y such that $f(A) \subseteq U$. Since f is (1, 2)*-sg-irresolute, $f^{-1}(U)$ is an (1,2)*-sg-open set containing A . Hence $\tau_{1,2}\text{-cl}(A) \subseteq f^{-1}(U)$ as A is (1,2)*- \ddot{g} -closed in X . Since f is (1,2)*- \ddot{g} -closed, $f(\tau_{1,2}\text{-cl}(A))$ is an (1,2)*- \ddot{g} -closed set contained in the (1,2)*-sg-open set U , which implies that $\tau_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) \subseteq U$ and hence $\tau_{1,2}\text{-cl}(f(A)) \subseteq U$. Therefore, $f(A)$ is an (1,2)*- \ddot{g} -closed set in Y .

The following example shows that the composition of two (1, 2)*- \ddot{g} -closed maps need not be a (1, 2)*- \ddot{g} -closed.

Example 3.7: Let X, Y and f be as in Example 3.2. Let $Z = \{a, b, c\}$ and $\eta_1 = \{\phi, Z, \{c\}\}$ and $\eta_2 = \{\phi, Z, \{a, b\}\}$. Then the sets in $\{\phi, Z, \{c\}, \{a, b\}\}$ are called $\eta_{1,2}$ -open and the sets in $\{\phi, Z, \{c\}, \{a, b\}\}$ are called $\eta_{1,2}$ -closed. Let $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity map. Then both f and g are (1, 2)*- \ddot{g} -closed maps but their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is not an (1,2)*- \ddot{g} -closed map, since for the $\tau_{1,2}$ -closed set $\{b, c\}$ in X , $(g \circ f)(\{b, c\}) = \{b, c\}$, which is not an (1, 2)*- \ddot{g} -closed set in Z .

Corollary 3.8: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be (1,2)*- \ddot{g} -closed and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be (1,2)*- \ddot{g} -closed and (1,2)*-sg-irresolute, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- \ddot{g} -closed.

Proof: Let A be a $\tau_{1,2}$ -closed set of X . Then by hypothesis $f(A)$ is an (1, 2)*- \ddot{g} -closed set in Y . Since g is both (1,2)*- \ddot{g} -closed and (1,2)*-sg-irresolute by Proposition 3.6, $g(f(A)) = (g \circ f)(A)$ is (1,2)*- \ddot{g} -closed in Z and therefore $g \circ f$ is (1,2)*- \ddot{g} -closed.

Proposition 3.9: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be (1, 2)*- \ddot{g} -closed maps where Y is a $T_{(1,2)*-\ddot{g}}$ -space. Then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1, 2)*- \ddot{g} -closed.

Proof: Let A be a $\tau_{1,2}$ -closed set of X . Then by assumption $f(A)$ is (1, 2)*- \ddot{g} -closed in Y . Since Y is a $T_{(1,2)*-\ddot{g}}$ -space, $f(A)$ is $\sigma_{1,2}$ -closed in Y and again by assumption $g(f(A))$ is (1,2)*- \ddot{g} -closed in Z . That is $(g \circ f)(A)$ is (1, 2)*- \ddot{g} -closed in Z and so $g \circ f$ is (1, 2)*- \ddot{g} -closed.

Proposition 3.10: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*- \ddot{g} -closed, $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- \ddot{g} -closed (resp. (1,2)*-g-closed, (1,2)*- ψ -closed, (1,2)*-sg-closed and (1,2)*-gs-closed) and Y is a $T_{(1,2)*-\ddot{g}}$ -space, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- \ddot{g} -closed (resp. (1,2)*-g-closed, (1,2)*- ψ -closed, (1,2)*-sg-closed and (1,2)*-gs-closed).

Proof: Similar to Proposition 3.9.

Proposition 3.11: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a (1,2)*-closed map and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be an (1,2)*- \ddot{g} -closed map, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*- \ddot{g} -closed.

Proof: Similar to Proposition 3.9.

Remark 3.12: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is an (1,2)*- \ddot{g} -closed and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*-closed, then their composition need not be an (1,2)*- \ddot{g} -closed map as seen from the following example.

Example 3.13: Let X, Y and f be as in Example 3.2. Let $Z = \{a, b, c\}$ and $\eta_1 = \{\phi, Z, \{a\}\}$ and $\eta_2 = \{\phi, Z, \{a, b\}\}$. Then the sets in $\{\phi, Z, \{a\}, \{a, b\}\}$ are called $\eta_{1,2}$ -open and the sets in $\{\phi, Z, \{c\}, \{b, c\}\}$ are called $\eta_{1,2}$ -closed. Let $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity map. Then f is an (1, 2)*- \ddot{g} -closed map and g is a (1, 2)*-closed map. But their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is not an (1,2)*- \ddot{g} -closed map, since for the $\tau_{1,2}$ -closed set $\{a, c\}$ in X , $(g \circ f)(\{a, c\}) = \{a, c\}$, which is not an (1,2)*- \ddot{g} -closed set in Z .

Definition 3.14: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

- (i) (1, 2)*- \ddot{g} -irresolute if the inverse image of every (1,2)*- \ddot{g} -closed set in Y is (1,2)*- \ddot{g} -closed in X .
- (ii) strongly (1,2)*- \ddot{g} -continuous if the inverse image of every (1,2)*- \ddot{g} -open set in Y is $\tau_{1,2}$ -open in X .

Theorem 3.15: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two maps such that their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is an (1,2)*- \ddot{g} -closed map. Then the following statements are true.

- (i) If f is (1, 2)*-continuous and surjective, then g is (1, 2)*- \ddot{g} -closed.
- (ii) If g is (1, 2)*- \ddot{g} -irresolute and injective, then f is (1, 2)*- \ddot{g} -closed.
- (iii) If f is (1, 2)*- \hat{g} -continuous, surjective and (X, τ) is a $T_{(1,2)*-\hat{g}}$ -space, then g is (1,2)*- \ddot{g} -closed.
- (iv) If g is strongly (1, 2)*- \ddot{g} -continuous and injective, then f is (1, 2)*-closed.

Proof:

(i) Let A be a $\sigma_{1,2}$ -closed set of Y . Since f is (1,2)*-continuous, $f^{-1}(A)$ is $\tau_{1,2}$ -closed in X and since $g \circ f$ is (1,2)*- \ddot{g} -closed, $(g \circ f)(f^{-1}(A))$ is (1,2)*- \ddot{g} -closed in Z . That is $g(A)$ is (1,2)*- \ddot{g} -closed in Z , since f is surjective. Therefore g is an (1, 2)*- \ddot{g} -closed map.

(ii) Let B be a $\tau_{1,2}$ -closed set of X . Since $g \circ f$ is (1, 2)*- \ddot{g} -closed, $(g \circ f)(B)$ is (1, 2)*- \ddot{g} -closed in Z . Since g is (1, 2)*- \ddot{g} -irresolute, $g^{-1}((g \circ f)(B))$ is (1,2)*- \ddot{g} -closed set in Y . That is $f(B)$ is (1,2)*- \ddot{g} -closed in Y , since g is injective. Thus f is an (1, 2)*- \ddot{g} -closed map.

(iii) Let C be a $\sigma_{1,2}$ -closed set of Y . Since f is (1, 2)*- \hat{g} -continuous, $f^{-1}(C)$ is (1, 2)*- \hat{g} -closed in X . Since X is a $T_{(1,2)*-\hat{g}}$ -space, $f^{-1}(C)$ is $\tau_{1,2}$ -closed in X and so as in (i), g is an (1,2)*- \ddot{g} -closed map.

(iv) Let D be a $\tau_{1,2}$ -closed set of X . Since $g \circ f$ is (1, 2)*- \ddot{g} -closed, $(g \circ f)(D)$ is (1, 2)*- \ddot{g} -closed in Z . Since g is strongly (1, 2)*- \ddot{g} -continuous, $g^{-1}((g \circ f)(D))$ is $\sigma_{1,2}$ -closed in Y . That is $f(D)$ is $\sigma_{1,2}$ -closed set in Y , since g is injective. Therefore f is a (1, 2)*-closed map.

In the next theorem we show that (1, 2)*-normality is preserved under (1, 2)*-continuous (1, 2)*- \ddot{g} -closed maps.

Theorem 3.16: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a (1,2)*-continuous, (1,2)*- \ddot{g} -closed map from a (1,2)*-normal space X onto a space Y , then Y is (1,2)*-normal.

Proof: Let A and B be two disjoint $\sigma_{1,2}$ -closed subsets of Y . Since f is (1, 2)*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2}$ -closed sets of X . Since X is (1, 2)*-normal, there exist disjoint $\tau_{1,2}$ -open sets U and V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is (1, 2)*- \ddot{g} -closed, by Theorem 3.5, there exist disjoint (1, 2)*- \ddot{g} -open sets G and H in Y such that $A \subseteq G, B \subseteq H, f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$. Since U and V are disjoint, $\sigma_{1,2}$ -int(G) and $\sigma_{1,2}$ -int(H) are disjoint $\sigma_{1,2}$ -open sets in Y . Since A is $\sigma_{1,2}$ -closed, A is (1, 2)*-sg-closed and therefore we have by Theorem 2.12, $A \subseteq \sigma_{1,2}$ -int(G). Similarly $B \subseteq \sigma_{1,2}$ -int(H) and hence Y is (1,2)*-normal.

Analogous to an (1, 2)*- \ddot{g} -closed map, we define an (1, 2)*- \ddot{g} -open map as follows:

Definition 3.17: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be an (1,2)*- \ddot{g} -open map if the image $f(A)$ is (1,2)*- \ddot{g} -open in Y for each $\tau_{1,2}$ -open set A in X .

Proposition 3.18: For any bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

- (i) $f^{-1} : (Y, \sigma_1, \sigma_2) \rightarrow (X, \tau_1, \tau_2)$ is (1,2)*- \ddot{g} -continuous.
- (ii) f is (1,2)*- \ddot{g} -open map.
- (iii) f is (1,2)*- \ddot{g} -closed map.

Proof:

(i) \Rightarrow (ii). Let U be an $\tau_{1,2}$ -open set of X . By assumption, $(f^{-1})^{-1}(U) = f(U)$ is (1, 2)*- \ddot{g} -open in Y and so f is (1, 2)*- \ddot{g} -open.

(ii) \Rightarrow (iii). Let F be a $\tau_{1,2}$ -closed set of X . Then F^c is $\tau_{1,2}$ -open set in X . By assumption, $f(F^c)$ is (1,2)*- \ddot{g} -open in Y . That is $f(F^c) = (f(F))^c$ is (1, 2)*- \ddot{g} -open in Y and therefore $f(F)$ is (1, 2)*- \ddot{g} -closed in Y . Hence f is (1, 2)*- \ddot{g} -closed.

(iii) \Rightarrow (i). Let F be a $\tau_{1,2}$ -closed set of X . By assumption, $f(F)$ is (1, 2)*- \ddot{g} -closed in Y . But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is (1, 2)*- \ddot{g} -continuous.

In the next two theorems, we obtain various characterizations of (1, 2)*- \ddot{g} -open maps.

Theorem 3.19: Assume that the collection of all (1, 2)*- \ddot{g} -open sets of Y is closed under arbitrary union. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a map. Then the following statements are equivalent:

- (i) f is an (1,2)*- \ddot{g} -open map.
- (ii) For a subset A of X , $f(\tau_{1,2}\text{-int}(A)) \subseteq (1,2)*\text{-}\ddot{g}\text{-int}(f(A))$.
- (iii) For each $x \in X$ and for each $\tau_{1,2}$ -neighborhood U of x in X , there exists an (1,2)*- \ddot{g} -neighborhood W of $f(x)$ in Y such that $W \subseteq f(U)$.

Proof:

(i) \Rightarrow (ii). Suppose f is (1,2)*- \ddot{g} -open. Let $A \subseteq X$. Then $\tau_{1,2}\text{-int}(A)$ is $\tau_{1,2}$ -open in X and so $f(\tau_{1,2}\text{-int}(A))$ is (1,2)*- \ddot{g} -open in Y . We have $f(\tau_{1,2}\text{-int}(A)) \subseteq f(A)$. Therefore by Proposition 2.9, $f(\tau_{1,2}\text{-int}(A)) \subseteq (1,2)*\text{-}\ddot{g}\text{-int}(f(A))$.

(ii) \Rightarrow (iii). Suppose (ii) holds. Let $x \in X$ and U be an arbitrary $\tau_{1,2}$ -neighborhood of x in X . Then there exists an $\tau_{1,2}$ -open set G such that $x \in G \subseteq U$. By assumption, $f(G) = f(\tau_{1,2}\text{-int}(G)) \subseteq (1,2)*\text{-}\ddot{g}\text{-int}(f(G))$. This implies $f(G) = (1,2)*\text{-}\ddot{g}\text{-int}(f(G))$. By Proposition 2.9, we have $f(G)$ is (1, 2)*- \ddot{g} -open in Y . Further, $f(x) \in f(G) \subseteq f(U)$ and so (iii) holds, by taking $W = f(G)$.

(iii) \Rightarrow (i). Suppose (iii) holds. Let U be any $\tau_{1,2}$ -open set in X , $x \in U$ and $f(x) = y$. Then $y \in f(U)$ and for each $y \in f(U)$, by assumption there exists an (1,2)*- \ddot{g} -neighborhood W_y of y in Y such that $W_y \subseteq f(U)$. Since W_y is an (1, 2)*- \ddot{g} -neighborhood of y , there exists an (1, 2)*- \ddot{g} -open set V_y in Y such that $y \in V_y \subseteq W_y$.

Therefore, $f(U) = \cup \{V_y : y \in f(U)\}$ is an (1,2)*- \ddot{g} -open set in Y . Thus f is an (1, 2)*- \ddot{g} -open map.

Theorem 3.20: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*- \ddot{g} -open if and only if for any subset S of Y and for any $\tau_{1,2}$ -closed set F containing $f^{-1}(S)$, there exists an (1,2)*- \ddot{g} -closed set K of Y containing S such that $f^{-1}(K) \subseteq F$.

Proof: Similar to Theorem 3.5.

Corollary 3.21: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*- \ddot{g} -open if and only if

$$f^{-1}((1,2)^*\text{-}\ddot{g}\text{-cl}(B)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B)) \text{ for each subset } B \text{ of } Y.$$

Proof: Suppose that f is (1, 2)*- \ddot{g} -open. Then for any $B \subseteq Y$, $f^{-1}(B) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$. By Theorem 3.20, there exists an (1, 2)*- \ddot{g} -closed set K of Y such that $B \subseteq K$ and $f^{-1}(K) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$.

Therefore, $f^{-1}((1,2)^*\text{-}\ddot{g}\text{-cl}(B)) \subseteq (f^{-1}(K)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(B))$, since K is an (1,2)*- \ddot{g} -closed set in Y .

Conversely, let S be any subset of Y and F be any $\tau_{1,2}$ -closed set containing $f^{-1}(S)$. Put $K = (1, 2)^*\text{-}\ddot{g}\text{-cl}(S)$. Then K is an (1, 2)*- \ddot{g} -closed set and $S \subseteq K$. By assumption, $f^{-1}(K) = f^{-1}((1,2)^*\text{-}\ddot{g}\text{-cl}(S)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(S)) \subseteq F$ and therefore by Theorem 3.20, f is (1,2)*- \ddot{g} -open.

Finally in this section, we define another new class of maps called (1, 2)*- \ddot{g}^* -closed maps which are stronger than (1, 2)*- \ddot{g} -closed maps.

Definition 3.22: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be (1,2)*- \ddot{g}^* -closed if the image $f(A)$ is (1,2)*- \ddot{g}^* -closed in Y for every (1,2)*- \ddot{g} -closed set A in X .

For example the map f in Example 3.2 is an (1, 2)*- \ddot{g}^* -closed map.

Remark 3.23: Since every $\tau_{1,2}$ -closed set is an (1,2)*- \ddot{g} -closed set we have (1,2)*- \ddot{g}^* -closed map is an (1,2)*- \ddot{g} -closed map. The converse is not true in general as seen from the following example.

Example 3.24: Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a, b\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed.

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is an (1, 2)*- \ddot{g} -closed but not (1, 2)*- \ddot{g}^* -closed map.

Since $\{a, c\}$ is (1, 2)*- \ddot{g} -closed set in X , but its image under f is $\{a, c\}$ which is not (1,2)*- \ddot{g}^* -closed set in Y .

Proposition 3.25: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*- \ddot{g}^* -closed if and only if $(1,2)^*\text{-}\ddot{g}\text{-cl}(f(A)) \subseteq f((1,2)^*\text{-}\ddot{g}\text{-cl}(A))$ for every subset A of X .

Proof: Similar to Proposition 3.3.

Analogous to (1, 2)*- \ddot{g}^* -closed map we can also define (1, 2)*- \ddot{g}^* -open map.

Proposition 3.26: For any bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

- (i) $f^{-1} : (Y, \sigma_1, \sigma_2) \rightarrow (X, \tau_1, \tau_2)$ is \ddot{g} -irresolute.
- (ii) f is (1,2)*- \ddot{g}^* -open map.
- (iii) f is (1,2)*- \ddot{g}^* -closed map.

Proof: Similar to Proposition 3.18.

Proposition 3.27: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-sg-irresolute and (1,2)*- \ddot{g} -closed, then it is an (1,2)*- \ddot{g}^* -closed map.

Proof: The proof follows from Proposition 3.6.

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