International Journal of Mathematical Archive-3(2), 2012, Page: 491-500
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON FUZZY SOFT MATRIX THEORY

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(Received on: 05-02-12; Accepted on: 27-02-12)

ABSTRACT

The purpose of this paper is to introduce the basic concept of fuzzy soft matrix theory. In our work, we have put forward the notions related to fuzzy soft matrices. Our work is an attempt to extend our earlier notion of fuzzy soft matrices.

Keywords: *Fuzzy Set, Soft Set, Fuzzy Soft Set, Fuzzy Soft Matrix, Product of Fuzzy Soft Matrices.*

1. INTRODUCTION

Fuzzy set theory proposed by Zadeh [**15**] at the University of California, Berkley in 1965 is a generalization of classical or crisp sets. It makes possible to describe vague notions and deals with the concepts and techniques which lay in the form of mathematical precision to human thought processes that in many ways are imprecise and ambiguous within the ambit of classical mathematics. In the Zadehian theory of fuzzy sets, it has been accepted that the classical set theoretic axioms of exclusion and contradiction are not satisfied. In this regard, Baruah [**3, 4**] proposed that two functions, namely fuzzy membership function and fuzzy reference function are necessary to represent a fuzzy set. Accordingly, Baruah [**3,4**] reintroduced the notion of complement of a fuzzy set in a different way and proved that indeed the set theoretic axioms of exclusion and contradiction are valid for fuzzy sets also.

In 1999, Molodtsov [**8**] introduced the theory of soft sets, which is a new mathematical approach to vagueness. In 2003, Maji et al. [**7**] studied the theory of soft sets initiated by Molodtsov [**8**] and developed several basic notions of Soft Set Theory. At present, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao [**13**] and Chen et al. [**5**] studied and improved the findings of Maji et al. [**7**]. In 2009, Ali et al.[**2**] gave some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets along with a new notion of relative complement of a soft set. Later in 2011, Neog and Sut [**11**] reintroduced the concept of complement of a soft set in such a way that the fundamental properties related to complement are satisfied also by soft sets.

In recent years the researchers have contributed a lot towards fuzzification of soft set theory. Maji et al. **[6]** initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan's Law etc. These results were further revised and improved by Ahmad and Kharal **[1]**. They defined arbitrary fuzzy soft union and intersection and proved De Morgan's Inclusions and De Morgan's Laws in Fuzzy Soft Set Theory. Neog and Sut [**10**] studied several basic properties of fuzzy soft sets and put forward a new notion of complement of a fuzzy soft set.

Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In [**12**], Neog and Sut proposed a matrix representation of a fuzzy soft set and successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems [**12,13**]. In this paper, we extend the notion of fuzzy soft matrices put

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forward in [**12**]. In our work, we are using the notion of extended fuzzy sets initiated by Baruah in [**4**]. Thus fuzzy sets in our work have been replaced by usual fuzzy sets with fuzzy reference function zero.

2. PRELIMINARIES

In this section, we recall some concepts and definitions which will be needed in the sequel.

Definition 2.1 [4]
Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ and $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ be two fuzzy sets defined over the same universe *U*.

Then the operations intersection and union are defined as

$$A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)=\{x,\min(\mu_1(x),\mu_3(x)),\max(\mu_2(x),\mu_4(x));x\in U\}$$

and $A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4)=\{x,\max(\mu_1(x),\mu_3(x)),\min(\mu_2(x),\mu_4(x));x\in U\}$.

Neog et al. [*****] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows-

Definition 2.2 [9]
Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ and $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ be two fuzzy sets defined over the same universe *U*. To avoid degenerate cases we assume that

$$\min(\mu_1(x),\mu_3(x))\geq\max(\mu_2(x),\mu_4(x))\forall x\in U\ .$$

Then the operations intersection and union are defined as

$$A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)=\{x,\min(\mu_1(x),\mu_3(x)),\max(\mu_2(x),\mu_4(x));x\in U\}$$

and $A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4)=\{x,\max(\mu_1(x),\mu_3(x)),\min(\mu_2(x),\mu_4(x));x\in U\}$.

Definition 2.3 [4]
For usual fuzzy sets $A(\mu,0)=\{x,\mu(x),0;x\in U\}$ and $B(1,\mu)=\{x,1,\mu(x);x\in U\}$ defined over the same universe *U*, we have

$$\begin{aligned}A(\mu,0)\cap B(1,\mu)&=\{x,\min(\mu(x),1),\max(0,\mu(x));x\in U\}\\&=\{x,\mu(x),\mu(x);x\in U\},\text{ which is nothing but the null set }\varphi.\end{aligned}$$

$$\begin{aligned}\text{And }A(\mu,0)\cup B(1,\mu)&=\{x,\max(\mu(x),1),\min(0,\mu(x));x\in U\}\\&=\{x,1,0;x\in U\},\text{ which is nothing but the universal set }U.\end{aligned}$$

This means if we define a fuzzy set $(A(\mu,0))^c=\{x,1,\mu(x);x\in U\}$, it is nothing but the complement of $A(\mu,0)=\{x,\mu(x),0;x\in U\}$.

Definition 2.4 [9]
Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ and $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ be two fuzzy sets defined over the same universe *U*. The fuzzy set $A(\mu_1,\mu_2)$ is a subset of the fuzzy set $B(\mu_3,\mu_4)$ if $\forall x\in U$, $\mu_1(x)\leq\mu_3(x)$ and $\mu_4(x)\leq\mu_2(x)$.

Two fuzzy sets $C=\{x,\mu_C(x):x\in U\}$ and $D=\{x,\mu_D(x):x\in U\}$ in the usual definition would be expressed as $C(\mu_C,0)=\{x,\mu_C(x),0;x\in U\}$ and $D(\mu_D,0)=\{x,\mu_D(x),0;x\in U\}$.

Accordingly, we have $C(\mu_C,0)\subseteq D(\mu_D,0)$ if $\forall x\in U$, $\mu_C(x)\leq\mu_D(x)$, Which can be obtained by putting $\mu_2(x)=\mu_4(x)=0$ in our new definition.

Definition 2.5 [8]
A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.
In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 2.6 [6]
A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$.

Definition 2.7 [1]
Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

Definition 2.8 [10]
A fuzzy soft set (*F*, *A*) over *U* is said to be null fuzzy soft set (with respect to the parameter set *A*), denoted by $\tilde{\varphi}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set φ. In other words, for a null fuzzy soft set (*F*, *A*), $\forall \varepsilon \in A, F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x); x \in U\}$. In the case of usual fuzzy sets, it is obvious that for a null fuzzy soft set (*F*, *A*), $\forall \varepsilon \in A, F(\varepsilon) = \{x, 0, 0; x \in U\}$.

Definition 2.9 [10]
A fuzzy soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set (with respect to the parameter set *A*), denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set *U*. In other words, for an absolute fuzzy soft set (*F*, *A*), $\forall \varepsilon \in A, F(\varepsilon) = \{x, 1, 0; x \in U\}$.

Definition 2.10 [6]
Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.11 [6]
Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Ahmad and Kharal [**1**] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows -

Definition 2.12 [1]
Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$. Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*,*C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Definition 2.13 [10]
The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$. In other words, $\forall \varepsilon \in A$, if $F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), 0; x \in U\}$, then $F^c(\varepsilon) = \{x, 1, \mu_{F(\varepsilon)}(x); x \in U\}$.

3. FUZZY SOFT MATRICES

Definition 3.1 [12] (Matrix Representation of a Fuzzy Soft Set)
Let $U = \{c_1, c_2, c_3, ...c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ...e_n\}$. Then the fuzzy soft set (F, E) can be expressed in matrix form as $A = [a_{ij}]_{m\times n}$ or simply by $[a_{ij}]$, i = 1,2,3…………..m ; j = 1,2,3……………..n and $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i in the fuzzy set $F(e_j)$ so that $\delta_{ij}(c_i) = \mu_{j1}(c_i) - \mu_{j2}(c_i)$ gives the fuzzy membership value of c_i. We shall identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all $m \times n$ fuzzy soft matrices over U will be denoted by $FSM_{m\times n}$. For usual fuzzy sets with fuzzy reference function 0, it is obvious to see that $a_{ij} = (\mu_{j1}(c_i), 0)\ \forall i, j$.

Definition 3.2 [12]
We define the membership value matrix corresponding to the matrix A as $MV(A) = [\delta_{(A)ij}]_{m\times n}$, where $\delta_{(A)ij} = \mu_{j1}(c_i) - \mu_{j2}(c_i)\ \ \forall i = 1,2,3......, m$ and $j = 1,2,3......, n$, where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i in the fuzzy set $F(e_j)$.

Definition 3.3
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. If $m \neq n$, then A is called a fuzzy soft rectangular matrix.

Definition 3.4
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. If $m = n$, then A is called a fuzzy soft square matrix.

Definition 3.5
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. If $m = 1$, then A is called a fuzzy soft row matrix. In other words, if the universe under consideration contains only one single element, we get a fuzzy soft row matrix.

Definition 3.6
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. If $n = 1$, then A is called a fuzzy soft column matrix. In other words, if the set of parameters under consideration contains only one single parameter, we get a fuzzy soft column matrix.

Definition 3.7
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. Then A is called a fuzzy soft zero (or null) matrix denoted by $[0]_{m\times n}$, or simply by $[0]$ if $\delta_{(A)ij} = 0$ for all i and j. For usual fuzzy sets, $\delta_{(A)ij} = \mu_{j1}(c_i)\ \forall i, j$.

Definition 3.8
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. Then A is called fuzzy soft diagonal matrix if $m = n$ and $a_{ij} = (\mu_j(c_i), \mu_j(c_i))$ for all $i \neq j$. In other words for a fuzzy soft diagonal matrix A, $\delta_{(A)ij} = 0$ $\forall i \neq j$.

Definition 3.9
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. Then A is called fuzzy soft scalar matrix if $m = n, a_{ij} = (\mu_j(c_i), \mu_j(c_i))$ for all $i \neq j$ and $\delta_{(A)ij} = \mu_{j1}(c_i) - \mu_{j2}(c_i) = \lambda \in [0,1]\ \forall\ i = j$. In other words for a fuzzy soft scalar matrix A, $\delta_{(A)ij} = \lambda \in [0,1]\ \forall i \neq j$.

Definition 3.10
Let $A=[a_{ij}]\in FSM_{m\times n}$, where $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$. Then A is called fuzzy soft unit or identity matrix if $m=n$, $a_{ij}=\left(\mu_j(c_i),\mu_j(c_i)\right)$ for all $i\neq j$ and $a_{ij}=(1,0)$ *i.e.* $\delta_{(A)ij}=1\ \forall\, i=j$.

Definition 3.11
Let $A=[a_{ij}]\in FSM_{m\times n}$, where $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$. Then A is called fuzzy soft upper triangular matrix if $m=n$, $a_{ij}=\left(\mu_j(c_i),\mu_j(c_i)\right)$ for all $i>j$. Thus for a fuzzy soft upper triangular matrix A, $\delta_{(A)ij}=0\ \forall i>j$.

Definition 3.12
Let $A=[a_{ij}]\in FSM_{m\times n}$, where $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$. Then A is called fuzzy soft lower triangular matrix if $m=n$, $a_{ij}=\left(\mu_j(c_i),\mu_j(c_i)\right)$ for all $i<j$. Thus for a fuzzy soft lower triangular matrix A, $\delta_{(A)ij}=0\ \forall i<j$.

A fuzzy soft matrix is said to be triangular if it is either fuzzy soft lower or fuzzy soft upper triangular matrix.

Definition 3.13
Let $A=[a_{ij}]\in FSM_{m\times m}$, where $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$, then the elements $a_{11},a_{12},\ldots\ldots\ldots\ldots,a_{mm}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of the fuzzy soft matrix.

Definition 3.14
Let the fuzzy soft matrices corresponding to the fuzzy soft sets (F,E) and (G,E) be $A=[a_{ij}]$, $B=[b_{ij}]\in FSM_{m\times n}$; $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$ and $b_{ij}=\left(\chi_{j1}(c_i),\chi_{j2}(c_i)\right)$, i =1,2,3…………..m ; j = 1,2,3……………..n. Then A and B are called fuzzy soft equal matrices denoted by $A=B$, if $\mu_{j1}(c_i)=\chi_{j1}(c_i)$ and $\mu_{j2}(c_i)=\chi_{j2}(c_i)\ \forall i,j$.

In [**13**], we have defined the 'addition (+)' operation between two fuzzy soft matrices as follows-

Definition 3.15 [13]
Let $U=\{c_1,c_2,c_3,\ldots\ldots\ldots\ldots\ldots\ldots c_m\}$ be the universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3,\ldots\ldots\ldots\ldots\ldots\ldots\ldots e_n\}$. Let the set of all $m\times n$ fuzzy soft matrices over U be $FSM_{m\times n}$.
Let $A,B\in FSM_{m\times n}$, where $A=\left[a_{ij}\right]_{m\times n}, a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$ and $B=\left[b_{ij}\right]_{m\times n}, b_{ij}=\left(\chi_{j1}(c_i),\chi_{j2}(c_i)\right)$. To avoid degenerate cases we assume that $\min\left(\mu_{j1}(c_i),\chi_{j1}(c_i)\right)\geq\max\left(\mu_{j2}(c_i),\chi_{j2}(c_i)\right)$ for all i and j. We define the operation 'addition (+)' between A and B as

$$A+B=C,\text{ where } C=\left[c_{ij}\right]_{m\times n},\qquad c_{ij}=\left(\max\left(\mu_{j1}(c_i),\chi_{j1}(c_i)\right),\min\left(\mu_{j2}(c_i),\chi_{j2}(c_i)\right)\right)$$

Example 3.1
Let $U=\{c_1,c_2,c_3,c_4\}$ be the universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3\}$. We consider the fuzzy soft sets

$$\begin{aligned}(F,E)\ &=\{F(e_1)=\{(c_1,0.3,0.0),(c_2,0.5,0.0),(c_3,0.6,0.0),(c_4,0.5,0.0)\},\\ &\quad F(e_2)=\{(c_1,0.7,0.0),(c_2,0.9,0.0),(c_3,0.7,0.0),(c_4,0.8,0.0)\},\\ &\quad F(e_3)=\{(c_1,0.6,0.0),(c_2,0.7,0.0),(c_3,0.7,0.0),(c_4,0.3,0.0)\}\}\end{aligned}$$

$$\begin{aligned}(G,E)\ &=\{G(e_1)=\{(c_1,0.8,0.0),(c_2,0.7,0.0),(c_3,0.5,0.0),(c_4,0.4,0.0)\},\\ &\quad G(e_2)=\{(c_1,0.9,0.0),(c_2,0.9,0.0),(c_3,0.8,0.0),(c_4,0.7,0.0)\},\\ &\quad G(e_3)=\{(c_1,0.5,0.0),(c_2,0.9,0.0),(c_3,0.6,0.0),(c_4,0.8,0.0)\}\}\end{aligned}$$

The fuzzy soft matrices representing these two fuzzy soft sets are respectively

$$A=\begin{bmatrix}(0.3,0.0) & (0.7,0.0) & (0.6,0.0)\\ (0.5,0.0) & (0.9,0.0) & (0.7,0.0)\\ (0.6,0.0) & (0.7,0.0) & (0.7,0.0)\\ (0.5,0.0) & (0.8,0.0) & (0.3,0.0)\end{bmatrix}_{4\times3} \text{ and } B=\begin{bmatrix}(0.8,0.0) & (0.9,0.0) & (0.5,0.0)\\ (0.7,0.0) & (0.9,0.0) & (0.9,0.0)\\ (0.5,0.0) & (0.8,0.0) & (0.6,0.0)\\ (0.4,0.0) & (0.7,0.0) & (0.8,0.0)\end{bmatrix}_{4\times3}$$

Here $A+B=\begin{bmatrix}(0.8,0.0) & (0.9,0.0) & (0.6,0.0)\\ (0.7,0.0) & (0.9,0.0) & (0.9,0.0)\\ (0.6,0.0) & (0.8,0.0) & (0.7,0.0)\\ (0.5,0.0) & (0.8,0.0) & (0.8,0.0)\end{bmatrix}_{4\times3}$

Proposition 3.1
Let $A,B,C\in FSM_{m\times n}$. Then the following results hold.

(i) $A+B=B+A$

(ii) $(A+B)+C=A+(B+C)$

(iii) $A+[0]=A=[0]+A$

Proof

(i) Let $A=\left[\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)\right]$, $B=\left[\left(\chi_{j1}(c_i),\chi_{j2}(c_i)\right)\right]$

Now, $A+B$

$$\begin{aligned}&=\left[\left(\max\left(\mu_{j1}(c_i),\chi_{j1}(c_i)\right),\min\left(\mu_{j2}(c_i),\chi_{j2}(c_i)\right)\right)\right]\\&=\left[\left(\max\left(\chi_{j1}(c_i),\mu_{j1}(c_i)\right),\min\left(\chi_{j2}(c_i),\mu_{j2}(c_i)\right)\right)\right]\\&=B+A\end{aligned}$$

(ii) Let $A=\left[\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)\right]$, $B=\left[\left(\chi_{j1}(c_i),\chi_{j2}(c_i)\right)\right]$, $C=\left[\left(\eta_{j1}(c_i),\eta_{j2}(c_i)\right)\right]$

Now, $(A+B)+C$

$$\begin{aligned}&=\left[\left(\max\left(\mu_{j1}(c_i),\chi_{j1}(c_i)\right),\min\left(\mu_{j2}(c_i),\chi_{j2}(c_i)\right)\right)\right]+\left[\left(\eta_{j1}(c_i),\eta_{j2}(c_i)\right)\right]\\&=\left[\left(\max\left(\left(\mu_{j1}(c_i),\chi_{j1}(c_i)\right),\eta_{j1}(c_i)\right),\min\left(\left(\mu_{j2}(c_i),\chi_{j2}(c_i)\right),\eta_{j2}(c_i)\right)\right)\right]\\&=\left[\left(\max\left(\mu_{j1}(c_i),\left(\chi_{j1}(c_i),\eta_{j1}(c_i)\right)\right),\min\left(\mu_{j2}(c_i),\left(\chi_{j2}(c_i),\eta_{j2}(c_i)\right)\right)\right)\right]\\&=A+\left(B+C\right)\end{aligned}$$

(iii) Let $A=\left[\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)\right]$, Also $[0]=\left[\left(\mu_{j}(c_i),\mu_{j}(c_i)\right)\right]$ so that $\delta_{(0)ij}=0\ \forall i,j$

It is clear that $\mu_j(c_i)=\mu_{j2}(c_i)\le\mu_{j1}(c_i)\ \ \forall i,j$.

Now, $A+[0]$

$$\begin{aligned}&=\left[\left(\max\left(\mu_{j1}(c_i),\mu_{j}(c_i)\right),\min\left(\mu_{j2}(c_i),\mu_{j}(c_i)\right)\right)\right]\\&=\left[\left(\max\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right),\min\left(\mu_{j2}(c_i),\mu_{j}(c_i)\right)\right)\right]\\&=\left[\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)\right]\\&=A\end{aligned}$$

Definition 3.16
Let $A=[a_{ij}]_{m\times n}$, $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i, so that $\delta_{ij}(c_i)=\mu_{j1}(c_i)-\mu_{j2}(c_i)$ gives the fuzzy membership value of c_i. Also let $k\in[0,1]$ be a scalar. Then kA is defined as -

$$\begin{aligned}kA&=k[a_{ij}]_{m\times n}\\&=k[\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)]_{m\times n}\\&=\left[\min\left(k,\mu_{j1}(c_i)\right),\min\left(k,\mu_{j2}(c_i)\right)\right]_{m\times n}\end{aligned}$$

Definition 3.17

Let $A = [a_{ij}]_{m\times n}$, $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i, so that $\delta_{ij}(c_i) = \mu_{j1}(c_i) - \mu_{j2}(c_i)$ gives the fuzzy membership value of c_i. Also let $B = [b_{jk}]_{n\times p}$, $b_{jk} = (\chi_{k1}(c_j), \chi_{k2}(c_j))$; where $\chi_{k1}(c_j)$ and $\chi_{k2}(c_j)$ represent the fuzzy membership function and fuzzy reference function respectively of c_j, so that $\delta_{jk}(c_j) = \chi_{k1}(c_j) - \chi_{k2}(c_j)$ gives the fuzzy membership value of c_j. We now define $A.B$, the product of A and B as,

$$
\begin{aligned}
A.B &= [d_{ik}]_{m\times p} \\
&= \left[\max\min(\mu_{j1}(c_i), \chi_{k1}(c_j)), \min\max(\mu_{j2}(c_i), \chi_{k2}(c_j))\right]_{m\times p}, 1 \le i \le m, 1 \le k \le p \text{ for } j = 1,2,\ldots..n
\end{aligned}
$$

Remark

(i) It is clear from the above definition that the fuzzy soft set represented by the matrix A is over a universe having 'm' elements and the corresponding set of parameters has 'n' parameters, whereas the fuzzy soft set represented by the matrix B is defined over a universe having 'n' elements and the corresponding set of parameters has 'p' parameters., *i.e*., the matrices A and B are defined over two different initial universes respectively.

(ii) Product of two fuzzy soft matrices A and B representing fuzzy soft sets over the same initial universe is defined only when the matrices are square matrices. We take one example below –

Example 3.2

Let $A = \begin{bmatrix} (0.3,0) & (0.4,0) & (0.3,0) \\ (0.2,0) & (0.9,0) & (0.2,0) \\ (0.5,0) & (0.1,0) & (1.0,0) \end{bmatrix}$ and $B = \begin{bmatrix} (0.1,0) & (0.4,0) & (0.3,0) \\ (0.2,0) & (0.1,0) & (0.6,0) \\ (0.3,0) & (0.7,0) & (0.0,0) \end{bmatrix}$ be two fuzzy soft square matrices representing two fuzzy soft sets defined over the same initial universe.

Then $A \cdot B = \begin{bmatrix} (0.3,0) & (0.3,0) & (0.4,0) \\ (0.2,0) & (0.2,0) & (0.6,0) \\ (0.3,0) & (0.7,0) & (0.3,0) \end{bmatrix}$ and $B \cdot A = \begin{bmatrix} (0.3,0) & (0.4,0) & (0.3,0) \\ (0.5,0) & (0.2,0) & (0.6,0) \\ (0.3,0) & (0.7,0) & (0.3,0) \end{bmatrix}$

What has been seen from the above example that $A.B \ne B.A$

(iii) If the product $A.B$ is defined then $B.A$ may not be defined. Let us consider the following example –

Example 3.3

Let $A = \begin{bmatrix} (0.75,0) & (0.40,0) & (0.90,0) & (0.75,0) \\ (0.40,0) & (0.50,0) & (0.30,0) & (0.40,0) \\ (0.70,0) & (0.40,0) & (0.60,0) & (0.30,0) \end{bmatrix}$ and $B = \begin{bmatrix} (0.85,0) & (0.75,0) \\ (0.25,0) & (0.50,0) \\ (0.55,0) & (0.45,0) \\ (0.30,0) & (0.45,0) \end{bmatrix}$

We have $A.B = \begin{bmatrix} (0.75,0) & (0.75,0) \\ (0.40,0) & (0.50,0) \\ (0.70,0) & (0.70,0) \end{bmatrix}$, but here $B.A$ is not defined as number of columns in B = 2, whereas number of rows in A = 3.

(iv) Using product of two fuzzy soft matrices, we can find out the positive integral powers of fuzzy soft square matrices. We take one example below –

Example 3.4

Let $A=\begin{bmatrix}(0.3,0) & (0.3,0) & (0.4,0)\\(0.2,0) & (0.2,0) & (0.6,0)\\(0.3,0) & (0.7,0) & (0.3,0)\end{bmatrix}$ be a fuzzy soft square matrix.

$$\text{Then } A\cdot A =\begin{bmatrix}(0.3,0) & (0.3,0) & (0.4,0)\\(0.2,0) & (0.2,0) & (0.6,0)\\(0.3,0) & (0.7,0) & (0.3,0)\end{bmatrix}\cdot\begin{bmatrix}(0.3,0) & (0.3,0) & (0.4,0)\\(0.2,0) & (0.2,0) & (0.6,0)\\(0.3,0) & (0.7,0) & (0.3,0)\end{bmatrix}$$

$$=\begin{bmatrix}(0.3,0) & (0.4,0) & (0.3,0)\\(0.3,0) & (0.6,0) & (0.3,0)\\(0.3,0) & (0.3,0) & (0.6,0)\end{bmatrix}$$

$$\text{We would write } A.A = A^2 =\begin{bmatrix}(0.3,0) & (0.4,0) & (0.3,0)\\(0.3,0) & (0.6,0) & (0.3,0)\\(0.3,0) & (0.3,0) & (0.6,0)\end{bmatrix}$$

Definition 3.18

Let $A=[a_{ij}]_{m\times n}$, $a_{ij}=(\mu_{j1}(c_i),\mu_{j2}(c_i))$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i, so that $\delta_{ij}(c_i)=\mu_{j1}(c_i)-\mu_{j2}(c_i)$ gives the fuzzy membership value of c_i. Then we define $A^T=[a_{ij}{}^T]_{n\times m}\in FSM_{n\times m}$, where $a_{ij}{}^T=a_{ji}$.

Proposition 3.2

Let $A,B\in FSM_{m\times n}$. Then the following results hold.

(i) $\left(A^T\right)^T=A$

(ii) $(A+B)^T=A^T+B^T$

Proof

(i) Let $A=[(\mu_{j1}(c_i),\mu_{j2}(c_i))]$

Now,
$$\begin{aligned}A^T &=[(\mu_{j1}(c_i),\mu_{j2}(c_i))]^T\\ &=[(\mu_{i1}(c_j),\mu_{i2}(c_j))]\\ \left(A^T\right)^T &=[(\mu_{i1}(c_j),\mu_{i2}(c_j))]^T\\ &=[(\mu_{j1}(c_i),\mu_{j2}(c_i))]=A\end{aligned}$$

(ii) Let $A=[(\mu_{j1}(c_i),\mu_{j2}(c_i))]$, $B=[(\chi_{j1}(c_i),\chi_{j2}(c_i))]$

Now,
$$\begin{aligned}A+B &=[(\max(\mu_{j1}(c_i),\chi_{j1}(c_i)),\min(\mu_{j2}(c_i),\chi_{j2}(c_i)))]\\ (A+B)^T &=[(\max(\mu_{j1}(c_i),\chi_{j1}(c_i)),\min(\mu_{j2}(c_i),\chi_{j2}(c_i)))]^T\\ &=[(\max(\mu_{i1}(c_j),\chi_{i1}(c_j)),\min(\mu_{i2}(c_j),\chi_{i2}(c_j)))]\\ &=[(\mu_{i1}(c_j),\mu_{i2}(c_j))]+[(\chi_{i1}(c_j),\chi_{i2}(c_j))]\\ &=[(\mu_{j1}(c_i),\mu_{j2}(c_i))]^T+[(\chi_{j1}(c_i),\chi_{j2}(c_i))]^T\\ &=A^T+B^T\end{aligned}$$

Definition 3.19

Let $A=[a_{ij}]_{m\times n}$, $a_{ij}=(\mu_{j1}(c_i),\mu_{j2}(c_i))$. Then A is said to be a fuzzy soft symmetric matrix if $A^T=A$

Example 3.5

Let $A = \begin{bmatrix} (0.1,0) & (0.4,0) & (0.2,0) \\ (0.4,0) & (0.5,0) & (0.6,0) \\ (0.2,0) & (0.6,0) & (0.3,0) \end{bmatrix}$ be a fuzzy soft square matrix. We see that

$$A^T = \begin{bmatrix} (0.1,0) & (0.4,0) & (0.2,0) \\ (0.4,0) & (0.5,0) & (0.6,0) \\ (0.2,0) & (0.6,0) & (0.3,0) \end{bmatrix} = A$$

By definition, A is a fuzzy soft symmetric matrix.

Definition 3.20

Let $A = [a_{ij}]_{m\times m}$, $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$. Then A is said to be a fuzzy soft idempotent matrix if $A^2 = A$

Example 3.6

Let $A = \begin{bmatrix} (0.3,0) & (0.4,0) & (0.2,0) \\ (0.2,0) & (0.5,0) & (0.2,0) \\ (0.3,0) & (0.5,0) & (0.3,0) \end{bmatrix}$ be a fuzzy soft square matrix.

Then $A \cdot A = A^2 = \begin{bmatrix} (0.3,0) & (0.4,0) & (0.2,0) \\ (0.2,0) & (0.5,0) & (0.2,0) \\ (0.3,0) & (0.5,0) & (0.3,0) \end{bmatrix} \begin{bmatrix} (0.3,0) & (0.4,0) & (0.2,0) \\ (0.2,0) & (0.5,0) & (0.2,0) \\ (0.3,0) & (0.5,0) & (0.3,0) \end{bmatrix}$

$$= \begin{bmatrix} (0.3,0) & (0.4,0) & (0.2,0) \\ (0.2,0) & (0.5,0) & (0.2,0) \\ (0.3,0) & (0.5,0) & (0.3,0) \end{bmatrix}$$

Thus $A . A = A^2 = \begin{bmatrix} (0.3,0) & (0.4,0) & (0.2,0) \\ (0.2,0) & (0.5,0) & (0.2,0) \\ (0.3,0) & (0.5,0) & (0.3,0) \end{bmatrix} = A$

It follows that A is a fuzzy soft idempotent matrix.

4. CONCLUSION

In this work, we have put forward the notions related to fuzzy soft matrices. Our work is in fact an attempt to extend our earlier notion of fuzzy soft matrices. Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result.

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