

NUMERICAL STUDY ON MHD MIXED CONVECTIVE FLOW
WITH DISPERSION AND CHEMICAL REACTION OVER A VERTICAL PLATE
IN NON-DARCY POROUS MEDIUM

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ABSTRACT

The effects of double dispersion and MHD on mixed convection heat and mass transfer for a Newtonian fluid over a vertical plate are discussed. The energy equation includes the viscous dissipation and thermal dispersion effects; concentration equation includes the chemical reaction and solute dispersion effects. The transformed governing equations are solved numerically using the fourth order Runge-Kutta method with the usual shooting technique. The velocity, temperature and concentration profiles are presented graphically, against the similarity variable η for different physical parameters. The rates of heat and mass transfer have also been discussed. The results obtained by us are in agreement with those obtained by Chamkha et al. [11], without the magnetic and dispersion terms.

Key words: Mixed convection, MHD, Double dispersion, viscous dissipation.

LIST OF SYMBOLS

c	Empirical constant
C	Concentration
c_p	Specific heat at constant pressure
f	Dimensionless stream function
F	Structural and thermo-physical parameter
g	Gravitational acceleration
M	Magnetic Parameter
J_w	Local mass flux
k	Molecular thermal conductivity
K	Permeability of the porous medium
k_d	Dispersion thermal conductivity
k_e	Effective thermal conductivity
Le	Lewis number
N	Buoyancy ratio
Nu_x	Local Nusselt number
p	Pressure
Pr	Prandtl number
q	Heat transfer rate
Ra	Rayleigh number
Re_x	Local Reynolds number
Sc	Schmidt number
Sh_x	Local Sherwood number

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T	Temperature
u, v	Velocity components in the x and y directions
x, y	Axes along and normal to the plate
α	Molecular thermal diffusivity
α_d	Dispersion diffusivity
α_x, α_y	Components of the thermal diffusivity in x and y directions
β_T	Thermal expansion coefficient
β_C	Solutal expansion coefficient
χ	Non-dimensional chemical reaction-porous medium parameter
ϕ	Dimensionless concentration
η	Similarity space variable
ν	Fluid kinematic viscosity
θ	Dimensionless temperature
ρ	Fluid density
ψ	Stream function
$\frac{Ra_x}{Pe_x}$	Mixed convection parameter
D	Thermal dispersion parameter
B	Solute dispersion parameter

SUBSCRIPTS

x, y	In the directions of x and y axes
w	Surface conditions
∞	Conditions away from the surface

SUPERSCRIPTS

'	Derivative with respect to η
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INTRODUCTION:

The study of convective flow through porous media has received a great deal of research interest over the last three decades due to its wide important applications in environmental, geophysical and energy related engineering problems. Mixed convection flow occurs frequently in nature. The temperature distribution varies layer to layer and these types of flows have wide applications in industry, agriculture and Oceanography. Heat and mass transfer for an electrically conducting fluid under the influence of MHD is considered to be of significant importance due to its applications in many engineering problems such as nuclear reactors and those dealing with liquid metals. Both the hydro magnetic flow and heat transfer in a viscous incompressible fluid over a moving continuous stretching surface is a significant type of flow has considerable practical applications. MHD laminar boundary layer flow over a wedge with suction or injection had been discussed by Kafoussias and Nanousis [1]. Anjalidevi and Kandasamy [2] have studied the effects of heat and mass transfer on nonlinear boundary layer flow over a wedge with suction or injection. The effect of induced magnetic field included in this analysis. Chamkha and Khaled [3] investigated the problem of coupled heat and mass transfer by MHD free convection from an inclined plate in the presence of internal heat generation or absorption. The effect of solutal and thermal dispersion effects in homogeneous and isotropic Darcian porous media has been analyzed by Dagan [4]. Postelnicu [5] studied the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Seddek [6] studied the finite element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydro magnetic flow with heat and mass transfer over a heat surface. El-Amin [7] studied the effects of chemical reaction and double dispersion on non-Darcy free convective heat and mass transfer in porous medium. Murti et al. [8] analyzed the effects of radiation, chemical reaction and double dispersion on heat and mass transfer in non-Darcy free convective flow. Amin et al. [9] studied the effects of chemical reaction and Double dispersion on Non-Darcy free convection heat and mass transfer. Murthy [10] analyzed the effect of double dispersion on mixed-convection heat and mass transfer in a non-Darcy porous medium. Chemical reaction and viscous dissipation effects on Darcy-Forchheimer mixed convection in a fluid saturated porous media has been studied by Chamkha et al. [11] The objective of this paper is to study the effects of double dispersion and MHD on heat and mass transfer with viscous dissipation over a vertical plate.

MATHEMATICAL FORMULATION

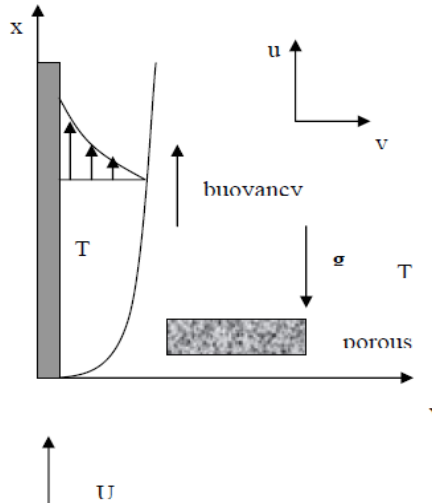


Fig.1. Schematic diagram of the problem

Consider the two dimensional study MHD mixed convection heat and mass transfer from a vertical plate embedded in a non-Darcy porous medium saturated with a Newtonian fluid. The coordinate system and flow model are shown in Fig.

(1) The x -coordinate is measured along the plate from its leading edge and the y -coordinate normal to it. A magnetic field is applied in the y -direction, the wall is maintained at constant temperature T_w and constant concentration C_w . The temperature and mass concentration of the ambient medium are assumed to be T_∞ and C_∞ respectively, under these assumptions and using the Boussinesq approximations, the boundary layer equations can be written as

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equation:

$$\left[1 + \left(\frac{K \sigma \mu_e^2 H_0^2}{\mu} \right) \right] \frac{\partial u}{\partial y} + \frac{c \sqrt{K}}{\nu} \frac{\partial}{\partial y} (u^2) = \pm \frac{gK}{\nu} \left(\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y} \right) \tag{2}$$

Energy Equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(\alpha_e \frac{\partial T}{\partial y} \right) + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \tag{3}$$

Concentration Equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial y} \left(D_e \frac{\partial C}{\partial y} \right) - \gamma (C - C_\infty) \tag{4}$$

$$\rho = \rho_\infty \{ 1 - \beta_T (T - T_\infty) - \beta_C (C - C_\infty) \}$$

and the boundary conditions of the problem are:

$$\left. \begin{aligned} y=0 : v=0, T=T_w, C=C_w \\ y \rightarrow \infty : u \rightarrow u_\infty, T=T_\infty, C=C_\infty \end{aligned} \right\} \tag{5}$$

where u and v are velocities in x and y directions, T is the temperature, K is the permeability constant, C is an empirical constant, ν is the kinematic viscosity, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_c is the coefficient of solute expansion, ρ is the density, C_p is the specific heat at constant pressure, M is the magnetic field. In equation (2) the “+” sign corresponds to the case of aiding buoyancy and the “-” sign corresponds to the case of opposing buoyancy flow, α_e is the thermal diffusivity and D_e is the mass diffusivity.

Introducing similarity variables and Similarity transformations

$$\psi = \alpha \sqrt{Pe_x} f(\eta), \eta = \frac{y}{x} \sqrt{Pe_x}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

where ψ the stream function that satisfies the continuity equation is defined as

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$

With the change of variables, Equation (1) is identically satisfied and Equations (2)-(4) are transformed, to

$$(1 + M + 2Ff')f'' = \pm \frac{Ra_x}{Pe_x} (\theta' + N\phi') \tag{6}$$

$$\theta'' + \frac{1}{2} f\theta' + Pr E_c (f'')^2 + D(f'\theta'' + f''\theta') = 0 \tag{7}$$

$$\frac{Pr}{Sc} \phi'' + \frac{1}{2} f\phi' + B(f'\phi'' + f''\phi') - \phi\chi = 0 \tag{8}$$

The corresponding dimensionless boundary conditions take the form

$$\begin{aligned} \eta = 0 & : f = 0, \theta = 1, \phi = 1 \\ \eta \rightarrow \infty & : f' \rightarrow 1, \theta \rightarrow 0, \phi \rightarrow 0 \end{aligned} \tag{9}$$

Here, the prime denotes differentiation with respect to η , $Ra_x = \frac{Kg\beta_T(T_w - T_\infty)x}{\nu\alpha}$ is a Local Rayleigh number,

$M = \frac{K\sigma\mu_e^2 H_0^2}{\mu}$ is the magnetic parameter, $Le = \frac{\alpha}{b}$ is the Lewis number, $Pe_x = \frac{u_\infty x}{\alpha}$ is the Peclet number,

$Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{b}$ is the Schmidt number, $N = \frac{\beta_c(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$ is the buoyancy ratio,

$E_c = \frac{u_\infty^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $F = \frac{c\sqrt{K}u_\infty}{\nu}$ is the inertia parameter, $D = \frac{\gamma d u_\infty}{\alpha}$ is the thermal

dispersion coefficient, $B = \frac{\zeta d u_\infty}{\alpha}$ is the solute dispersion coefficient and $\chi = \frac{\gamma x}{u_\infty}$ is the chemical reaction

parameter. The local heat transfer rate which is one of the primary interests of the study is given by

$$q_w = -k_e \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0} = -(k + k_d) \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0}, \text{ where } k_e \text{ is the effective thermal conductivity of the porous medium}$$

which is the sum of molecular thermal conductivity k and the dispersion thermal conductivity k_d .

The local Nusselt number Nu_x is defined as $Nu_x = \frac{q_w x}{k_e(T_w - T_\infty)}$. Now the set of primary variables which describes

the problem may be replaced with another set of dimensionless variables. This includes a dimensionless variable which

is related to heat transfer and is given by $\frac{Nu_x}{(Pe_x)^{\frac{1}{2}}} = -[1 + Df'(0)]\theta'(0)$. The local mass flux at the wall is given

by $j_w = -D_y \left(\frac{\partial C}{\partial y} \right) \Big|_{y=0}$ defines another dimensionless variable that is the local Sherwood number, Sh_x is defined

as $Sh_x = \frac{j_w x}{D_1 (c_w - c_\infty)}$ it may be also defined as another dimensionless variable, i.e.,

$$\frac{Sh_x}{(Pe_x)^2} = -[1 + B f'(0)] \phi'(0) .$$

SOLUTION PROCEDURE:

The dimensionless equations (6), (7), (8) together with the boundary conditions (9) are solved numerically by means of the fourth order Runge-Kutta method coupled with the shooting technique. The solution, thus, obtained is matched with the given values of $f'(\infty), \theta(\infty), \phi(\infty)$. In addition, the boundary condition $\eta \rightarrow \infty$ is approximated by $\eta_{\max} = 6$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. This choice of η_{\max} helps to compare the present results with those of the earlier researchers.

RESULT AND DISCUSSION:

The system of ordinary differential equations (6)-(8) along with the boundary conditions (9) has been solved numerically using Runge-Kutta method of order four with shooting technique. Comparison with the existing result from the literature shows a favorable agreement with Chamkha et. al [11]. In the absence of magnetic parameter (M), thermal dispersion (D) and solute dispersion (B). Extensive calculations have been performed to obtain the flow, temperature and concentration fields for various physical parameters. The velocity, temperature and concentration profiles for different values of magnetic parameter (M), thermal dispersion (D), solute dispersion (B) and mixed convection parameters (Ra/Pe) are shown in Figures 2-4. Figure 2. Shows that velocity in the boundary layer is larger than the external velocity in case of aiding flow. For a fixed value of mixed convection parameter in case of aiding flow velocity increases with an increase in magnetic parameter. It is clear from the figure.3 that temperature profile and boundary layer thickness decreases with an increase in mixed convection parameter in both aiding and opposing flows. From figure.4 it is observed that concentration profile, boundary layer thickness decreases with an increase in mixed convection. But this increment is less in the presence of magnetic parameter and double dispersion than absence. From fig.5 it is observed that velocity profile decreases with an increase in chemical reaction parameter. This increment is more in the presence of magnetic parameter than its absence. It is observed from the fig.6 that temperature boundary layer increases with an increase in chemical reaction in the presence of magnetic parameter and double dispersion. This increment is more in the absence of the first case. Concentration boundary layer decreases more with an increase in chemical reaction parameter in the presence of magnetic field and dispersion as shown in fig.7. In fig.8, we see the behavior of velocity profile according to the variation of the inertia parameter. It is clear from this figure that the increase in the inertia parameter reduces the velocity. From fig.9, it can be observed that in case of aiding flow an increase in inertia parameter increases the temperature in the boundary layer. It is clear from the fig.10 that concentration variation is very less in presence of magnetic parameter and double dispersion than its absence. Figure.11. shows the influence of viscous dissipation on dimensionless velocity, a rise in viscosity increases the velocity in the boundary layer. This increment is more in the absence of magnetic parameter and double dispersion than its presence. Figure.12 illustrates that variation of dimensionless temperature function versus η with various values of viscous dissipation (Ec). The temperature of the thermal boundary layer increases with an increase in viscous

dissipation parameter, much difference is not observed in concentration profile. In fig.13 we observe that velocity within the boundary decreases with an increase in Schmidt number. It is observed from the fig.14 that with an increase in Schmidt numbers the change in temperature within the boundary is negligible. Fig.15 shows that an increase in Schmidt number decreases the concentration boundary layer in the presence of magnetic field and double dispersion. The velocity, temperature and concentration profiles for different values of buoyancy ratio are shown in figs. 16 - 18. In fig.16 it is clear that with an increase in aiding buoyancy velocity decreases within the boundary. From figs.17 and 18, we observe that with an increase in buoyancy ratio temperature increases in thermal boundary layer and concentration also increases within its boundary layer. The effects of mixed convection parameter on Nusselt number for various values of chemical reaction parameter are shown in fig.19. It is observed from the figure that heat transfer decreases with an increase in chemical reaction parameter within the boundary layer. It is observed from the fig.20 that mass transfer rate increases with an increase in chemical reaction parameter. For a fixed value of chemical reaction parameter Sherwood number increases with an increase in magnetic parameter. From fig.21 it is clear that Nusselt number decreases with an increase in viscous dissipation parameter, but this increment is less in the presence of

magnetic parameter and double dispersion than their absence. We observed from fig.22 that Sherwood number decreases with an increase in viscosity parameter within the boundary layer.

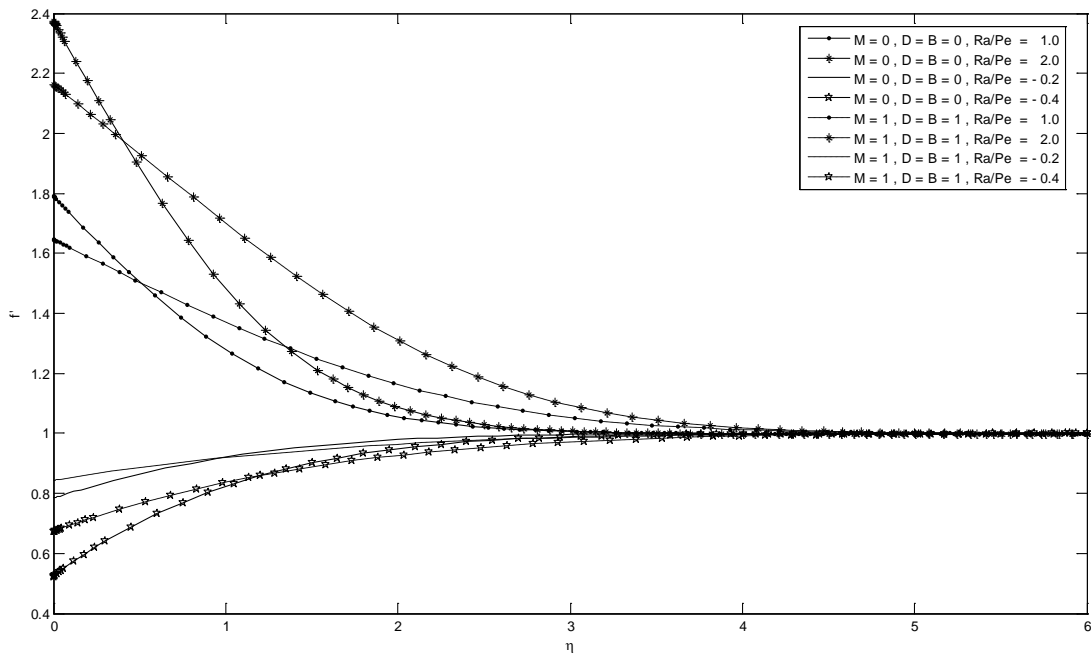


Fig.2: Velocity profiles for different values of $M, D, B, \frac{Ra}{Pe}$
 $(N = 2, F = 1, \chi = 0.4, Pr = 0.73, E_c = 0.5, Sc = 1)$

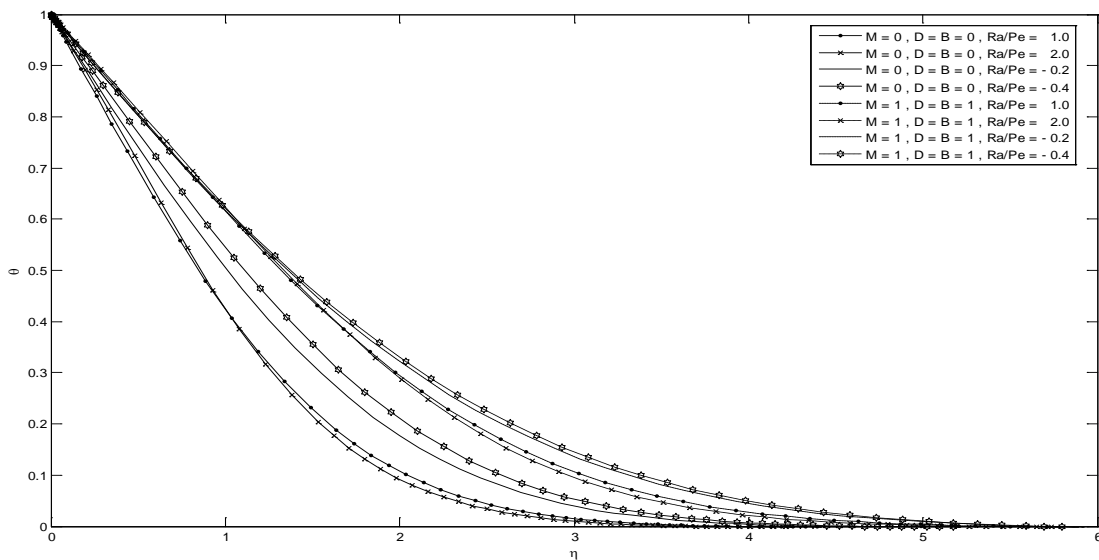


Fig.3: Temperature profiles for different values of $M, D, B, \frac{Ra}{Pe}$
 $(N = 2, F = 1, \chi = 0.4, Pr = 0.73, E_c = 0.5, Sc = 1)$

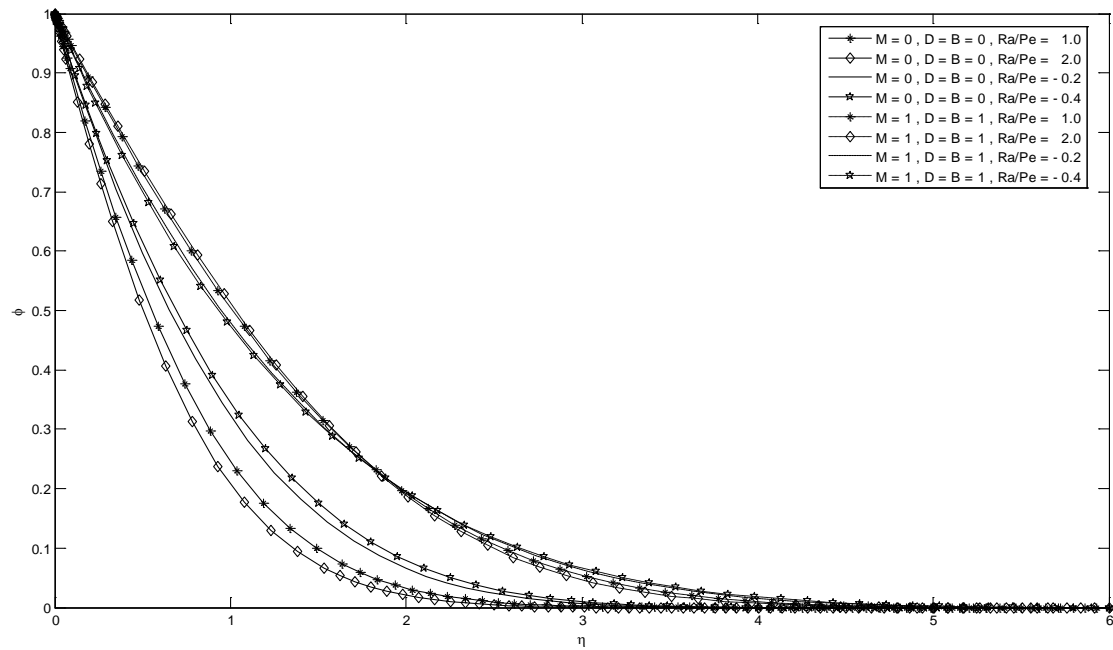


Fig.4: Concentration profiles for different values of $M, D, B, \frac{Ra}{Pe}$
 $(N = 2, F = 1, \chi = 0.4, Pr = 0.73, E_c = 0.5, Sc = 1)$

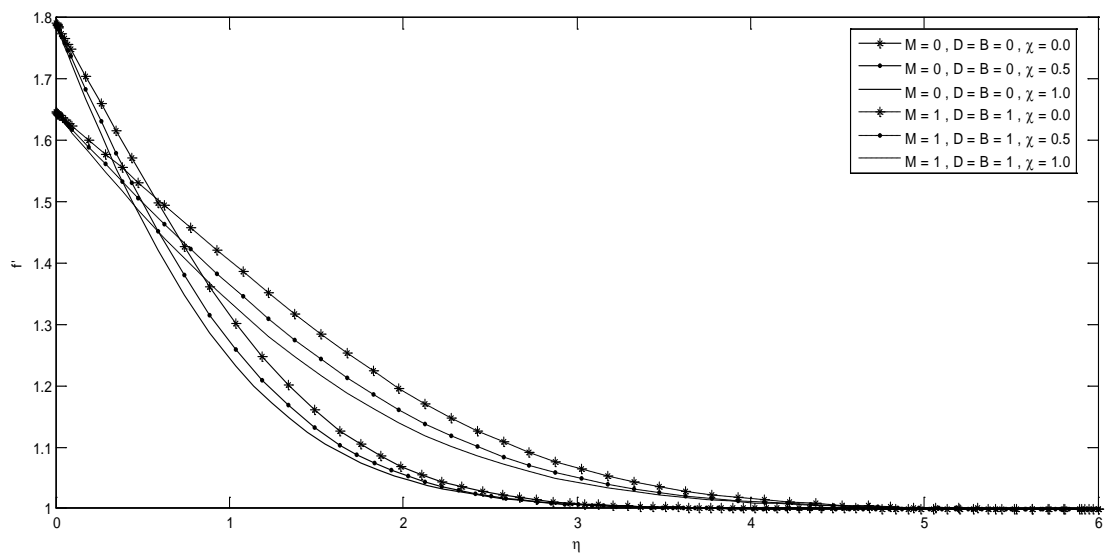


Fig.5: Variation of velocity for different values of M, D, B, χ
 $(N = 2, F = 1, \frac{Ra}{Pe} = 1, Pr = 0.73, E_c = 0.5, Sc = 1)$

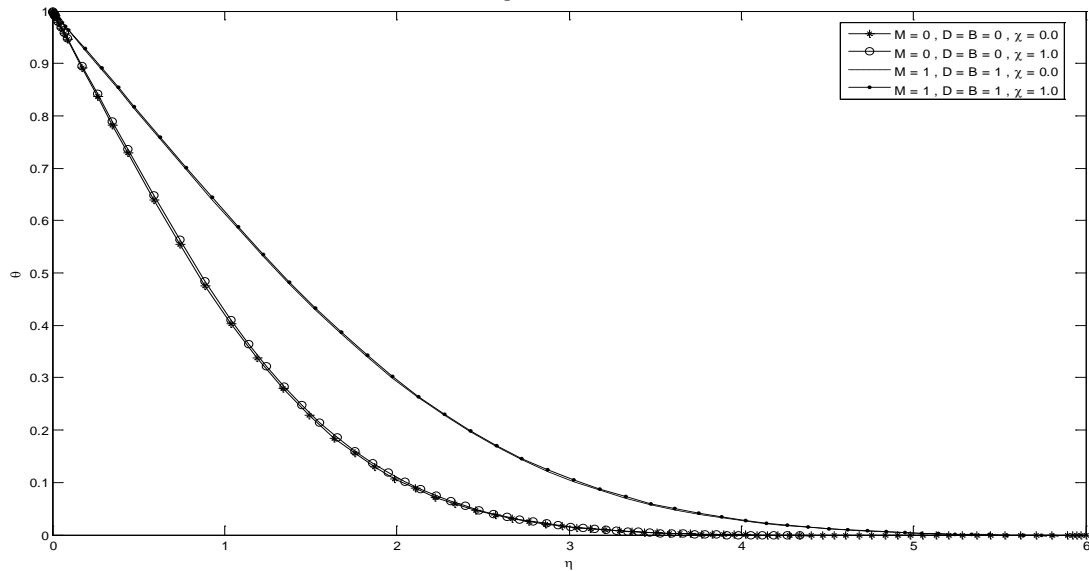


Fig.6: Temperature profile variation for different values of M, D, B, χ

$$\left(N = 2, F = 1, \frac{Ra}{Pe} = 1, Pr = 0.73, E_c = 0.5, Sc = 1 \right)$$

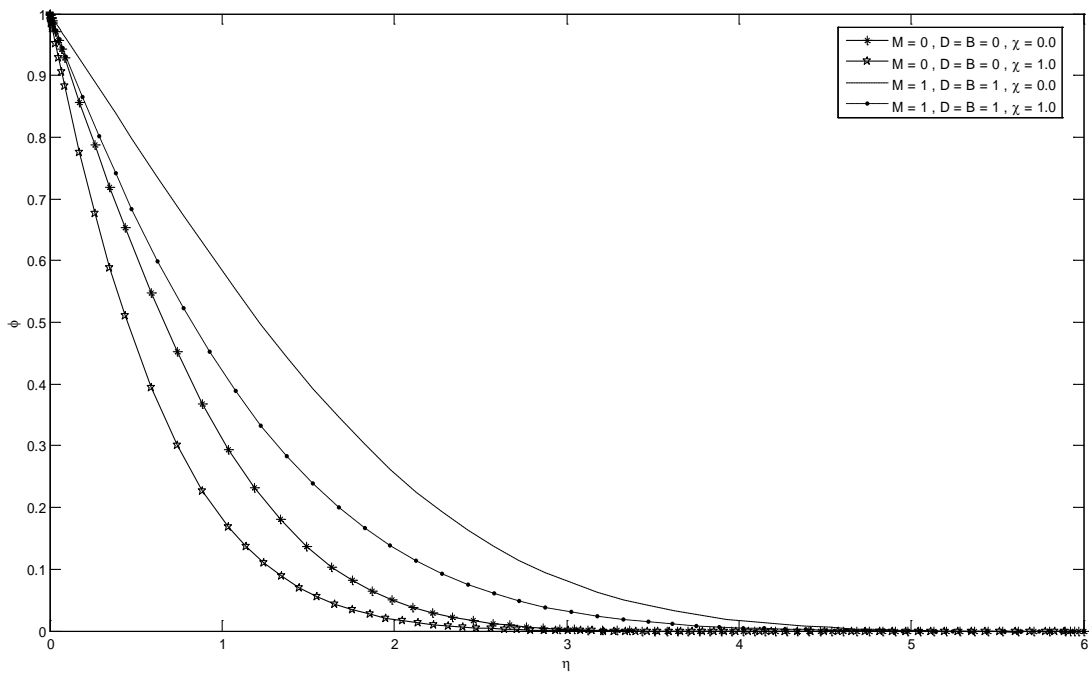


Fig.7: Concentration variations for different values of M, D, B, χ

$$\left(N = 2, F = 1, Ra/Pe = 1, Pr = 0.73, E_c = 0.5, Sc = 1 \right)$$

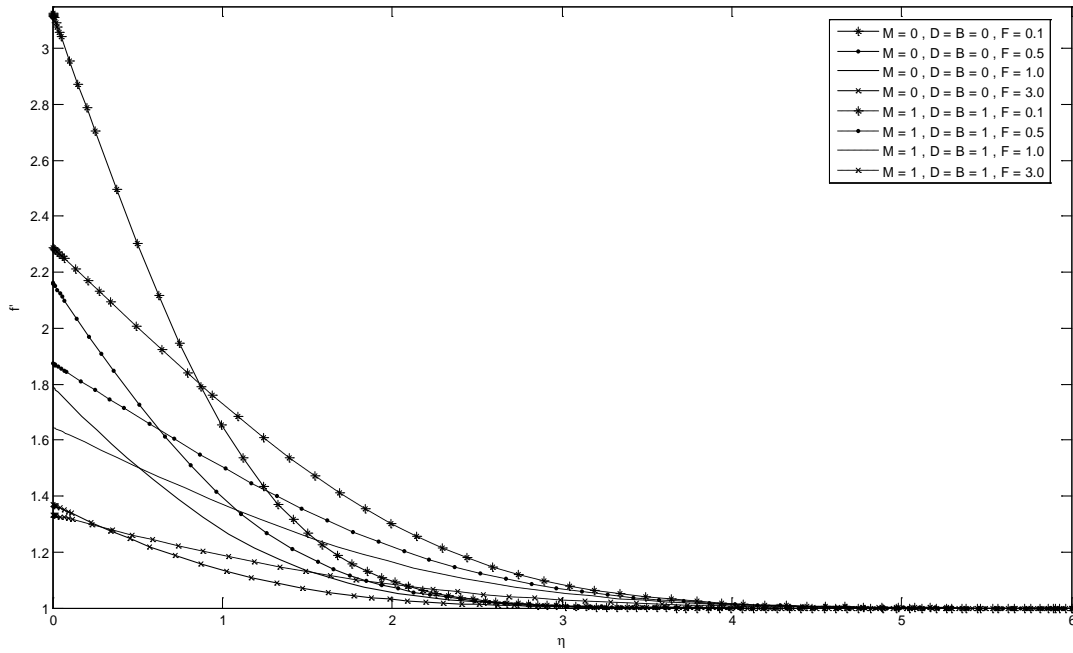


Fig.8: Effects of M, D, B and F on velocity profile
($N = 2, \chi = 0.4, Ra/Pe = 1, Pr = 0.73, E_c = 0.5, Sc = 1$)

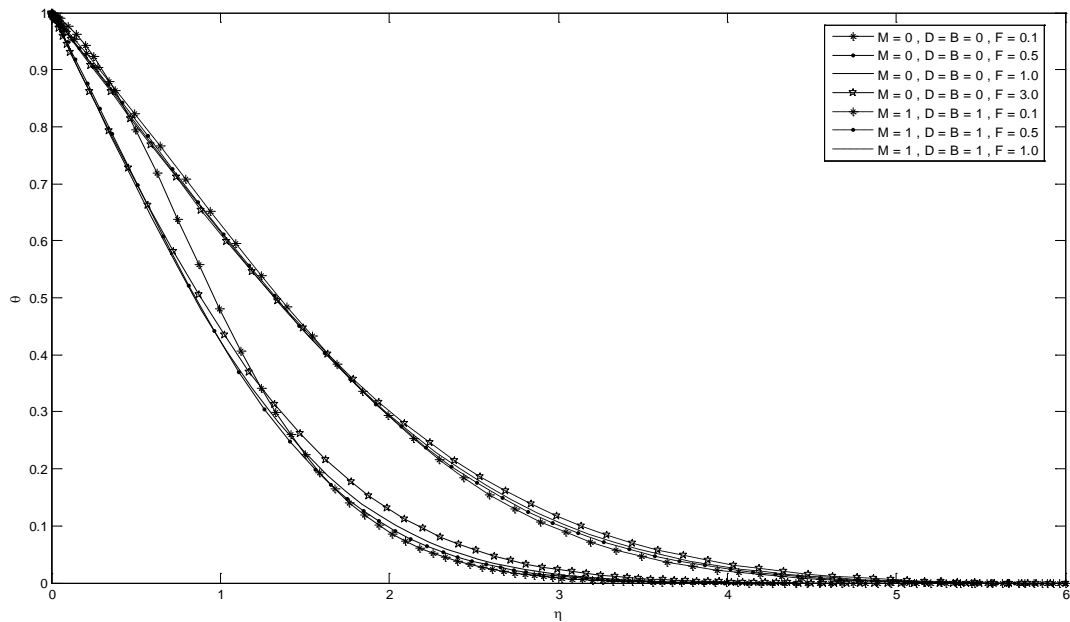


Fig.9: Effects of M, D, B and F on Temperature profile
($N = 2, \chi = 0.4, Ra/Pe = 1, Pr = 0.73, E_c = 0.5, Sc = 1$)

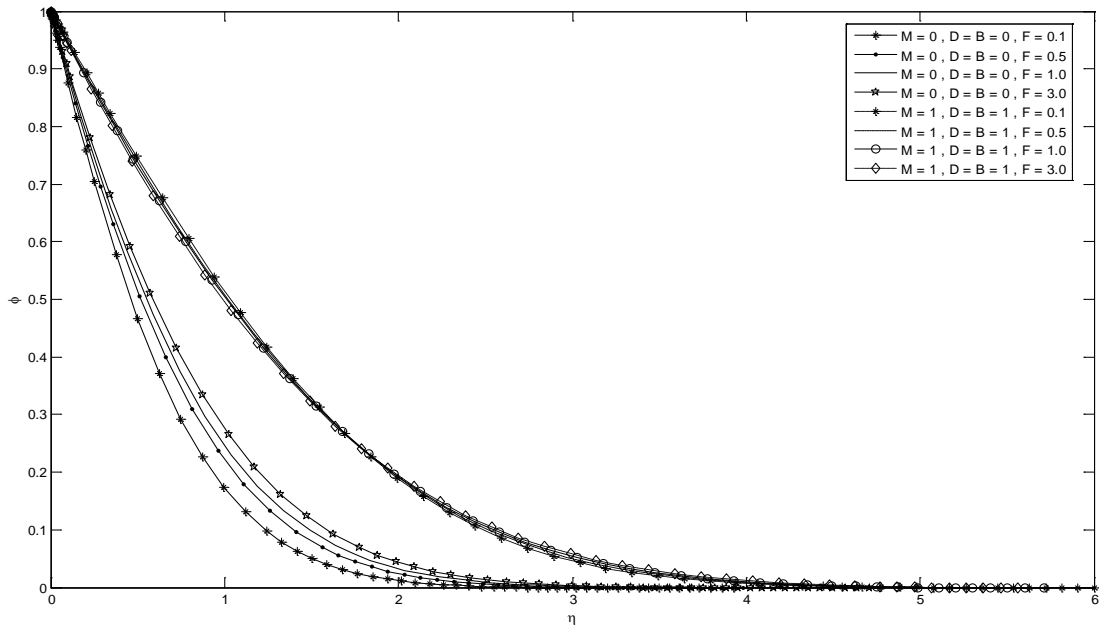


Fig.10: Effects of M, D, B and F on concentration profile
 ($N = 2, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.5, Sc = 1$)

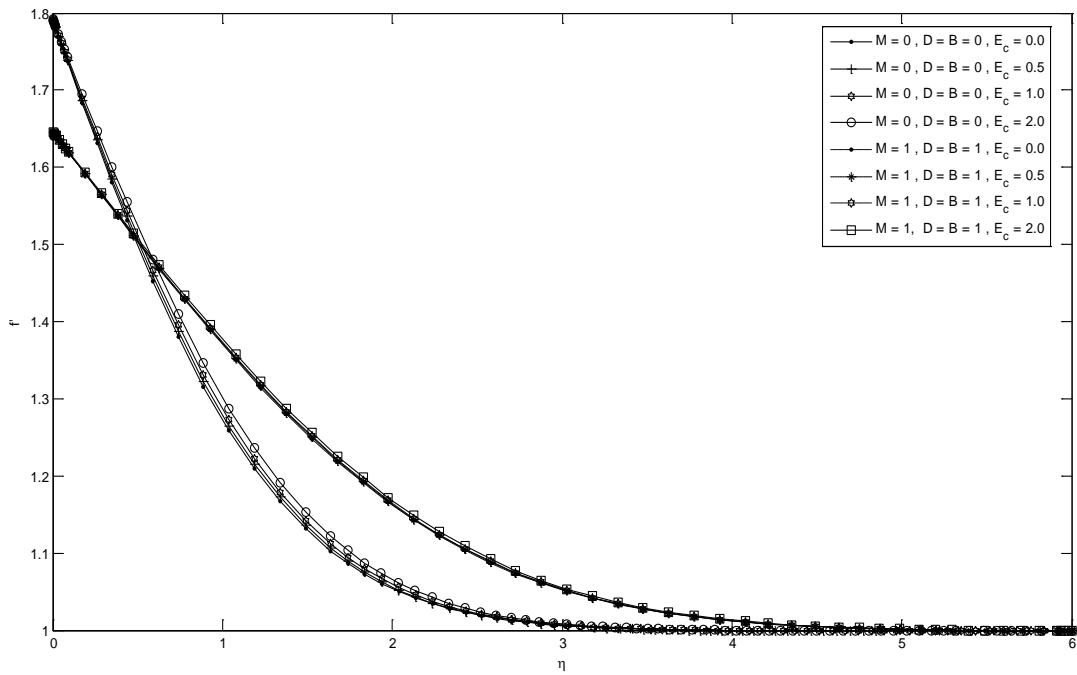


Fig.11: Velocity profiles for various values of M, D, B and E_c
 ($N = 2, F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, Sc = 1$)

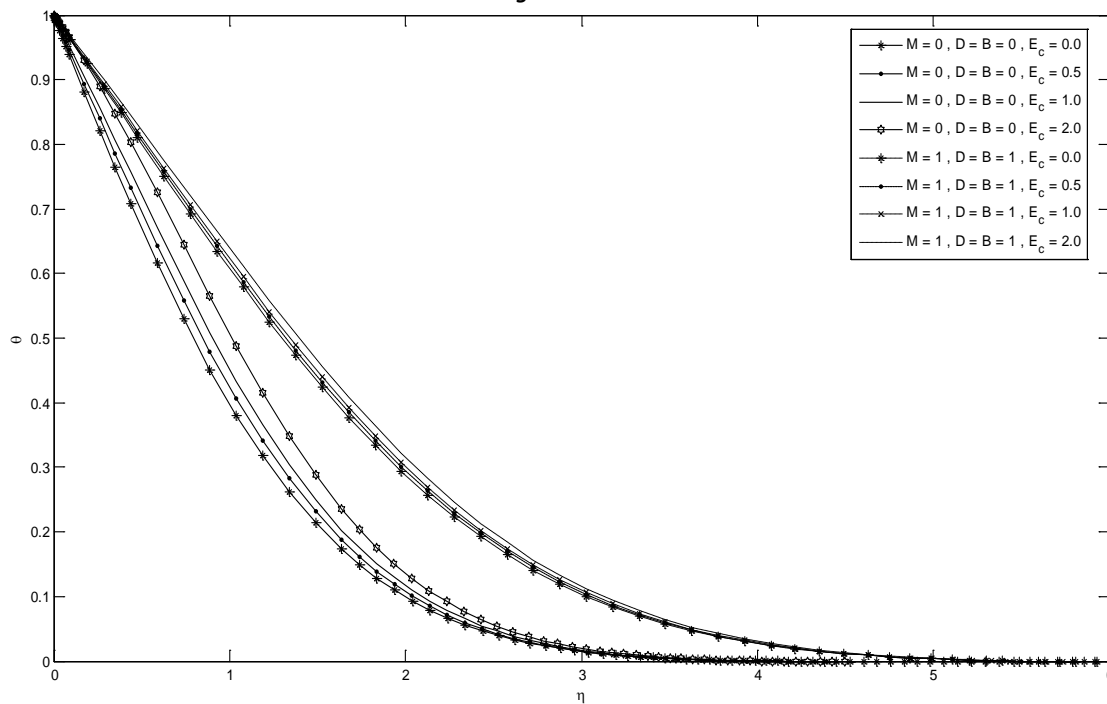


Fig.12: Temperature profiles for various values of M, D, B and E_c
 ($N = 2, F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, Sc = 1$)

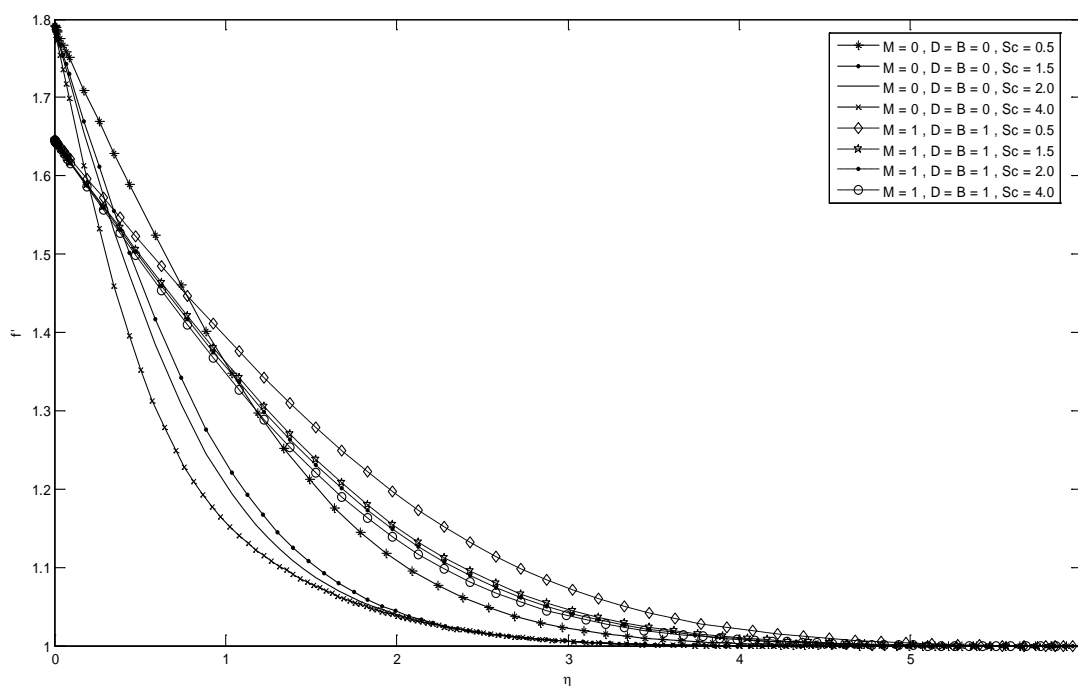


Fig.13: Velocity profiles for various values of M, D, B and Sc
 ($N = 2, F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.73$)

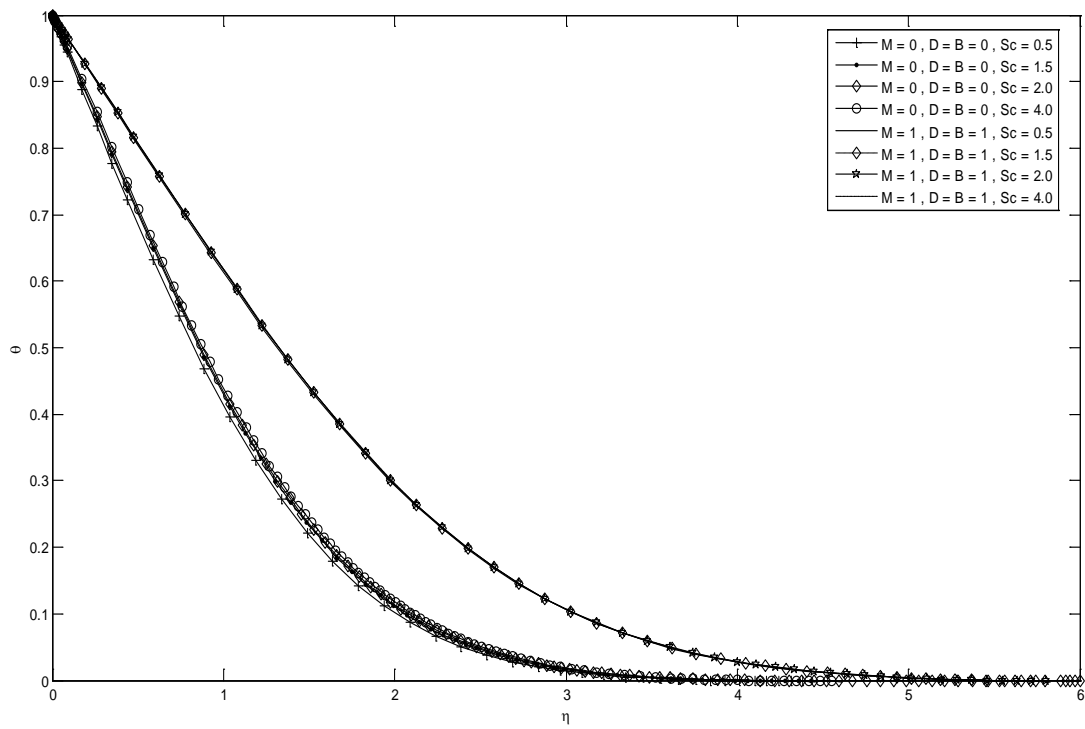


Fig.14. Temperature profiles for various values of M, D, B and Sc
 ($N = 2, F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.73$)

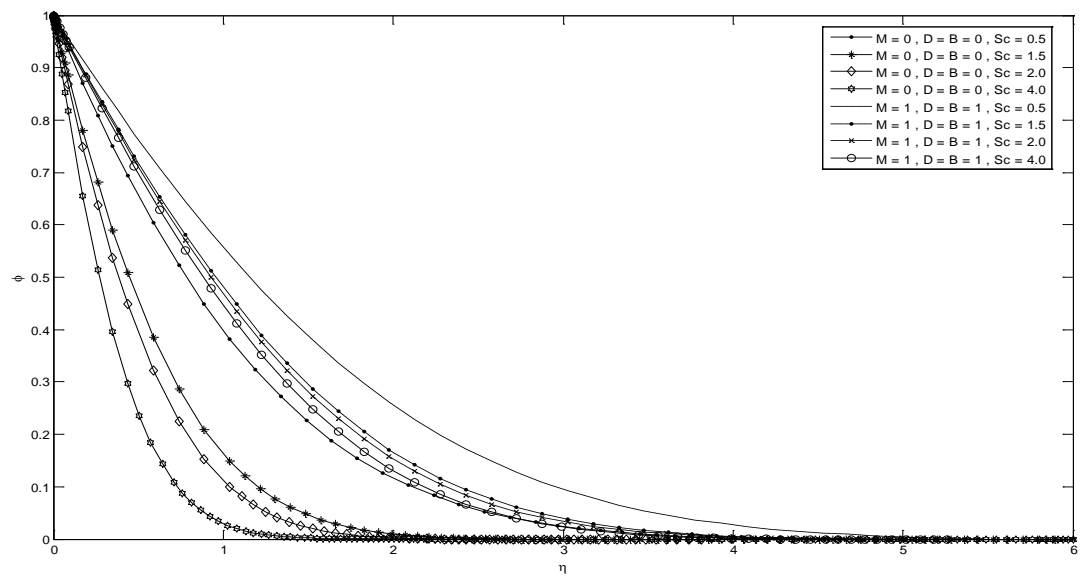


Fig.15: Concentration profiles for various values of M, D, B and Sc
 ($N = 2, F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.73$)

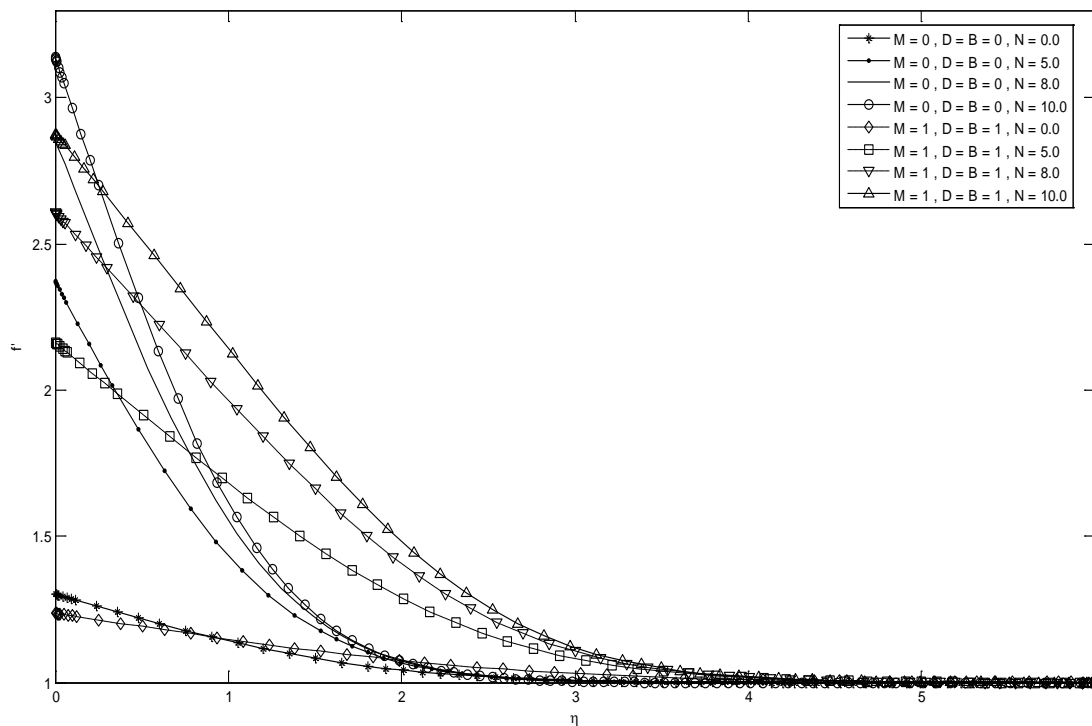


Fig.16: Effects of M, D, B and N on velocity profile
 ($F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.73, Sc = 1$)

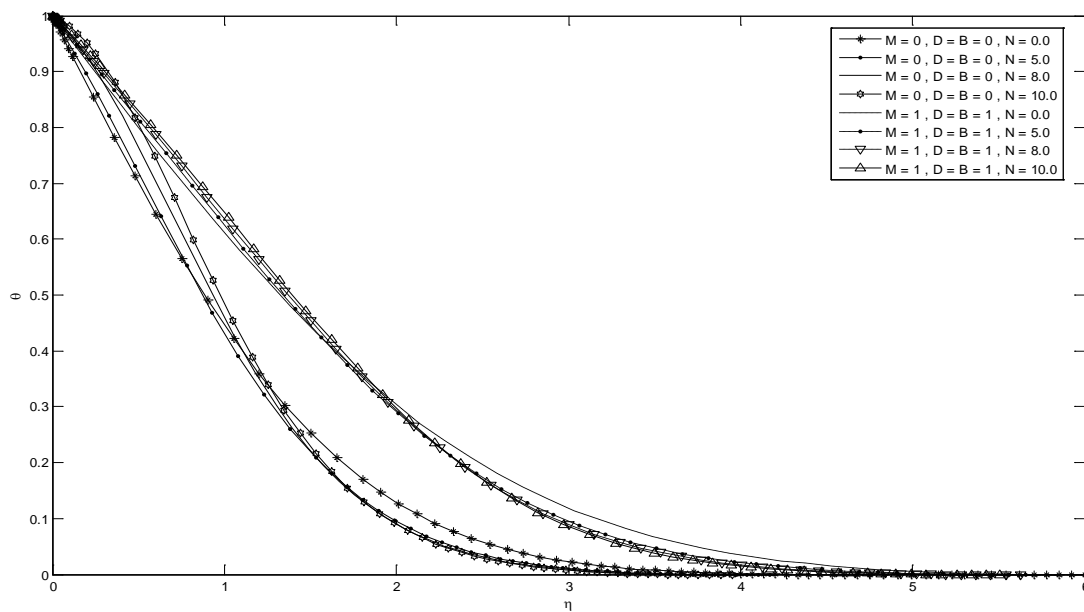


Fig.17: Effects of M, D, B and N on temperature profile
 ($F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.73, Sc = 1$)

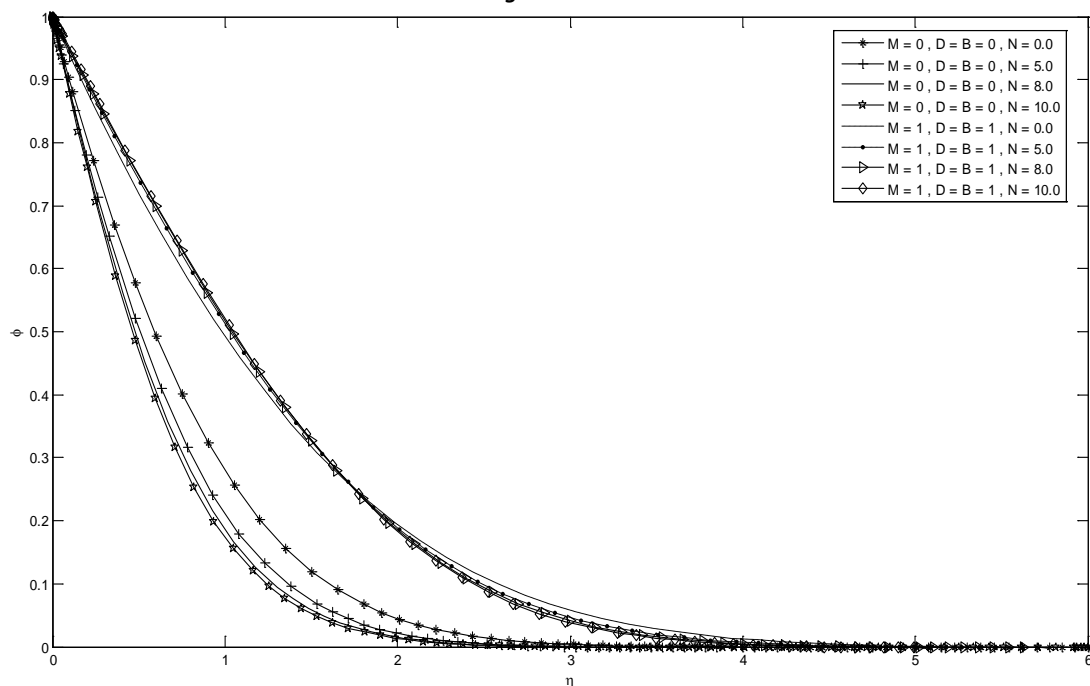


Fig.18: Effects of M, D, B and N on concentration profile
 ($F = 1, \chi = 0.4, Ra / Pe = 1, Pr = 0.73, E_c = 0.73, Sc = 1$)

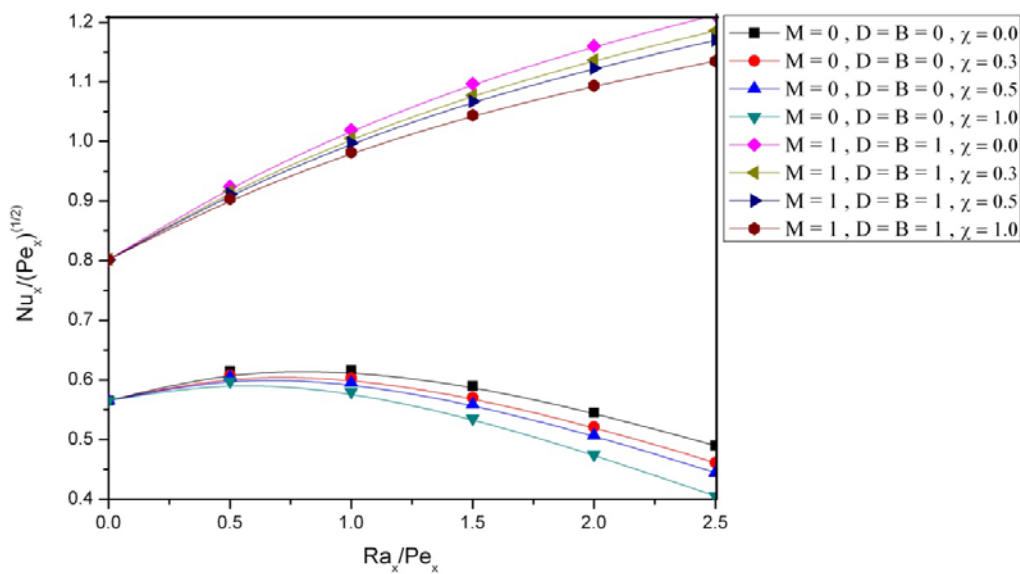


Fig.19: Effect of Mixed convection on Nusselt number for different values of M, D, B and χ
 ($N = 2, F = 1, Ra / Pe = 1, Pr = 0.73, E_c = 0.73, Sc = 1$)

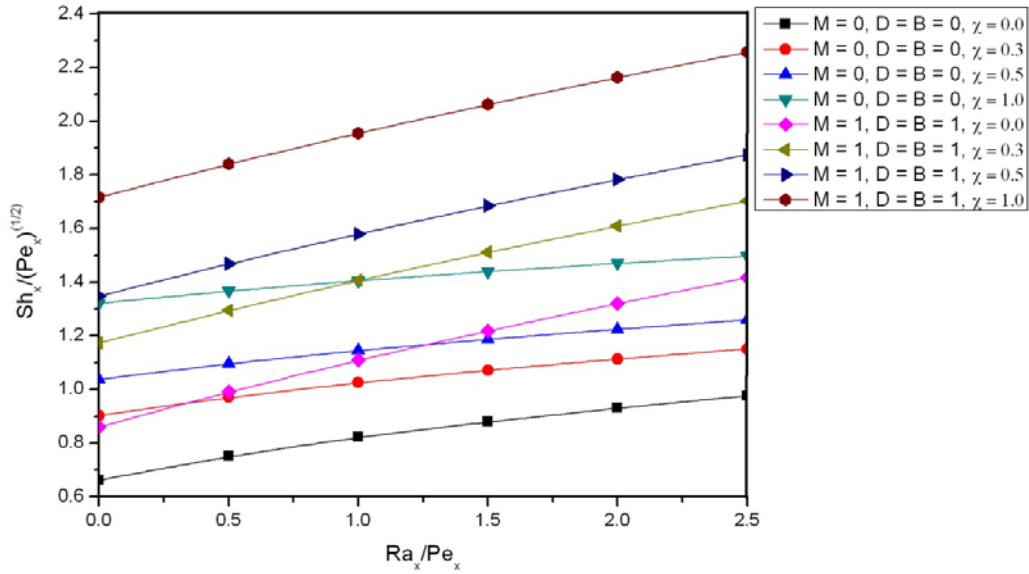


Fig.20: Effect of Mixed convection on Sherwood number for different values of M, D, B and χ
 ($N = 2, F = 1, Ra / Pe = 1, Pr = 0.73, E_c = 0.73, Sc = 1$)

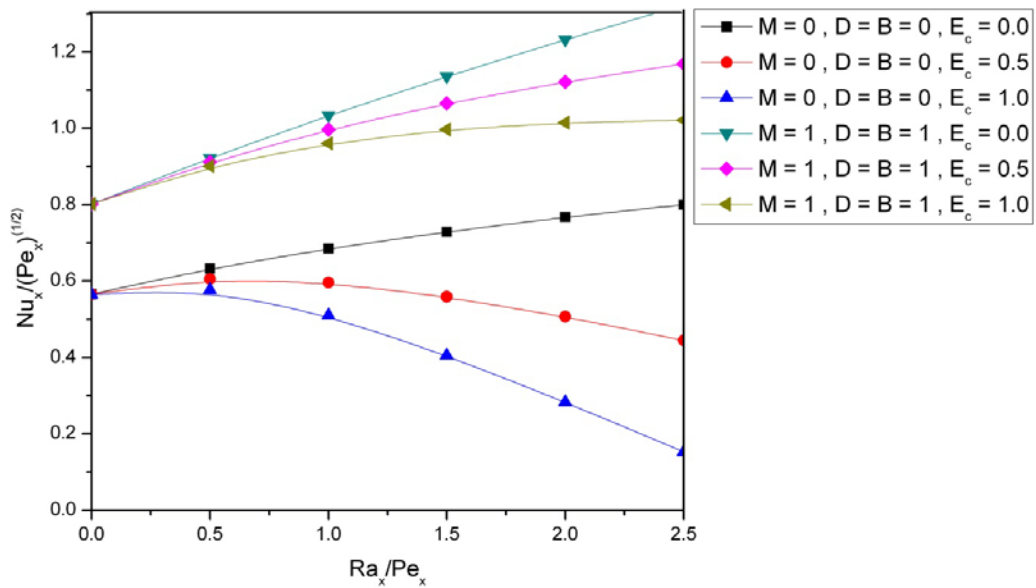


Fig.21: Effect of Mixed convection on Nusselt number for different values of M, D, B and E_c
 ($N = 2, F = 1, Ra / Pe = 1, Pr = 0.73, \chi = 0.4, Sc = 1$)

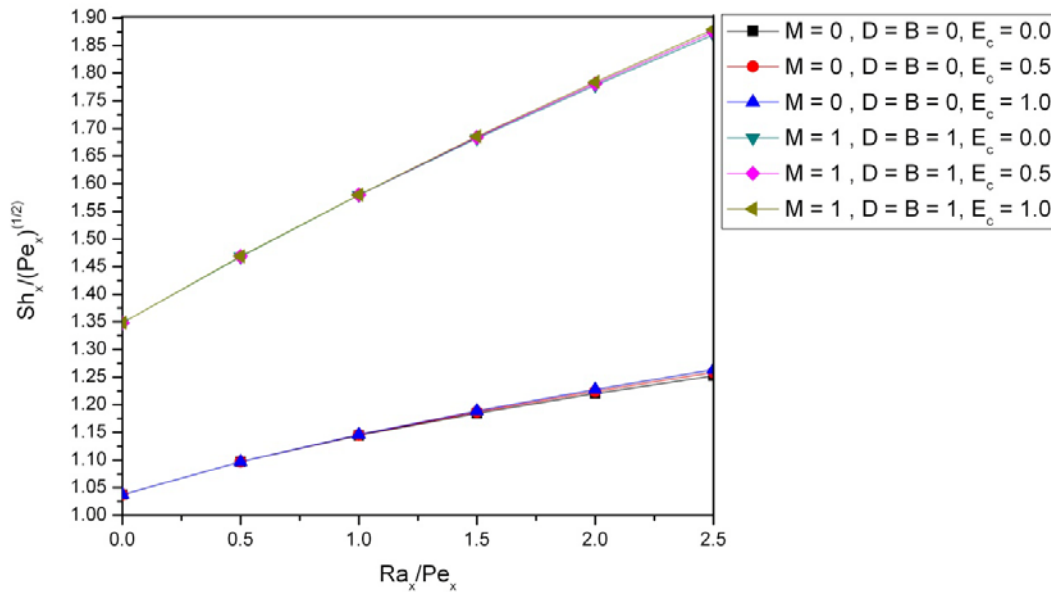


Fig.22: Effect of Mixed convection on Sherwood number for different values of M, D, B and E_c
 $(N = 2, F = 1, Ra / Pe = 1, Pr = 0.73, \chi = 0.4, Sc = 1)$

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